Example: Sample space

What is the sample space when a coin is tossed until a tail or three heads appear?

$S = \{T, HT, HHT, HHH\}$

Example: Describe the sample space consisting of all points inside a circle of radius 5 centered at the origin?

$S = \{(x,y) \mid x^2 + y^2 \leq 5\}$

Example - Probability of an event.

* What is the probability of getting a total of 8 when a pair of fair dice are tossed?

$$S = \{(1,1),(1,2),(1,3),(1,4),(1,5),(1,6),$$
$$(2,1),(2,2),\ldots\ (6,6)\}$$

How many elements in sample space? 36

Lets call A the event that we get a total 8

$$A = \{(2,6),(3,5),(4,4),(5,3),$$
$$(6,2)\}$$

$P(A) = 5/36$ since dice is fair

* What is the probability of a total 1? 0

* What is the probability of getting a total $\neq 8$?

$$P(A') = 1 - P(A) = 31/36$$

* What is the probability of getting a total of 11?

$B = \{(5,6), (6,5)\}$

A and B are mutually exclusive. So the probability of getting either a 8 or 11 for the total is $P(A \cup B) = \frac{5}{36} + \frac{2}{36} = 7/36$

* What is the probability of getting a 3 on the first dice?

$C = \{(3,1), (3,2), (3,3), (3,4), (3,5), (3,6)\}$

$P(C) = 6/36 = 1/6$

* What is the probability of getting either a total of 8 or ~~3 or 2~~ first dice = 3?

$P(A \cup C) = P(A) + P(C) - P(A \cap C)$

$A \cap C = \{(3,5)\}$ so $P(A \cap C) = 1/36$

$P(A \cup C) = 5/36 + 6/36 - 1/36 = 10/36$

We can verify this result by listing the elements of $A \cup C$

$$A \cup C = \{(2,6), (3,5), (4,4), (5,3), (6,2), (3,1), (3,2), (3,3), (3,4), (3,6)\}$$

$$P(A \cup C) = 10/36$$

Example : Venn diagrams and probability.

Three companies A, B and C provide cell-phone coverage in a rural area. For a randomly chosen location in this area, the probability of coverage for the first two companies are

$P(A) = 0.8 \quad P(B) = 0.75$

We also know $P(A \cup B) = 0.9$ and $P(B \cap C) = 0.45$.

* What is the probability of not having coverage from company A?

$$P(A') = 1 - P(A) = 0.2$$

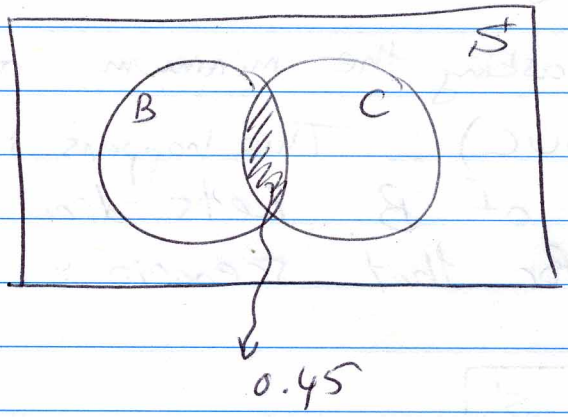
* What is the probability of having coverage both from company A and B?

$$P(A \cap B) = P(A) + P(B) - P(A \cup B)$$
$$= 0.8 + 0.75 - 0.9 = 0.65$$

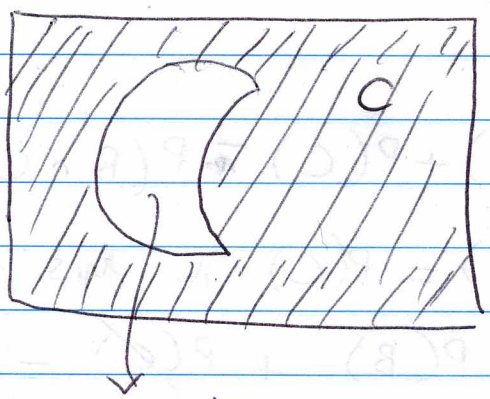
* Company A claims it has better coverage than company C. Can you verify this?

Lets find the maximum possible value for $P(C)$.

The only information we have relating to C is $P(B \cap C) = 0.45$. Lets draw a Venn diagram



C would be largest if it includes all the area outside B in addition to their intersection



What is the probability of this area?

$$P(B) - P(B \cap C) = 0.75 - 0.45 = 0.3$$

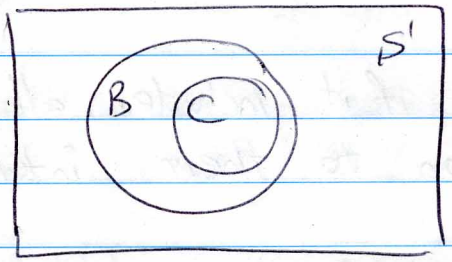
Then since C is the complement of this

$$\max P(C) = 1 - 0.3 = 0.7$$

Company A's claim is true.

* If you own two cell phones; one from company B and one from company C. What is your worst case coverage?

This question is asking the minimum possible value for $P(B \cup C)$. This happens if C is a subset of B. Lets draw the Venn diagram for that scenario:



Intuitively we know $P(B \cup C) = P(B)$

Also

$$P(B \cup C) = P(B) + P(C) - P(B \cap C)$$

what is $P(B \cap C) = P(C)$ in this case

so $P(B \cup C) = P(B) + P(C) - P(C)$

$$\min P(B \cup C) = P(B) = 0.75$$