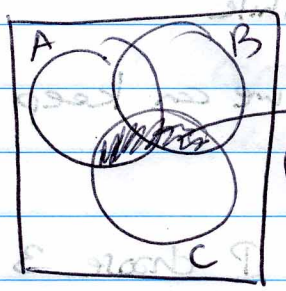
Conditional probability examples

Let A, B and C be 3 events. Then:

- $P(A \cap B | C) = \frac{P(A \cap B \cap C)}{P(C)}$
- $P(A | B \cap C) = \frac{P(A \cap B \cap C)}{P(B \cap C)}$
- $P(A \cup B | C) = \frac{P((A \cup B) \cap C)}{P(C)}$ (*)



$(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$P((A \cap C) \cup (B \cap C)) = P(A \cap C) + P(B \cap C) - P(A \cap B \cap C)$

Substitute into (*)

$$P(A \cup B | C) = \frac{P(A \cap C)}{P(C)} + \frac{P(B \cap C)}{P(C)} - \frac{P(A \cap B \cap C)}{P(C)}$$

$$= P(A|C) + P(B|C) - P(A \cap B | C)$$