Example

$A \cap B$ think of an "AND" gate

Let $A = \{1, 2, 5, 6\}$, $B = \{2, 4, 6\}$

$S = \{1, 2, 3, 4, 5, 6\}$

So the input A to the AND gate is ON if the outcome is in the list of outcomes in the definition of A. Similarly for B.

outcome	A	B	$A \cap B$
2, 6	ON	ON	ON
1, 5	ON	OFF	OFF
4	OFF	ON	OFF
3	OFF	OFF	OFF

→ This is the line that constitutes the list of outcomes in $A \cap B$, so $A \cap B = \{2, 6\}$

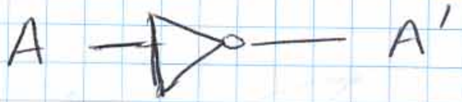
$A \cup B$ think of an "OR" gate

outcome	A	B	$A \cup B$
2, 6	ON	ON	ON
1, 5	ON	OFF	ON
4	OFF	ON	ON
3	OFF	OFF	OFF

→ These lines all constitute the list of outcomes in $A \cup B$, so

$A \cup B = \{2, 6, 1, 5, 4\}$

A' think of a NOT gate



Note: $(A')' = A$

outcome	A	A'
1, 2, 5, 6	ON	OFF
3, 4	OFF	ON

→ this is the line that lists the outcomes in A'

$A' = \{3, 4\}$

Notice that $A \cap B = B \cap A$ and $A \cup B = B \cup A$

Define a third event $C = \{3, 6\}$

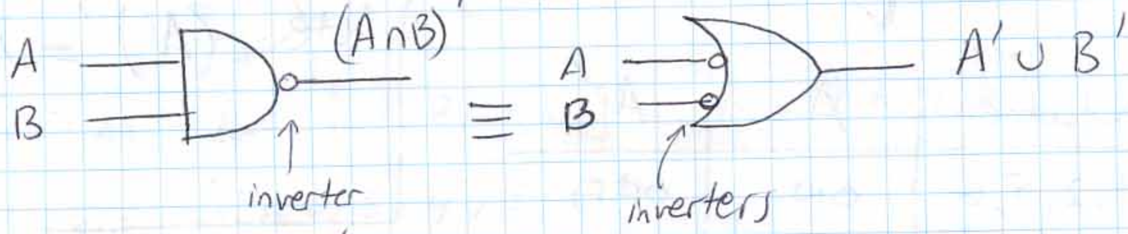
$A \cap B \cap C = (A \cap B) \cap C = A \cap (B \cap C)$
$= (A \cap C) \cap B$

$A \cup B \cup C = (A \cup B) \cup C = A \cup (B \cup C)$
$= (A \cup C) \cup B$

outcomes	A	B	C	$A \cap B \cap C$	$A \cup B \cup C$
6	ON	ON	ON	ON	ON
2	ON	ON	OFF	OFF	ON
none	ON	OFF	ON	OFF	ON
1, 5	ON	OFF	OFF	OFF	ON
none	OFF	ON	ON	OFF	ON
4	OFF	ON	OFF	OFF	ON
3	OFF	OFF	ON	OFF	ON
none	OFF	OFF	OFF	OFF	OFF
				$\{6\}$	$\{6, 2, 1, 5, 4, 3\} = S$

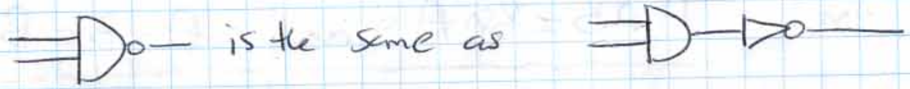
can get these from Venn diagram

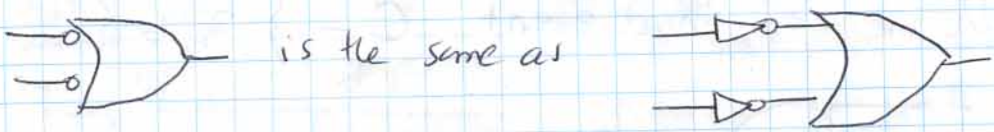
De Morgan's laws:

① 

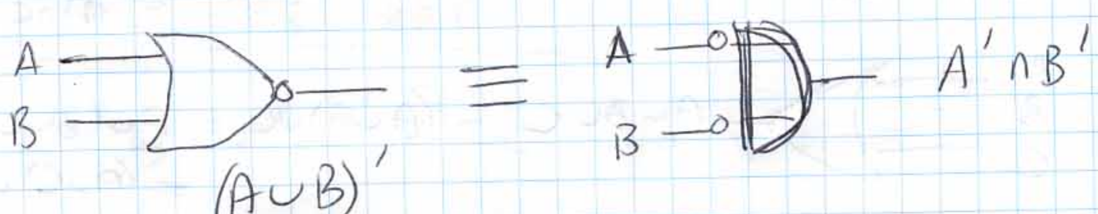
AND NOT gate

$$(A \cap B)' \equiv A' \cup B' \quad \left(\begin{array}{c}\text{Verify from}\\ \text{Venn diagram}\end{array}\right)$$





---

② 

$$(A \cup B)' = A' \cap B'$$

Distributivity : $(A \cup B) \cap C = (A \cap C) \cup (B \cap C)$

$(A \cap B) \cup C = (A \cup C) \cap (B \cup C)$

Other properties: $A \cup (A \cap B) = A$

$A \cap (A \cup B) = A$

$A \cap A' = \phi, \quad A \cup A' = S', \quad A \cap \phi = \phi, \quad A \cup \phi = A$