PID Control

Objectives

The objective of this lab is to study basic design issues for proportional-integral-derivative control laws. Emphasis is placed on transient responses and steady-state errors. The first control problem consists in the regulation of velocity for brush DC motors and is solved using proportional-integral control. The second problem consists in the regulation of position and requires derivative compensation in the form of velocity feedback.

Introduction

In the lab on first-order systems, the response of a brush DC motor with the voltage v (V or volts) considered as an input and the angular velocity ω (rad/s or s^{-1}) considered as an output was found to be approximately described by a model

$$P(s) = \frac{\omega(s)}{v(s)} = \frac{k}{(s+a)}.$$
(8.16)

A proportional control law (P) consists in having

$$v = k_P(r - \omega), \tag{8.17}$$

where r is the reference input for the velocity, in rad/s. k_P is called the *proportional gain*. The resulting closed-loop transfer function is given by

$$P_{CL}(s) = \frac{\omega(s)}{r(s)} = \frac{kk_P}{s+a+kk_P}.$$
(8.18)

Note that the closed-loop pole is given by $-a - kk_P$. In theory, it would appear that the closedloop pole could be moved arbitrarily far in the left-half plane through the use of a sufficiently large proportional gain. The response of the system could be made arbitrarily fast in that manner. As this lab will show, there are limits on the gains that can be applied, however. These limits are due to effects that are neglected in the model (such as the inductance of the motor and the limit on the voltage), but are nevertheless present in the physical system.

Proportional-integral control for velocity tracking

The DC gain in (8.18) is equal to $kk_P/(a + kk_P)$. For large k_P , this gain approaches 1, but large gains are impractical. Therefore, it is useful to modify the control law in order to adjust the DC gain. Specifically, replacing (8.17) by

$$v = k_P (k_F r - \omega), \tag{8.19}$$

yields a closed-loop transfer function

$$P_{CL}(s) = \frac{\omega(s)}{r(s)} = \frac{k_F k k_P}{s + a + k k_P}.$$
(8.20)

The closed-loop pole is equal to the original one, but the DC gain can now be adjusted to 1 by setting

$$k_F = \frac{a + kk_P}{kk_P}.$$
(8.21)

We will call k_F the feedforward gain.

Despite the capability of adjusting the feedforward gain k_F in order to obtain a DC gain of 1, perfect tracking of reference inputs is usually not achieved because the parameters of the system are not exactly known or may vary, and because disturbances may affect the response of the system. These problems can be resolved through the use of a *proportional-integral* (PI) control law of the form

$$v = k_P(r-\omega) + k_I \int (r-\omega)dt, \qquad (8.22)$$

where k_P and k_I are called the *proportional gain* and the *integral gain*, respectively. Then, the closed-loop transfer function becomes

$$P_{CL}(s) = \frac{\omega(s)}{r(s)} = \frac{kk_P(s + \frac{k_I}{k_P})}{s^2 + (a + kk_P)s + kk_I}.$$
(8.23)

The DC gain is equal to 1, regardless of what the parameters of the system or of the control law are. Of course, it should be remembered that the DC gain reflects the steady-state conditions only if the closed-loop system is stable, *i.e.*, if the poles of (8.23) are all in the open left-half plane. Generally, the responses cannot be made as fast for a PI control law, so that the benefit of a zero steady-state error has to be weighted against that of the speed of response.

Proportional-integral-derivative control for position tracking

To control *position*, instead of velocity, it is common to use of *proportional-integral-derivative* (PID) control law

$$v = k_P(r-\theta) + k_I \int (r-\theta)dt + k_D \frac{d}{dt}(r-\theta).$$
(8.24)

Note that the derivative term can be viewed as a proportional feedback acting on the velocity error. In general, derivative feedback improves the stability and the damping of the closed-loop system.

In practice, the control law (8.24) is often modified in two ways. First, the derivative action is applied only to the output θ , and not to the reference input. This is done because reference inputs often change in steps, and the derivative is then either zero or not defined (infinite). Second, a feedforward gain is often applied to the reference input. This is not done to adjust the DC gain (as for the control law without integral term), but rather to place the zero of the closed-loop transfer function. This will be explained shortly. The modified control law is given by

$$v = k_P(k_F r - \theta) + k_I \int (r - \theta) dt - k_D \frac{d}{dt} \theta.$$
(8.25)

The closed-loop transfer function for the system with transfer function

$$P(s) = \frac{\theta(s)}{v(s)} = \frac{k}{s(s+a)},\tag{8.26}$$

and the PID control law (8.25), is given by

$$P_{CL}(s) = \frac{\theta(s)}{r(s)} = \frac{kk_F k_P (s + \frac{\kappa_I}{k_F k_P})}{s^3 + (a + kk_D)s^2 + kk_P s + kk_I}.$$
(8.27)

Note that the closed-loop transfer function (8.27) has three poles. There is also a zero at $-k_I/(k_F k_P)$. For the original control law with $k_F = 1$, the zero may have a small magnitude compared to the closed-loop poles, yielding overshoot in the step response even if the closed-loop poles were well-damped. Reducing the value of k_F allows one to push the zero farther in the left-half plane and to improve the step response.

Pre-lab

Derive equation (8.20) and calculate values of k_P and k_F such that the closed-loop pole is at an arbitrary location -b and such that the DC gain is 1. Specialize the results to the cases b = 2a, b = 6a, and b = 11a. Calculate the specific values of the gains for the DC motor (a = 100, k = 1000) for all three cases.

Derive equation (8.23) and calculate values of k_P and k_I such that the closed-loop poles are both at an arbitrary location -b. Specialize the results to the case where b = a, and to the specific values of the DC motor.

Derive the transfer function given in (8.27) and calculate the values of the PID parameters such that all three poles are placed at some -b. Calculate the parameters that correspond to b = a, and also for the specific motor parameters (a = 100, k = 1000).

Experiments

Equipment needed

You will need a brush DC motor, a dual power amplifier, and a cable rack.

Preliminary testing

Carry out the usual testing procedure. Transfer the file LAB4.C to your account, and create an executable program for the file.

Proportional control

The program LAB4 lets you enter values for the proportional, integral, and feedforward gains. Then, you enter a value for the reference input, and may change it in real-time. The reference input is a reference speed, given in rpm. First, experiment with proportional control by letting $k_I = 0$. Set k_P and k_F according to the pre-lab calculations, and apply a reference input that steps from 0 to 1000 rpm and then to 2000 rpm and then back to zero. Repeat the experiment for all three cases and plot the results. Discuss what happens when the gain k_P becomes large.

Proportional-integral control

Apply the values calculated for the PI control law, setting $k_F = 1$ in the program. You may also experiment with other values of k_P and k_I , in particular those resulting in faster responses. Plot the results for your best experiment.

Proportional-integral-derivative (PID) control

Adjust the program LAB4.C in order to:

- Read the derivative parameter k_D from the keyboard (change a *printf* statement and add a *scanf* statement in *user_init*).
- Read the reference position in degrees (change a *printf* statement in *user_init* and compute r in rad in *user_task*).
- Implement the PID control law for position (change the computation of *u* in *user_task*).
- Declare the new variables at the beginning of the program.

Once this is done, you may apply the calculated values of the PID parameters, with $k_F = 1$, and a step of reference input of 90°. The settling time should be approximately 100ms, with an overshoot of the response. Adjusting the parameter k_F should yield a better response. Plot the results for a few values of k_F on a single graph. Indicate what value of k_F gives the best response (minimum settling time with negligible overshoot).

DEMONSTRATE YOUR FINAL EXPERIMENT TO THE TA

Report at a glance

Be sure to include:

- Pre-lab calculations.
- Plots of the responses with the proportional control law, for the three cases.
- Plot of the response with the proportional-integral control law, with the values of the gains that were used.
- Plot of responses with PID control law, and a few values of k_F .
- Written note from the TA that the program worked.
- Comments.