ECE 3510 Root Locus Design Crib Sheet

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Using 2nd-order approximation: $\frac{N(s)}{(s+a)^2+b^2} = \frac{N(s)}{s^2+2\cdot a\cdot s+a^2+b^2} = \frac{N(s)}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^2}$

$\omega_n^2 = a^2+b^2$ $\qquad \omega_n$ = natural frequency

$\zeta\cdot\omega_n = a$

$\zeta = \frac{a}{\omega_n} = \frac{a}{\sqrt{a^2+b^2}}$ = damping factor $\qquad \zeta = \sin\left(\operatorname{atan}\left(\frac{a}{b}\right)\right)$

Overshoot: $\quad OS = e^{-\pi\cdot\frac{a}{b}} \qquad \%OS = 100\%\cdot e^{-\pi\cdot\frac{a}{b}} \qquad \frac{a}{b} = \frac{\ln(OS)}{-\pi}$

angle of constant damping line: $90\cdot deg + \operatorname{atan}\left(\frac{a}{b}\right)$

2% settling time: $\quad T_s = \frac{4}{a} = \frac{4}{\zeta\cdot\omega_n}$

Time of first peak: $\quad T_p = \frac{\pi}{b}$

Static error constant (position): $\quad K_p = \lim_{s\to 0} K\cdot C(s)\cdot G(s) \qquad e_{step}(\infty) = e_{step} = \frac{1}{1+K_p}$ $\qquad$ Nise p378

Lag compensation improves K_p, K_v and K_a by $\frac{z_c}{p_c}$ $\qquad$ IE: $K_{pc} \simeq K_{puc}\cdot\frac{z_c}{p_c}$

Searching along a line of constant damping:

Try s values, choosing b: $\quad s = -\left(\frac{a}{b}\cdot b\right) + b\cdot j$ $\qquad$ Test: $\arg(G(s))\ \pm 180^o$ $\quad$ or $\quad -\operatorname{Re}(G(s)) >> \operatorname{Im}(G(s))$

Linear interpolation: $\quad$ new b = $b_1 - \frac{b_2 - b_1}{\operatorname{Im}(G(s_2)) - \operatorname{Im}(G(s_1))}\cdot\operatorname{Im}(G(s_1))$

Can also try "a" values with slight modification of the above.

Weird forms from Nise book:

$\sigma_d = a \qquad \%OS = 100\%\cdot e^{\frac{-\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}$ $\quad$ p195

$\omega_d = b \qquad \zeta = \frac{-\ln(OS)}{\sqrt{\pi^2+(\ln(OS))^2}}$ $\quad$ p195 $\qquad T_p = \frac{\pi}{\omega_n\cdot\sqrt{1-\zeta^2}}$ $\quad$ p194

p378 $\quad$ Static error constant (ramp): (velocity) $\quad K_v = \lim_{s\to 0} s\cdot K\cdot C(s)\cdot G(s) \qquad e_{ramp} = \frac{1}{K_v}$

Static error constant (parabola): (acceleration) $\quad K_a = \lim_{s\to 0} s^2\cdot K\cdot C(s)\cdot G(s) \qquad e_{parabola} = \frac{1}{K_a}$