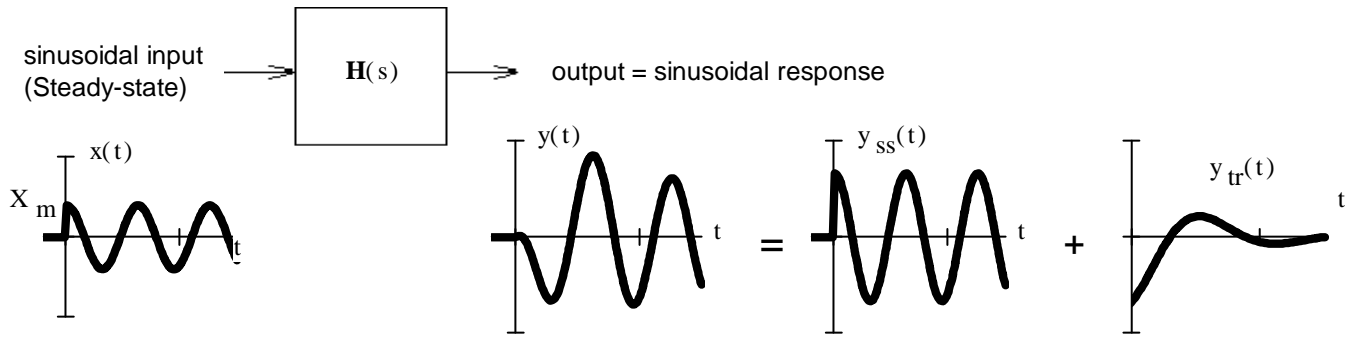


The sinusoidal response of a system is the output when the input is a sinusoidal (which starts at time = 0).

System Sinusoidal Response



Complete step response = steady-state response + transient response

$$Y(s) = X(s) \cdot H(s) = X_m \cdot H(j\omega) \cdot u(t) + Y_{tr}(s)$$

$H(j\omega)$ = phasor-type transfer function

Sinusoidal Input

$$\cos(\omega \cdot t) \cdot u(t) \iff \frac{s}{s^2 + \omega^2}$$

$$\sin(\omega \cdot t) \cdot u(t) \iff \frac{\omega}{s^2 + \omega^2}$$

General sinusoidal input: $[X_{mc} \cdot \cos(\omega \cdot t) + X_{ms} \cdot (\sin(\omega \cdot t) \cdot u(t))] \cdot u(t)$

$$X(s) = \frac{X_{mc} \cdot s + X_{ms} \cdot \omega}{s^2 + \omega^2}$$

OR $X_m \cdot \cos(\omega \cdot t + \theta) \cdot u(t) = [X_{mc} \cdot \cos(\omega \cdot t) + X_{ms} \cdot (\sin(\omega \cdot t) \cdot u(t))] \cdot u(t)$

$$X_{mc} = X_m \cdot \cos(\theta) \quad X_{ms} = -X_m \cdot \sin(\theta) \quad \text{note that the sine carries the opposite sign as you might expect.}$$

Steady-State Response & $H(j\omega)$

$$Y(s) = X(s) \cdot H(s) = \frac{X_{mc} \cdot s + X_{ms} \cdot \omega}{s^2 + \omega^2} \cdot H(s) = \text{Complete sinusoidal response}$$

$$= \frac{A \cdot s + B \cdot \omega}{s^2 + \omega^2} + \frac{C}{(s)} + \frac{D}{(s)} + \frac{E}{(s)} + \dots$$

partial fraction expansion: $Y(s) = \frac{X_{mc} \cdot s + X_{ms} \cdot \omega}{s^2 + \omega^2} \cdot H(s) = \frac{A \cdot s + B \cdot \omega}{s^2 + \omega^2} + \left[\frac{C}{(s)} + \frac{D}{(s)} + \frac{E}{(s)} \right] \cdot s$

steady-state response + transient response

$$Y_{ss}(s) + Y_{tr}(s)$$

multiply both sides by: $(s^2 + \omega^2)$

$$(X_{mc} \cdot s + X_{ms} \cdot \omega) \cdot H(s) = A \cdot s + B \cdot \omega + \left[\frac{C}{(s)} + \frac{D}{(s)} + \frac{E}{(s)} \right] \cdot (s^2 + \omega^2)$$

set $s := j\omega$

$$(X_{mc} \cdot j\omega + X_{ms} \cdot \omega) \cdot H(j\omega) = A \cdot j\omega + B \cdot \omega + \left[\frac{C}{(s)} + \frac{D}{(s)} + \frac{E}{(s)} \right] \cdot 0$$

divide both sides by $j\omega$

$$(X_{mc} - X_{ms} \cdot j) \cdot H(j\omega)$$

$$X(\omega) \cdot H(j\omega) = A - B \cdot j = Y_{ss}(\omega) = \text{steady-state response in phasor form}$$

(real is cosine, imaginary is -sine)

$X(\omega)$ = the input expressed in phasor form NOT $X(s)$ with $s := \omega$ or $s := j\omega$ (that would be ∞)

$H(j\omega)$ = the steady-state sinusoidal transfer function

= phasor-type transfer function

The **transient part** would be found by finishing the partial-fraction expansion. Sinusoidal Response Notes p1

Steady-State Response by Phasors

Expression of signals as phasors

T = Period

f = frequency, cycles / second $f = \frac{1}{T} = \frac{\omega}{2 \cdot \pi}$

ω = radian frequency, radians/sec $\omega = 2 \cdot \pi \cdot f$

A = amplitude

Phase: $\phi = -\frac{\Delta t}{T} \cdot 360\text{-deg}$ or: $\phi = -\frac{\Delta t}{T} \cdot 2 \cdot \pi \cdot \text{rad}$

$y(t) = A \cdot \cos(\omega \cdot t + \phi)$

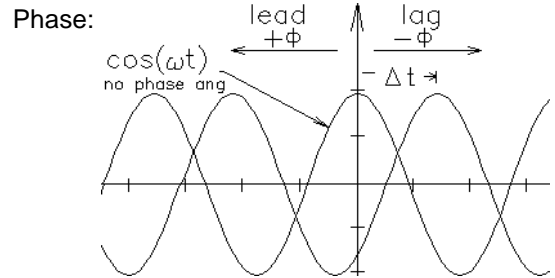
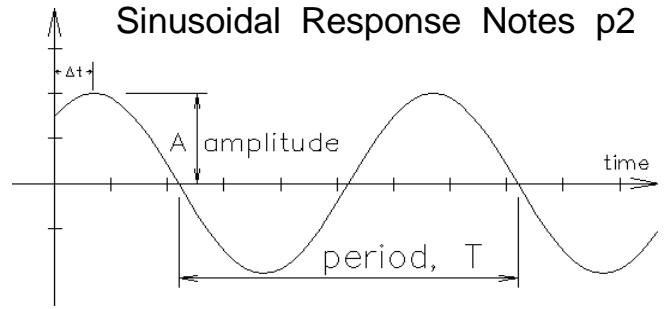
voltage: $v(t) = V_p \cdot \cos(\omega \cdot t + \phi)$

current: $i(t) = I_p \cdot \cos(\omega \cdot t + \phi)$

Phasor

$V(\omega) = V_p \cdot e^{j \cdot \phi}$

$I(\omega) = I_p \cdot e^{j \cdot \phi}$



Ex1 Let's assume the input to your system is $v_1(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 15\text{-deg})$ $V_1(\omega) = 3.2 \cdot V \cdot e^{j \cdot 15\text{-deg}}$

or: $V_1(\omega) = 3.2V \angle 15^\circ$

or: $V_1(\omega) = (3.091 + 0.828j) \cdot V$

In rectangular form:

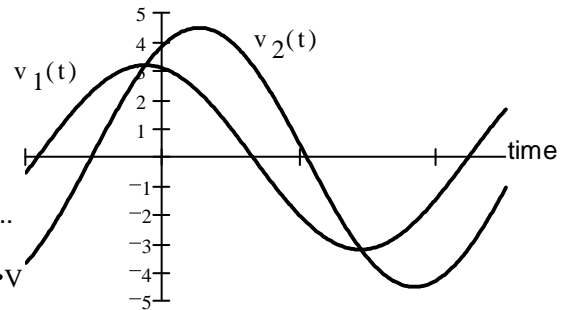
$3.2 \cdot V \cdot \cos(15\text{-deg}) = 3.091 \cdot V$

$3.2 \cdot V \cdot \sin(15\text{-deg}) = 0.828 \cdot V$

Ex2 What if a signal is the sum of two sinusoids.

$v_1(t) = 3.2 \cdot V \cdot \cos(\omega \cdot t + 15\text{-deg})$

$v_2(t) = 4.5 \cdot V \cdot \sin(\omega \cdot t + 60\text{-deg})$ $v_3(t) = v_1(t) + v_2(t)$



I'm going to drop the (ω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

From Ex1: $V_1 := 3.2 \cdot V \cdot e^{j \cdot 15\text{-deg}} = 3.2V \angle 15^\circ$ $V_1 = 3.091 + 0.828j \cdot V$

Phasors are based on cosines, so express $v_2(t)$ as a cosine. Remember: $\sin(\omega t) = \cos(\omega \cdot t - 90\text{-deg})$

So: $v_2(t) = 4.5 \cdot V \cdot \cos(\omega \cdot t + 60\text{-deg} - 90\text{-deg}) = 4.5 \cdot V \cdot \cos(\omega \cdot t - 30\text{-deg})$

$V_2 = 4.5V \angle -30^\circ$ or: $V_2 := 4.5 \cdot V \cdot e^{-j \cdot 30\text{-deg}}$

$4.5 \cdot V \cdot \cos(-30\text{-deg}) = 3.897 \cdot V$

$4.5 \cdot V \cdot \sin(-30\text{-deg}) = -2.25 \cdot V$

$V_2 = 3.897 - 2.25j \cdot V$ \

$V_1 = 3.091 + 0.828j \cdot V$ / } add

$V_3 := V_1 + V_2$

$V_3 = 6.988 - 1.422j \cdot V$ sum

Add real parts: $3.897 + 3.091 = 6.988$

Add imaginary parts: $-2.25 + 0.828 = -1.422$

Change V_3 back to polar coordinates:

$\sqrt{6.988^2 + 1.422^2} = 7.131$

$\text{atan}\left(\frac{-1.422}{6.988}\right) = -11.502 \cdot \text{deg}$

OR, in Mathcad notation (you'll see these in future solutions):

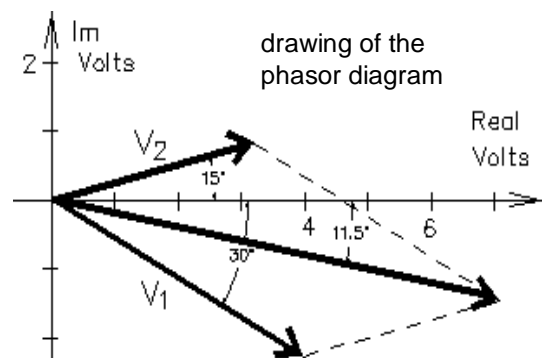
$|V_3| = 7.131 \cdot V$

$\text{arg}(V_3) = -11.5 \cdot \text{deg}$

$V_3(\omega) = 7.131V \angle -11.5^\circ$ or: $V_3(\omega) = 7.131 \cdot V \cdot e^{-j \cdot 11.5\text{-deg}}$

V_3 may also be converted back to the time domain:

$v_3(t) = v_1(t) + v_2(t) = 7.13 \cdot \cos(\omega \cdot t - 11.5\text{-deg}) \cdot V$



Magnitude and Phase of transfer functions With steady-state sinusoidal inputs which start at $t = 0$

Ex3 a) Find the magnitude and phase of the following transfer function at this frequency: $\omega := 2 \cdot \frac{\text{rad}}{\text{sec}}$

$$\mathbf{H}(s) = \frac{2 \cdot s^2 + 5 \cdot s + 20}{s^2 + 1 \cdot s + 10} = \frac{2 \cdot s^2 + \frac{5}{\text{sec}} \cdot s + \frac{20}{\text{sec}^2}}{s^2 + \frac{1}{\text{sec}} \cdot s + \frac{10}{\text{sec}^2}} \quad \text{Expressed with proper units}$$

$$s := j \cdot \omega$$

$$\mathbf{H}(j \cdot \omega) = \frac{2 \cdot (j \cdot \omega)^2 + 5 \cdot (j \cdot \omega) + 20}{(j \cdot \omega)^2 + \frac{1}{\text{sec}} \cdot (j \cdot \omega) + \frac{10}{\text{sec}^2}} = \frac{(20 - 2 \cdot \omega^2) + (5 \cdot \omega) \cdot j}{(10 - \omega^2) + (1 \cdot \omega) \cdot j} = \frac{(20 - 2 \cdot 2^2) + (5 \cdot 2) \cdot j}{(10 - 2^2) + (1 \cdot 2) \cdot j} = \frac{12 + 10 \cdot j}{6 + 2 \cdot j}$$

without units

$$|\mathbf{H}(j \cdot \omega)| = M = \frac{\sqrt{12^2 + 10^2}}{\sqrt{6^2 + 2^2}} = 2.47 \quad \angle \mathbf{H}(j \cdot \omega) = \text{atan}\left(\frac{10}{12}\right) - \text{atan}\left(\frac{2}{6}\right) = 21.37 \cdot \text{deg} \quad \mathbf{H}(j \cdot \omega) = 2.47 \angle 21.37^\circ$$

b) Find the steady-state sinusoidal output if the input is: $3.2 \cdot V \cdot \cos(2 \cdot t + 15 \cdot \text{deg})$ $\mathbf{V}_{\text{in}} := 3.2 \cdot V \cdot e^{j \cdot 15 \cdot \text{deg}} = 3.2V \angle 15^\circ$

$$\mathbf{V}_{\text{outss}}(\omega) = \mathbf{V}_{\text{in}}(\omega) \cdot \mathbf{H}(j \cdot \omega) = (3.2 \cdot V \cdot e^{j \cdot 15 \cdot \text{deg}}) \cdot (2.47 \cdot e^{j \cdot 21.37 \cdot \text{deg}}) = 3.2 \cdot V \cdot 2.47 \cdot e^{j \cdot (15 \cdot \text{deg} + 21.37 \cdot \text{deg})}$$

$$= 7.904 \cdot V \cdot e^{j \cdot 36.37 \cdot \text{deg}} = 6.364 + 4.687j \cdot V$$

$$v_{\text{outss}}(t) = 7.904 \cdot V \cdot \cos(2 \cdot t + 36.37 \cdot \text{deg}) \cdot u(t) = (6.364 \cdot \cos(2 \cdot t) - 4.687 \cdot \sin(2 \cdot t)) \cdot u(t)$$

note that the sine carries the opposite sign as the imaginary part of phasor.

Ex4 a) Find the magnitude and phase of the following transfer function at this frequency: $f := 5 \cdot \text{Hz}$ $\omega := 2 \cdot \pi \cdot f$

$$\mathbf{H}(s) = \frac{s^2 + \frac{20}{\text{sec}} \cdot s + \frac{1000}{\text{sec}^2}}{s^2 + \frac{10}{\text{sec}} \cdot s + \frac{800}{\text{sec}^2}} \quad \omega = 31.42 \cdot \frac{\text{rad}}{\text{sec}}$$

$$s := j \cdot \omega = j \cdot 31.42 \cdot \frac{\text{rad}}{\text{sec}} \quad \mathbf{H}(j \cdot \omega) = \frac{(j \cdot \omega)^2 + \frac{20}{\text{sec}} \cdot (j \cdot \omega) + \frac{1000}{\text{sec}^2}}{(j \cdot \omega)^2 + \frac{10}{\text{sec}} \cdot (j \cdot \omega) + \frac{800}{\text{sec}^2}} = \frac{(j \cdot 31.42)^2 + 20 \cdot (j \cdot 31.42) + 1000}{(j \cdot 31.42)^2 + 10 \cdot (j \cdot 31.42) + 800} \quad \text{without units}$$

$$= \frac{13.04 + 628.319 \cdot j}{-186.96 + 314.159 \cdot j} = 1.459 - 0.91j$$

$$|\mathbf{H}(j \cdot \omega)| = M = \sqrt{1.459^2 + 0.91^2} = 1.72 \quad \angle \mathbf{H}(j \cdot \omega) = \text{atan}\left(\frac{-0.91}{1.459}\right) = -31.95 \cdot \text{deg} \quad \mathbf{H}(j \cdot \omega) = 1.72 \angle -31.95^\circ$$

b) Find the steady-state sinusoidal output if the input is: $x(t) = 4 \cdot \cos(2 \cdot \pi \cdot 5 \cdot \text{Hz} \cdot t)$

$$\mathbf{X}(\omega) = 4 + 0j \quad \text{and then} \quad \mathbf{Y}(\omega) = 4 \cdot (1.459 - 0.91 \cdot j) = 5.836 - 3.64j$$

Note that you can use the rectangular form of $\mathbf{H}(j \cdot \omega)$

$$y(t) = \left(5.836 \cdot \cos\left(31.42 \cdot \frac{\text{rad}}{\text{sec}} \cdot t\right) + 3.64 \cdot \sin\left(31.42 \cdot \frac{\text{rad}}{\text{sec}} \cdot t\right) \right) \cdot u(t) \quad \text{note that the sine carries the opposite sign as the imaginary part of phasor.}$$

$$\sqrt{5.836^2 + 3.64^2} = 6.878 \quad \text{atan}\left(\frac{-3.64}{5.836}\right) = -31.95 \cdot \text{deg} \quad y(t) = 6.88 \cdot \cos\left(31.42 \cdot \frac{\text{rad}}{\text{sec}} \cdot t - 32 \cdot \text{deg}\right) \cdot u(t)$$

Impedances

Resistor

$$v_R = i_R \cdot R$$

$$V_R(s) = R \cdot I(s)$$

$$Z_R = R$$

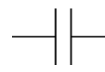
Inductor

$$v_L(t) = L \frac{d}{dt} i_L(t)$$

$$V_L(s) = s \cdot L \cdot I_L(s)$$

$$Z_L = L \cdot s$$

Capacitor



$$i_C(t) = C \cdot \frac{d}{dt} v_C(t)$$

$$I_C(s) = C \cdot s \cdot V_C(s)$$

$$V_C(\omega) = \frac{1}{C \cdot s} \cdot I(\omega)$$

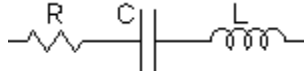
$$Z_C = \frac{1}{C \cdot s}$$

series:



$$Z_{eq} = Z_1 + Z_2 + Z_3 + \dots$$

Example:

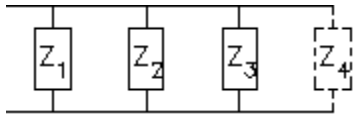


$$Z_{eq} = R + \frac{1}{C \cdot s} + L \cdot s$$

Voltage divider:

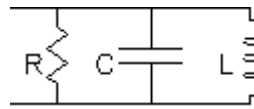
$$V_{Zn} = V_{total} \cdot \frac{Z_n}{Z_1 + Z_2 + Z_3 + \dots}$$

parallel:



$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots}$$

Example:

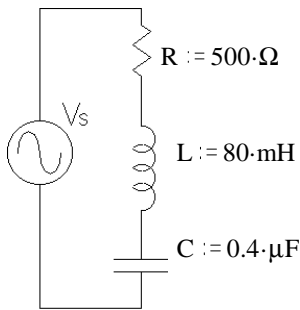


Current divider:

$$I_{Zn} = I_{total} \cdot \frac{\frac{1}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \dots}$$

$$Z_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{\frac{1}{C \cdot s}} + \frac{1}{L \cdot s}} = \frac{1}{\frac{1}{R} + C \cdot s + \frac{1}{L \cdot s}}$$

Ex5



a) Find the steady-state V_R and $v_R(t)$ given $v_S(t)$ is a 12 Vpp cosine wave at: $f = 2 \text{ kHz}$

$$V_S(\omega) = 6 \cdot V \cdot e^{j0} \quad \omega = 2 \cdot \pi \cdot f \quad \omega = 12566 \cdot \frac{\text{rad}}{\text{sec}}$$

Transfer function for V_R as the output: $H(s) = \frac{V_R(s)}{V_S(s)} = \frac{R}{R + L \cdot s + \frac{1}{C \cdot s}}$

$$V_R(\omega) = 6 \cdot V \cdot \frac{R}{R + L(j \cdot \omega) + \frac{1}{C(j \cdot \omega)}} = 1.666 - 2.687j \cdot V \quad V_R = 3.163V \angle -58.2^\circ$$

$$v_R(t) = 3.163 \cdot V \cdot \cos\left(12566 \cdot \frac{\text{rad}}{\text{sec}} \cdot t - 58.2 \cdot \text{deg}\right)$$

b) Find the current: $\frac{I(s)}{V(s)} = \frac{1}{Z(s)} \quad I(s) = \frac{V(s)}{Z(s)} \quad s := j \cdot \omega = j \cdot 12566 \cdot \frac{\text{rad}}{\text{sec}}$

$$I := \frac{6 \cdot V \cdot e^{j0}}{R + L(j \cdot \omega) + \frac{1}{C(j \cdot \omega)}} = \frac{6 \cdot V}{500 + 0.080 \cdot (j \cdot 12566) + \frac{1}{0.4 \cdot 10^{-6} \cdot (j \cdot 12566)}} = \frac{6 \cdot V}{500 + 1005.3 \cdot j - 198.95 \cdot j} = \frac{6 \cdot V}{500 + 806.366 \cdot j}$$

magnitude: $\frac{6 \cdot V}{948.8 \cdot \Omega} = 6.324 \text{ mA}$

angle: $0 \cdot \text{deg} - 58.2 \cdot \text{deg} = -58.2 \cdot \text{deg} \quad I = 6.324 \text{ mA} \angle -58.2^\circ$

$$\sqrt{500^2 + 806.366^2} = 948.802$$

$$\text{atan}\left(\frac{806.366}{500}\right) = 58.198 \cdot \text{deg}$$

c) Draw a phasor diagram of all the voltages.

$$V_L = I \cdot Z_L \quad 6.324 \text{ mA} \cdot 1005 \cdot \Omega = 6.356 \cdot V$$

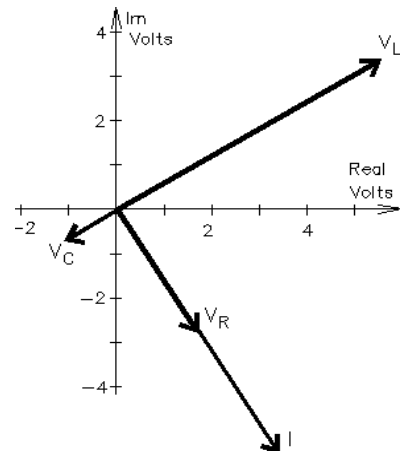
$$-58.2 \cdot \text{deg} + 90 \cdot \text{deg} = 31.8 \cdot \text{deg}$$

$$V_L = 6.356V \angle 31.8^\circ$$

$$V_C = I \cdot Z_C \quad 6.324 \text{ mA} \cdot (-199) \cdot \Omega = -1.258 \cdot V$$

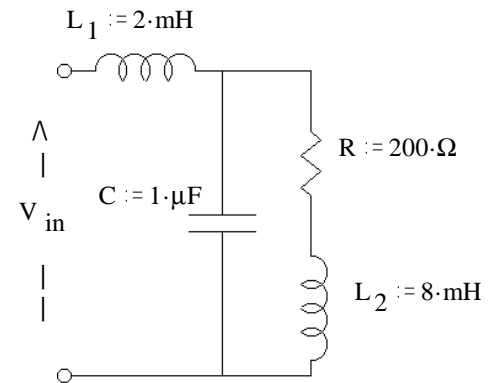
$$-58.2 \cdot \text{deg} + (90) \cdot \text{deg} = 31.8 \cdot \text{deg}$$

$$V_C = -1.258V \angle 31.8^\circ = 1.258V \angle -148.2^\circ$$



Sinusoidal Response Notes p5

- Ex6** a) Find the steady-state V_C and $v_C(t)$
 given $v_{in}(t)$ is a 12 Vp cosine wave at: $f := 2.5 \cdot \text{kHz}$
 with a 20° leading phase angle. $V_{in} := 12 \cdot V \cdot e^{j \cdot 20 \cdot \text{deg}}$



Transfer function for V_C as the output: $H(s) = \frac{V_C(s)}{V_{in}(s)}$

$$H(s) = \frac{\frac{1}{R + L_2 \cdot s}}{L_1 \cdot s + \frac{1}{\frac{1}{R + L_2 \cdot s} + C \cdot s}} = \frac{\left(\frac{1}{R + L_2 \cdot s} + C \cdot s \right)}{\left(\frac{1}{R + L_2 \cdot s} + C \cdot s \right)}$$

$$= \frac{1}{L_1 \cdot s \cdot \left(\frac{1}{R + L_2 \cdot s} + C \cdot s \right) + 1} = \frac{1}{L_1 \cdot (j \cdot \omega) \cdot \left[\frac{1}{R + L_2 \cdot (j \cdot \omega)} + C \cdot (j \cdot \omega) \right] + 1}$$

$$f = 2.5 \cdot \text{kHz} \quad \omega := 2 \cdot \pi \cdot f \quad \omega = 15708 \cdot \frac{\text{rad}}{\text{sec}}$$

$$H(j\omega) = \frac{1}{0.002 \cdot (j \cdot 15708) \cdot \left[\frac{1}{200 + 0.008 \cdot (j \cdot 15708)} + 10^{-6} \cdot (j \cdot 15708) \right] + 1}$$

$$= \frac{1}{31.416 \cdot j \cdot [3.585 \cdot 10^{-3} - 2.252 \cdot 10^{-3} \cdot j + (j \cdot 0.015708)] + 1} = \frac{1}{1 - 0.423 + 0.113 \cdot j} = \frac{1}{0.588 \cdot e^{j \cdot 11.039 \cdot \text{deg}}}$$

$$= 1.7 \cdot e^{-j \cdot 11.039 \cdot \text{deg}}$$

$$V_C = V_{in}(\omega) \cdot H(j\omega) = 12 \cdot V \cdot e^{j \cdot 20 \cdot \text{deg}} \cdot 1.7 \cdot e^{-j \cdot 11.039 \cdot \text{deg}} = 12 \cdot V \cdot 1.7 \angle 20 - 11.039^\circ = 20.4V \angle 8.96^\circ$$

$$v_C(t) = 20.4 \cdot V \cdot \cos\left(15708 \cdot \frac{\text{rad}}{\text{sec}} \cdot t + 8.96 \cdot \text{deg}\right)$$

- a) Find the steady-state I_{L2} and $i_{L2}(t)$.

Transfer function for I_{L2} as the output: $H(s) = \frac{I_{L2}(s)}{V_{in}(s)} = \frac{\left(\frac{V_C(s)}{R + L_2 \cdot s} \right)}{V_{in}(s)} = \frac{V_C(s)}{V_{in}(s)} \cdot \frac{1}{R + L_2 \cdot s}$

$$H(s) = \frac{1}{L_1 \cdot s \cdot \left(\frac{1}{R + L_2 \cdot s} + C \cdot s \right) + 1} \cdot \frac{1}{(R + L_2 \cdot s)} = \frac{1}{L_1 \cdot s + L_1 \cdot s \cdot C \cdot s \cdot (R + L_2 \cdot s) + (R + L_2 \cdot s)}$$

$$H(j\omega) = \frac{1}{L_1 \cdot (j \cdot \omega) + L_1 \cdot C \cdot (j \cdot \omega)^2 \cdot [R + L_2 \cdot (j \cdot \omega)] + [R + L_2 \cdot (j \cdot \omega)]} = 5.249 \cdot 10^{-3} - 4.926 \cdot 10^{-3} \cdot j \quad \frac{1}{\Omega} = \frac{7.198}{\text{k}\Omega} \cdot e^{-j \cdot 43.181 \cdot \text{deg}}$$

$$I_{L2} = V_{in}(\omega) \cdot H(j\omega) = 12 \cdot V \cdot e^{j \cdot 20 \cdot \text{deg}} \cdot \left(\frac{7.198}{\text{k}\Omega} \cdot e^{-j \cdot 43.181 \cdot \text{deg}} \right) = 12 \cdot V \cdot \frac{7.198}{\text{k}\Omega} \angle 20 - 43.181^\circ = 86.38 \text{mA} \angle -23.18^\circ$$

$$i_{L2}(t) = 86.38 \cdot \text{mA} \cdot \cos\left(15708 \cdot \frac{\text{rad}}{\text{sec}} \cdot t - 23.18 \cdot \text{deg}\right)$$

Sinusoidal Response Notes p5

Sinusoidal Response Notes p6

Ex7 This system: $H(s) = \frac{s+20}{s+5}$ Has this input: $x(t) = 4 \cdot \sin(12 \cdot t - 40 \cdot \text{deg}) \cdot u(t)$

a) Use steady-state AC analysis to find the steady-state output. $y_{ss}(t) = ?$

$$\text{AC steady-state } H(j\omega) = H(j \cdot 12) = \frac{j \cdot 12 + 20}{j \cdot 12 + 5} = \frac{\sqrt{12^2 + 20^2} \cdot e^{j \cdot \text{atan}\left(\frac{12}{20}\right)}}{\sqrt{12^2 + 5^2} \cdot e^{j \cdot \text{atan}\left(\frac{12}{5}\right)}} = \frac{23.324 \cdot e^{j \cdot 30.964 \cdot \text{deg}}}{13 \cdot e^{j \cdot 67.38 \cdot \text{deg}}}$$

$$= 1.794 \angle -36.416 \cdot \text{deg}$$

$$X(\omega) = 4 \cdot e^{-j \cdot 130 \cdot \text{deg}} \quad \text{Note } 90^\circ \text{ phase-lag because it's given as a sine wave}$$

$$Y_{ss}(\omega) = 4 \cdot 1.794 = 7.176 \angle -130 - 36.416 = -166.416$$

$$= 7.176 \angle -166.416 \cdot \text{deg} \quad 7.176 \cdot e^{-j \cdot 166.416 \cdot \text{deg}} = -6.975 - 1.685j$$

$$y_{ss}(t) = 7.176 \cdot \cos(12 \cdot t - 166.416 \cdot \text{deg}) \cdot u(t) = (-6.975 \cdot \cos(12 \cdot t) + 1.685 \cdot \sin(12 \cdot t)) \cdot u(t)$$

b) Express the output, and separate into 3 partial fractions that you can find in the laplace transform table without using complex numbers. Show what they are, but don't find the coefficients.

Find the input as a sum of a pure cosine and sine $4 \cdot e^{-j \cdot 130 \cdot \text{deg}} = -2.571 - 3.064j$

$$4 \cdot \cos(-130 \cdot \text{deg}) = -2.571 \quad 4 \cdot \sin(-130 \cdot \text{deg}) = -3.064$$

$$\text{so: } 4 \cdot \sin(12 \cdot t - 40 \cdot \text{deg}) \cdot u(t) = (-2.571 \cdot \cos(12 \cdot t) + 3.064 \cdot \sin(12 \cdot t)) \cdot u(t)$$

$$\& \quad X(s) = \frac{-2.571 \cdot s}{s^2 + 12^2} + \frac{3.064 \cdot 12}{s^2 + 12^2}$$

$$Y(s) = \frac{-2.571 \cdot s + 3.064 \cdot 12}{s^2 + 144} \cdot \frac{(s+20)}{(s+5)} = \frac{A}{s+5} + \frac{B \cdot s}{s^2 + 144} + \frac{C \cdot 12}{s^2 + 144}$$

c) Continue with the partial fraction expansion just far enough to find the **transient** coefficient as a number.

$$(3.064 \cdot 12 - 2.571 \cdot s) \cdot (s+20) = A \cdot (s^2 + 144) + B \cdot s \cdot (s+5) + C \cdot 12 \cdot (s+5)$$

let $s := -5$

$$(3.064 \cdot 12 - 2.571 \cdot (-5)) \cdot (-5 + 20) = A \cdot (5^2 + 144) + 0 + 0$$

$$A := \frac{(s+20) \cdot (3.064 \cdot 12 - 2.571 \cdot s)}{25 + 144} \quad A = 4.404$$

d) Express the complete (both transient and steady-state) output as a function of time. $y(t) = ?$

$$y(t) = (4.404 \cdot e^{-5 \cdot t} + 7.176 \cdot \cos(12 \cdot t - 166.416 \cdot \text{deg})) \cdot u(t)$$

$$7.176 \cdot \cos(-166.416 \cdot \text{deg}) = -6.975$$

Either answer

$$7.176 \cdot \sin(-166.416 \cdot \text{deg}) = -1.685$$

$$y(t) = (4.404 \cdot e^{-5 \cdot t} - 6.975 \cdot \cos(12 \cdot t) + 1.685 \cdot \sin(12 \cdot t)) \cdot u(t)$$

e) What is the time constant of the transient part this expression? $\tau = ? = \frac{1}{5}$

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