Sinusoidal Response Notes

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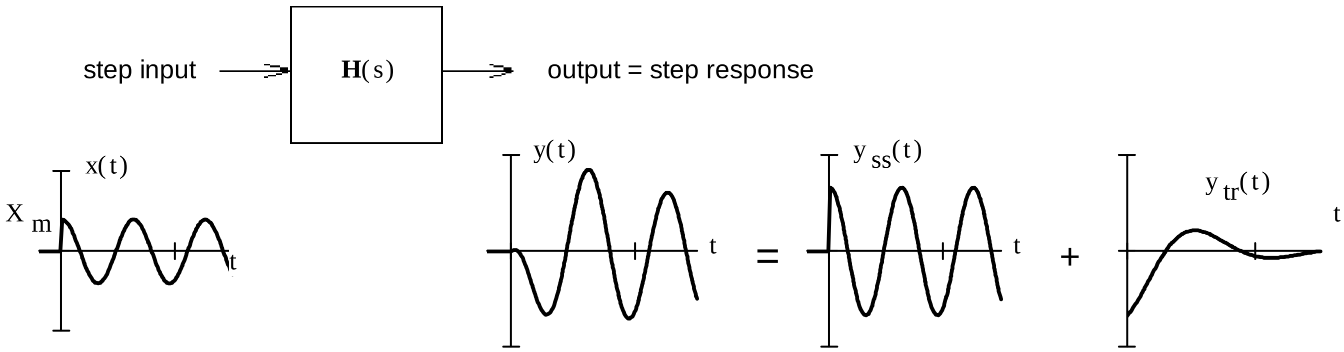
1/29/13

For BIBO Systems

The sinusoidal response of a system is the output when the input is a sinusoidal (which starts at time = 0).

System Sinusoidal Response



Complete step response = steady-state response + transient response

$$\mathbf{Y}(s) = \mathbf{X}(s)\cdot\mathbf{H}(s) = X_m\cdot\mathbf{H}(j\omega)\cdot u(t) + \mathbf{Y}_{tr}(s)$$

$\mathbf{H}(j\omega)$ = phasor-type transfer function

Sinusoidal Input

$$\cos(\omega\cdot t)\cdot u(t) \iff \frac{s}{s^2+\omega^2} \qquad \sin(\omega\cdot t)\cdot u(t) \iff \frac{b}{s^2+\omega^2}$$

General sinusoidal input: $\left[X_{mc}\cdot\cos(\omega\cdot t) + X_{ms}\cdot(\sin(\omega\cdot t)\cdot u(t))\right]\cdot u(t)$ $\qquad \mathbf{X}(s) = \dfrac{X_{mc}\cdot s + X_{ms}\cdot\omega}{s^2+\omega^2}$

Steady-State Response & $\mathbf{H}(j\omega)$

$$\mathbf{Y}(s) = \mathbf{X}(s)\cdot\mathbf{H}(s) = \frac{X_{mc}\cdot s + X_{ms}\cdot\omega}{s^2+\omega^2}\cdot\mathbf{H}(s) = \text{Complete sinusoidal response}$$

$$= \frac{A\cdot s + B\cdot\omega}{s^2+\omega^2} + \frac{C}{(\blacksquare)} + \frac{D}{(\blacksquare)} + \frac{E}{(\blacksquare)} + \ldots$$

partial fraction expansion: $\mathbf{Y}(s) = \dfrac{X_{mc}\cdot s + X_{ms}\cdot\omega}{s^2+\omega^2}\cdot\mathbf{H}(s) = \dfrac{A\cdot s + B\cdot\omega}{s^2+\omega^2} + \left[\dfrac{C}{(\blacksquare)} + \dfrac{D}{(\blacksquare)} + \dfrac{E}{(\blacksquare)}\right]\cdot s$

steady-state response + transient response

$\mathbf{Y}_{ss}(s) + \mathbf{Y}_{tr}(s)$

multiply both sides by: $\left(s^2+\omega^2\right)$ $\qquad \left(X_{mc}\cdot s + X_{ms}\cdot\omega\right)\cdot\mathbf{H}(s) = A\cdot s + B\cdot\omega + \left[\dfrac{C}{(\blacksquare)} + \dfrac{D}{(\blacksquare)} + \dfrac{E}{(\blacksquare)}\right]\cdot\left(s^2+\omega^2\right)$

set $s := j\omega$ $\qquad \left(X_{mc}\cdot j\omega + X_{ms}\cdot\omega\right)\cdot\mathbf{H}(j\omega) = A\cdot j\omega + B\cdot\omega + \left[\dfrac{C}{(\blacksquare)} + \dfrac{D}{(\blacksquare)} + \dfrac{E}{(\blacksquare)}\right]\cdot 0$

$\mathbf{X}(j\omega)\cdot\mathbf{H}(j\omega) = A\cdot j\omega + B\cdot\omega = \mathbf{Y}_{ss}(s)$ = steady-state response in phasor form

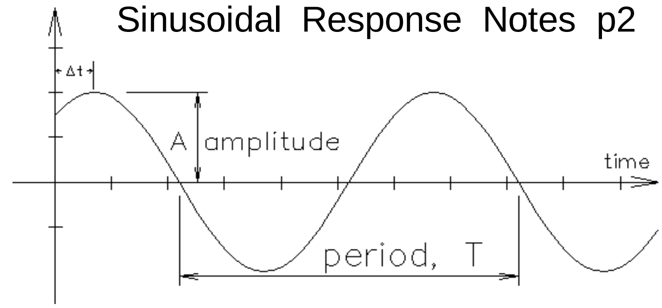
$\mathbf{X}(j\omega)$ = the input expressed in phasor form

$\mathbf{H}(j\omega)$ = the steady-state sinusoidal transfer function

= phasor-type transfer function

The **transient part** would be found by finishing the partial-fraction expansion.

Steady-State Response by Phasors



Expression of signals as phasors

T = Period

f = frequency, cycles / second $\quad f = \frac{1}{T} = \frac{\omega}{2\cdot\pi}$

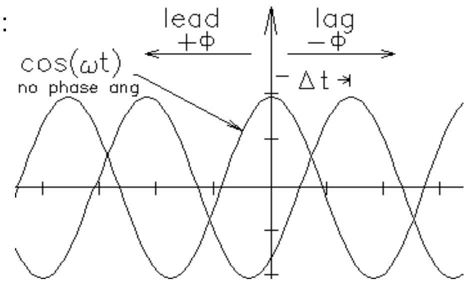
ω = radian frequency, radians/sec $\quad \omega = 2\cdot\pi\cdot f$

A = amplitude

Phase: $\phi = -\frac{\Delta t}{T}\cdot 360\cdot deg$ or: $\phi = -\frac{\Delta t}{T}\cdot 2\cdot\pi\cdot rad$

$y(t) = A\cdot\cos(\omega\cdot t + \phi)$

Phase:



		Phasor
voltage:	$v(t) = V_p\cdot\cos(\omega\cdot t + \phi)$	$\mathbf{V}(j\omega) = V_p\cdot e^{j\cdot\phi}$
current:	$i(t) = I_p\cdot\cos(\omega\cdot t + \phi)$	$\mathbf{I}(j\omega) = I_p\cdot e^{j\cdot\phi}$

Ex1 Let's assume the input to your system is $v_1(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$ $\quad \mathbf{V_1}(j\omega) = 3.2\cdot V\cdot e^{j\cdot 15\cdot deg}$

or: $\mathbf{V_1}(j\omega) = 3.2V\ \underline{/15^o}$

In rectangular form:

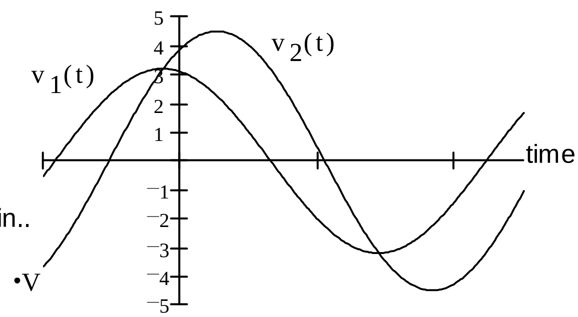
$3.2\cdot V\cdot\cos(15\cdot deg) = 3.091\cdot V \qquad 3.2\cdot V\cdot\sin(15\cdot deg) = 0.828\cdot V$

or: $\mathbf{V_1}(j\omega) = (3.091 + 0.828\cdot j)\cdot V$

Ex2 What if a signal is the sum of two sinusoids.

$v_1(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 15\cdot deg)$

$v_2(t) = 4.5\cdot V\cdot\sin(\omega\cdot t + 60\cdot deg) \qquad v_3(t) = v_1(t) + v_2(t)$



I'm going to drop the (jω) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain..

From Ex1: $\mathbf{V_1} := 3.2\cdot V\cdot e^{j\cdot 15\cdot deg} = 3.2V\ \underline{/15^o} \qquad \mathbf{V_1} = 3.091 + 0.828j\ \cdot V$

Phasors are based on cosines, so express $v_2(t)$ as a cosine. Remember: $\sin(\omega t) = \cos(\omega\cdot t - 90\cdot deg)$

So: $v_2(t) = 4.5\cdot V\cdot\cos(\omega\cdot t + 60\cdot deg - 90\cdot deg) = 4.5\cdot V\cdot\cos(\omega\cdot t - 30\cdot deg)$

$\mathbf{V_2} = 4.5V\ \underline{/-30^o}$ or: $\mathbf{V_2} := 4.5\cdot V\cdot e^{-j\cdot 30\cdot deg}$

$4.5\cdot V\cdot\cos(-30\cdot deg) = 3.897\cdot V \qquad 4.5\cdot V\cdot\sin(-30\cdot deg) = -2.25\cdot V$

Add real parts: $3.897 + 3.091 = 6.988$

Add imaginary parts: $-2.25 + 0.828 = -1.422$

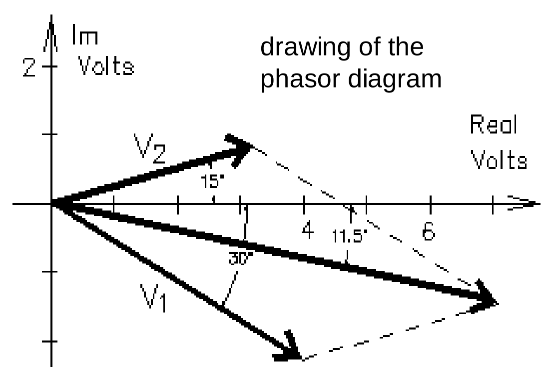
$\mathbf{V_2} = 3.897 - 2.25j\ \cdot V$ \
$\mathbf{V_1} = 3.091 + 0.828j\ \cdot V$ } add
/

$\mathbf{V_3} := \mathbf{V_1} + \mathbf{V_2}$

$\mathbf{V_3} = 6.988 - 1.422j\ \cdot V$ sum

Change $\mathbf{V_3}$ back to polar coordinates:

$\sqrt{6.988^2 + 1.422^2} = 7.131 \qquad \operatorname{atan}\left(\frac{-1.422}{6.988}\right) = -11.502\cdot deg$



drawing of the phasor diagram

OR, in Mathcad notation (you'll see these in future solutions):

$\left|\mathbf{V_3}\right| = 7.131\cdot V \qquad \arg(\mathbf{V_3}) = -11.5\cdot deg$

$\mathbf{V_3}(j\omega) = 7.131V\ \underline{/-11.5^o}$ or: $\mathbf{V_3}(j\omega) = 7.131\cdot V\cdot e^{-j\cdot 11.5\cdot deg}$

$\mathbf{V_3}$ may also be converted back to the time domain:

$v_3(t) = v_1(t) + v_2(t) = 7.13\cdot\cos(\omega\cdot t - 11.5\cdot deg)\cdot V$

Magnitude and Phase of transfer functions With steady-state sinusoidal inputs

Ex3 a) Find the magnitude and phase of the following transfer function at this frequency: $\omega := 2\cdot\frac{rad}{sec}$

$$\mathbf{H}(s) = \frac{2\cdot s^2 + 5\cdot s + 20}{s^2 + 1\cdot s + 10} = \frac{2\cdot s^2 + \frac{5}{sec}\cdot s + \frac{20}{sec^2}}{s^2 + \frac{1}{sec}\cdot s + \frac{10}{sec^2}}$$ Expressed with proper units

$s := j\cdot\omega$

$$\mathbf{H}(j\cdot\omega) = \frac{2\cdot(j\cdot\omega)^2 + 5\cdot(j\cdot\omega) + 20}{(j\cdot\omega)^2 + \frac{1}{sec}\cdot(j\cdot\omega) + \frac{10}{sec^2}} = \frac{(20 - 2\cdot\omega^2) + (5\cdot\omega)\cdot j}{(10 - \omega^2) + (1\cdot\omega)\cdot j} = \frac{(20 - 2\cdot 2^2) + (5\cdot 2)\cdot j}{(10 - 2^2) + (1\cdot 2)\cdot j} = \frac{12 + 10\cdot j}{6 + 2\cdot j}$$

without units

$$|\mathbf{H}(j\cdot\omega)| = M = \frac{\sqrt{12^2 + 10^2}}{\sqrt{6^2 + 2^2}} = 2.47 \qquad \underline{/\,H(j\omega)} = \operatorname{atan}\left(\frac{10}{12}\right) - \operatorname{atan}\left(\frac{2}{6}\right) = 21.37\cdot deg$$

b) Find the steady-state sinusoidal output if the input is: $3.2\cdot V\cdot\cos(2\cdot t + 15\cdot deg)$ $\quad \mathbf{V}_{in} := 3.2\cdot V\cdot e^{j\cdot 15\cdot deg} = 3.2V\ \underline{/15^o}$

$$\mathbf{V}_{outss}(j\omega) = \mathbf{V}_{in}(j\omega)\cdot\mathbf{H}(j\cdot\omega) = \left(3.2\cdot V\cdot e^{j\cdot 15\cdot deg}\right)\cdot\left(2.47\cdot e^{j\cdot 21.37\cdot deg}\right) = 3\cdot V\cdot 2.47\cdot e^{j\cdot(15\cdot deg + 21.37\cdot deg)}$$

$$= 7.41\cdot V\cdot e^{j\cdot 36.37\cdot deg} = 5.967 + 4.394j\ \cdot V$$

$$v_{outss}(t) = 7.41\cdot V\cdot\cos(2\cdot t + 36.37\cdot deg)$$

Ex4 a) Find the magnitude and phase of the following transfer function at this frequency: $f := 5\cdot Hz \qquad \omega := 2\cdot\pi\cdot f$

$$\omega = 31.42\cdot\frac{rad}{sec}$$

$$\mathbf{H}(s) = \frac{s^2 + \frac{20}{sec}\cdot s + \frac{1000}{sec^2}}{s^2 + \frac{10}{sec}\cdot s + \frac{800}{sec^2}}$$

$$s := j\cdot\omega = j\cdot 31.42\cdot\frac{rad}{sec} \qquad \mathbf{H}(j\cdot\omega) = \frac{(j\cdot\omega)^2 + \frac{20}{sec}\cdot(j\cdot\omega) + \frac{1000}{sec^2}}{(j\cdot\omega)^2 + \frac{10}{sec}\cdot(j\cdot\omega) + \frac{800}{sec^2}} = \frac{(j\cdot 31.42)^2 + 20\cdot(j\cdot 31.42) + 1000}{(j\cdot 31.42)^2 + 10\cdot(j\cdot 31.42) + 800}$$ without units

$$= \frac{13.04 + 628.319\cdot j}{-186.96 + 314.159\cdot j} = 1.459 - 0.91j$$

$$|\mathbf{H}(j\cdot\omega)| = M = \sqrt{1.459^2 + 0.91^2} = 1.72 \qquad \underline{/\,H(j\omega)} = \operatorname{atan}\left(\frac{-0.91}{1.459}\right) = -31.95\cdot deg$$

b) Find the steady-state sinusoidal output if the input is: $x(t) = 4\cdot\cos(2\cdot\pi\cdot 5\cdot Hz\cdot t)$

$\mathbf{X}(\omega) = 4 + 0j$ and then $\mathbf{Y}(\omega) = 4\cdot(1.459 - 0.91\cdot j) = 5.836 - 3.64j$

Note that you can use the rectangular form of $\mathbf{H}(j\cdot\omega)$

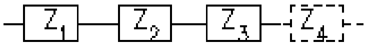
$$y(t) = 5.836\cdot\cos\left(31.42\cdot\frac{rad}{sec}\cdot t\right) + 3.64\cdot\sin\left(31.42\cdot\frac{rad}{sec}\cdot t\right)$$

note that the sine carries the opposite sign as the imaginary part.

$$\sqrt{5.836^2 + 3.64^2} = 6.878 \qquad \operatorname{atan}\left(\frac{-3.64}{5.836}\right) = -31.95\cdot deg \qquad y(t) = 6.88\cdot\cos\left(31.42\cdot\frac{rad}{sec}\cdot t - 32\cdot deg\right)$$

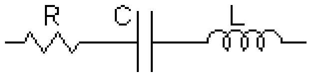
Impedances

series:



$$Z_{eq} = Z_1 + Z_2 + Z_3 + \ldots$$

Example:

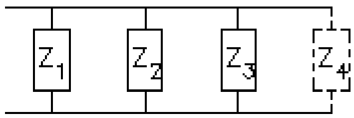


$$Z_{eq} = R + \frac{1}{C\cdot s} + L\cdot s$$

Voltage divider:

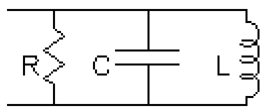
$$V_{Zn} = V_{total}\cdot\frac{Z_n}{Z_1 + Z_2 + Z_3 + \ldots}$$

parallel:



$$Z_{eq} = \frac{1}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots}$$

Example:

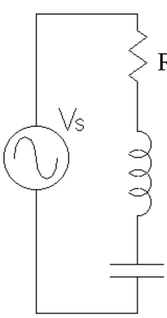


$$Z_{eq} = \frac{1}{\frac{1}{R} + \frac{1}{\frac{1}{C\cdot s}} + \frac{1}{L\cdot s}} = \frac{1}{\frac{1}{R} + C\cdot s + \frac{1}{L\cdot s}}$$

Current divider:

$$I_{Zn} = I_{total}\cdot\frac{\frac{1}{Z_n}}{\frac{1}{Z_1} + \frac{1}{Z_2} + \frac{1}{Z_3} + \ldots}$$

Ex4



$R := 500\cdot\Omega$

$L := 80\cdot mH$

$C := 0.4\cdot\mu F$

a) Find the steady-state $\mathbf{V_R}$ and $v_R(t)$ given $v_S(t)$ is a 12 Vpp cosine wave at: $f := 2\cdot kHz$

$$\mathbf{V}_S(j\omega) = 6\cdot V\cdot e^{j0} \qquad \omega := 2\cdot\pi\cdot f \qquad \omega = 12566\cdot\frac{rad}{sec}$$

Transfer function for $\mathbf{V_R}$ as the output: $\mathbf{H}(s) = \frac{\mathbf{V}_R(s)}{\mathbf{V}_S(s)} = \frac{R}{R + L\cdot s + \frac{1}{C\cdot s}}$

$$\mathbf{V}_R(j\omega) = 6\cdot V\cdot\frac{R}{R + L\cdot(j\cdot\omega) + \frac{1}{C\cdot(j\cdot\omega)}} = 1.666 - 2.687j \cdot V \qquad \mathbf{V_R} = 3.163V\ \underline{/-58.2}^o$$

$$v_R(t) = 3.163\cdot V\cdot\cos\left(12566\cdot\frac{rad}{sec}\cdot t - 58.2\cdot deg\right)$$

b) Find the current: $\frac{\mathbf{I}(s)}{\mathbf{V}(s)} = \frac{1}{\mathbf{Z}(s)} \qquad \mathbf{I}(s) = \frac{\mathbf{V}(s)}{\mathbf{Z}(s)} \qquad s := j\cdot\omega = j\cdot 12566\cdot\frac{rad}{sec}$

$$\mathbf{I} := \frac{6\cdot V\cdot e^{j\cdot 0}}{R + L\cdot(j\cdot\omega) + \frac{1}{C\cdot(j\cdot\omega)}} = \frac{6\cdot V}{500 + 0.080\cdot(j\cdot 12566) + \frac{1}{0.4\cdot 10^{-6}\cdot(j\cdot 12566)}} = \frac{6\cdot V}{500 + 1005.3\cdot j - 198.95\cdot j} = \frac{6\cdot V}{500 + 806.366\cdot j}$$

$$\sqrt{500^2 + 806.366^2} = 948.802$$

$$\operatorname{atan}\left(\frac{806.366}{500}\right) = 58.198\cdot deg$$

magnitude: $\frac{6\cdot V}{948.8\cdot\Omega} = 6.324\cdot mA$

angle: $0\cdot deg - 58.2\cdot deg = -58.2\cdot deg \qquad \mathbf{I} = 6.324mA\ \underline{/-58.2}^o$

c) Draw a phasor diagram of all the voltages.

$\mathbf{V_L} = \mathbf{I}\cdot\mathbf{Z_L}$ $\quad 6.324\cdot mA\cdot 1005\cdot\Omega = 6.356\cdot V$

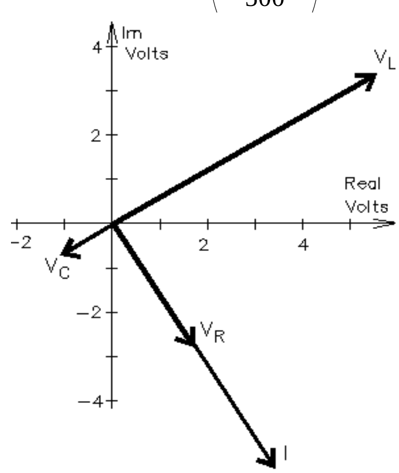
$-58.2\cdot deg + 90\cdot deg = 31.8\cdot deg$

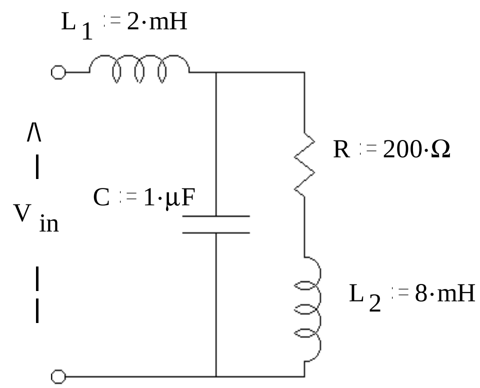
$\mathbf{V_L} = 6.356V\ \underline{/\ 31.8}^o$

$\mathbf{V_C} = \mathbf{I}\cdot\mathbf{Z_C}$ $\quad 6.324\cdot mA\cdot(-199)\cdot\Omega = -1.258\cdot V$

$-58.2\cdot deg + (90)\cdot deg = 31.8\cdot deg$

$\mathbf{V_C} = -1.258V\ \underline{/\ 31.8}^o = 1.258V\ \underline{/-148.2}^o$





Ex5 a) Find the steady-state $\mathbf{V_C}$ and $v_C(t)$ given $v_{in}(t)$ is a 12 Vp cosine wave at: $f := 2.5\cdot kHz$ with a 20º leading phase angle.

Transfer function for $\mathbf{V_C}$ as the output: $\mathbf{H}(s) = \dfrac{\mathbf{V_C}(s)}{\mathbf{V_{in}}(s)}$

$$\mathbf{H}(s) = \frac{\dfrac{1}{\dfrac{1}{R+L_2\cdot s}+C\cdot s}}{L_1\cdot s+\dfrac{1}{\dfrac{1}{R+L_2\cdot s}+C\cdot s}} \qquad \frac{\left(\dfrac{1}{R+L_2\cdot s}+C\cdot s\right)}{\left(\dfrac{1}{R+L_2\cdot s}+C\cdot s\right)}$$

$$= \frac{1}{L_1\cdot s\cdot\left(\dfrac{1}{R+L_2\cdot s}+C\cdot s\right)+1}$$

$\mathbf{V_{in}} := 12\cdot V\cdot e^{j\cdot 20\cdot deg}$ $\quad \omega := 2\cdot\pi\cdot f \quad \omega = 15708\cdot\dfrac{rad}{sec}$

$$\mathbf{H}(j\omega) = \frac{1}{L_1\cdot(j\cdot\omega)\cdot\left[\dfrac{1}{R+L_2\cdot(j\cdot\omega)}+C\cdot(j\cdot\omega)\right]+1} = \frac{1}{0.002\cdot(j\cdot 15708)\cdot\left[\dfrac{1}{200+0.008\cdot(j\cdot 15708)}+10^{-6}\cdot(j\cdot 15708)\right]+1}$$

$$= \frac{1}{31.416\cdot j\ \cdot\left[3.585\cdot 10^{-3}-2.252\cdot 10^{-3}\cdot j\ +(j\cdot 0.015708)\right]+1} = \frac{1}{1-0.423+0.113\cdot j} = \frac{1}{0.588\cdot e^{j\cdot 11.039\cdot deg}}$$

$$= 1.7\cdot e^{-j\cdot 11.039\cdot deg}$$

$$\mathbf{V_C} = \mathbf{V_{in}}(j\omega)\cdot\mathbf{H}(j\omega) = 12\cdot V\cdot e^{j\cdot 20\cdot deg}\cdot 1.7\cdot e^{-j\cdot 11.039\cdot deg} = 12\cdot V\cdot 1.7\ \underline{/\ 20-11.039^o} = 20.4V\ \underline{/\ 8.96^o}$$

$$v_c(t) = 20.4\cdot V\cdot\cos\left(15708\cdot\frac{rad}{sec}\cdot t+8.96\cdot deg\right)$$

a) Find the steady-state $\mathbf{I_{L2}}$ and $i_{L2}(t)$.

Transfer function for $\mathbf{I_{L2}}$ as the output: $\mathbf{H}(s) = \dfrac{\mathbf{I_{L2}}(s)}{\mathbf{V_{in}}(s)} = \dfrac{\left(\dfrac{\mathbf{V_C}(s)}{R+L_2\cdot s}\right)}{\mathbf{V_{in}}(s)} = \dfrac{\mathbf{V_C}(s)}{\mathbf{V_{in}}(s)}\cdot\dfrac{1}{R+L_2\cdot s}$

$$\mathbf{H}(s) = \frac{1}{L_1\cdot s\cdot\left(\dfrac{1}{R+L_2\cdot s}+C\cdot s\right)+1}\cdot\frac{1}{(R+L_2\cdot s)} = \frac{1}{L_1\cdot s+L_1\cdot s\cdot C\cdot s\cdot(R+L_2\cdot s)+(R+L_2\cdot s)}$$

$$\mathbf{H}(j\omega) = \frac{1}{L_1\cdot(j\cdot\omega)+L_1\cdot C\cdot(j\cdot\omega)^2\cdot\left[R+L_2\cdot(j\cdot\omega)\right]+\left[R+L_2\cdot(j\cdot\omega)\right]} = 5.249\bullet 10^{-3}-4.926\bullet 10^{-3}j \quad \bullet\frac{1}{\Omega} = \frac{7.198}{k\Omega}\cdot e^{-j\cdot 43.181\cdot deg}$$

$$\mathbf{I_{L2}} = \mathbf{V_{in}}(j\omega)\cdot\mathbf{H}(j\omega) = 12\cdot V\cdot e^{j\cdot 20\cdot deg}\cdot\left(\frac{7.198}{k\Omega}\cdot e^{-j\cdot 43.181\cdot deg}\right) = 12\cdot V\cdot\frac{7.198}{k\Omega}\ \underline{/\ 20-43.181^o} = 86.38mA\ \underline{/\ -23.18^o}$$

$$i_{L2}(t) = 86.38\cdot mA\cdot\cos\left(15708\cdot\frac{rad}{sec}\cdot t-23.18\cdot deg\right)$$

Ex6 This system: $\mathbf{H}(s) = \frac{s+20}{s+5}$ Has this input: $x(t) = 4\cdot\sin(12\cdot t - 40\cdot deg)\cdot u(t)$

a) Use steady-state AC analysis to find the steady-state output. $y_{ss}(t) = ?$

$$\text{AC steady-state } \mathbf{H}(j\omega) = \mathbf{H}(j\cdot 12) = \frac{j\cdot 12+20}{j\cdot 12+5} = \frac{\sqrt{12^2+20^2}\cdot e^{j\cdot atan\left(\frac{12}{20}\right)}}{\sqrt{12^2+5^2}\cdot e^{j\cdot atan\left(\frac{12}{5}\right)}} = \frac{23.324\cdot e^{j\cdot 30.964\cdot deg}}{13\cdot e^{j\cdot 67.38\cdot deg}}$$

$$= 1.794 \angle -36.416\cdot deg$$

$\mathbf{X}(j\omega) = 4\cdot e^{-j\cdot 130\cdot deg}$ Note 90º phase-lag because it's given as a sine wave

$$\mathbf{Y}_{ss}(j\omega) = 4\cdot 1.794 = 7.176 \angle -130 - 36.416 = -166.416$$

$$= 7.176 \angle -166.416\cdot deg$$

$$y_{ss}(t) = 7.176\cdot\cos(12\cdot t - 166.416\cdot deg)$$

b) Express the output, and separate into 3 partial fractions that you can find in the laplace transform table without using complex numbers. Show what they are, but don't find the coefficients.

Find the input as a sum of a pure sine and cosine $-4\cdot\sin(-130\cdot deg) = 3.064$ $4\cdot\cos(-130\cdot deg) = -2.571$

so: $4\cdot\sin(12\cdot t - 40\cdot deg)\cdot u(t) = (3.064\cdot\sin(12\cdot t) - 2.571\cdot\cos(12\cdot t))\cdot u(t)$

$$\mathbf{Y}(s) = \frac{3.064\cdot 12 - 2.571\cdot s}{s^2+144}\cdot\frac{(s+20)}{(s+5)} = \frac{A}{s+5} + \frac{B\cdot s}{\left(s^2+144\right)} + \frac{C\cdot 12}{\left(s^2+144\right)}$$

c) Continue with the partial fraction expansion just far enough to find the **transient** coefficient as a number.

$$(3.064\cdot 12 - 2.571\cdot s)\cdot(s+20) = A\cdot\left(s^2+144\right) + B\cdot s\cdot(s+5) + C\cdot 12\cdot(s+5)$$

let $s := -5$

$$(3.064\cdot 12 - 2.571\cdot(-5))\cdot(-5+20) = A\cdot\left(5^2+144\right) + 0 + 0$$

$$A := \frac{(s+20)\cdot(3.064\cdot 12 - 2.571\cdot s)}{25+144} \qquad A = 4.404$$

d) Express the complete (both transient and steady-state) output as a function of time. $y(t) = ?$

$$y(t) = \left(4.404\cdot e^{-5\cdot t} + 7.176\cdot\cos(12\cdot t - 166.416\cdot deg)\right)\cdot u(t)$$

$7.176\cdot\cos(-166.416\cdot deg) = -6.975$

$-7.176\cdot\sin(-166.416\cdot deg) = 1.685$

Either answer

$$y(t) = \left(4.404\cdot e^{-5\cdot t} - 6.975\cdot\cos(12\cdot t) + 1.685\cdot\sin(12\cdot t)\right)\cdot u(t)$$

e) What is the time constant of the transient part this expression? $\tau = ? \quad = \frac{1}{5}$