ECE 3510 Unconventional Root Locus A. Stolp 3/17/20

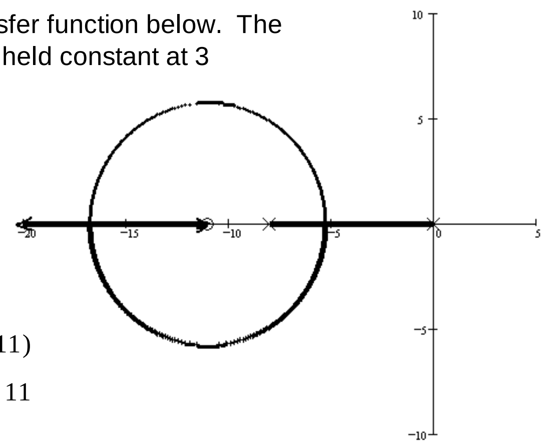
A regular root locus plot is very useful if you want to see how the positions of the closed-loop poles of a feedback system are affected by simple proportional gain in the system. But... what if you want to see how these poles are affected by some other variable in the system, like the filter time-constant in the Phase-Locked-Loop lab? Could we use some of the same concepts to see the affects of some other variable? The answer is yes, we just have to hold the gain constant and rearrange things a bit. An "unconventional root-locus plot" is like a regular root locus plot except that the gain is held constant and the plot shows how the closed-loop poles move as the result of changing some variable *other* than gain.

To create an unconventional root locus plot:

1. Determine the gain factor if it can be adjusted, and make it part of the open-loop transfer function, $G(s)$.
   Hold it constant at some number.
2. Determine the denominator of the closed-loop transfer function, $H(s)$. Let's call it $D_H(s)$.
3. Rearrange $D_H(s)$ into this form: $D'(s) + x \cdot N'(s)$ where x is the variable for which you want to draw the root locus.
   Notice that x occupies exactly the same position the gain would normally occupy. Normal: $D_G(s) + k \cdot N_G(s)$
   Note: If you cannot rearrange $D_H(s)$ into this form, then you cannot use this method to create an root locus plot for the variable x. Now: $D'(s) + x \cdot N'(s)$
4. Now simply draw a root locus as though $D'(s)$ was the open-loop denominator and $N'(s)$ was the open-loop numerator.

Ex.1 Sketch the unconventional root-locus plot for the open-loop transfer function below. The root-locus should be plotted for an increasing a. The gain will be held constant at 3

$$G(s) = \frac{(s + 2 \cdot a)}{(s + 5) \cdot (s + a)}$$



The denominator of the closed-loop transfer function:

$$(s + 5) \cdot (s + a) + 3 \cdot (s + 2 \cdot a) = s^2 + s \cdot a + 8 \cdot s + 11 \cdot a$$

$$= s^2 + 8 \cdot s + s \cdot a + 11 \cdot a \qquad = s \cdot (s + 8) + a \cdot (s + 11)$$

$$D'(s) = s \cdot (s + 8) \qquad N'(s) = (s + 11)$$

poles at 0 and -8 zero at -11

Ex.2 Sketch the unconventional root-locus plot for the open-loop transfer function below. The root-locus should be plotted for an increasing g.

$$G(s) = \frac{k \cdot s \cdot (g \cdot s + 1)}{(s + 4 \cdot g) \cdot (s + 5)} \qquad k = 2 \quad \text{and is constant}$$

The denominator of the closed-loop transfer function:

$$(s + 4 \cdot g) \cdot (s + 5) + 2 \cdot s \cdot (g \cdot s + 1)$$

$$s^2 + 5 \cdot s + 4 \cdot g \cdot s + 20 \cdot g + 2 \cdot g \cdot s^2 + 2 \cdot s$$

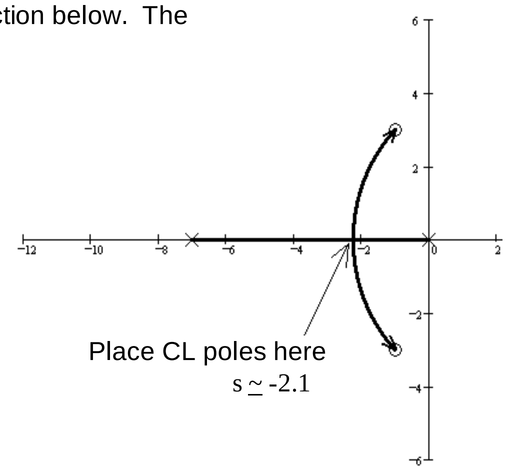
$$s^2 + 7 \cdot s + 2 \cdot g \cdot s^2 + 4 \cdot g \cdot s + 20 \cdot g$$

$$s \cdot (s + 7) + g \cdot \left(2 \cdot s^2 + 4 \cdot s + 20\right)$$

$$D'(s) \qquad g \cdot N'(s)$$

$$s \cdot (s + 7) + g \cdot 2 \cdot \left(s^2 + 2 \cdot s + 10\right)$$

$$a := -1 \qquad b = \sqrt{10 - 1^2} = 3$$



The arrow points to a desirable place for the closed-loop poles for minimal ringing and the shortest settling time.

To find the value of g needed: $0 = s \cdot (s + 7) + 2 \cdot g \cdot \left(s^2 + 2 \cdot s + 10\right)$ solve for $g = \dfrac{-\left(s^2 + 7 \cdot s\right)}{\left(2 \cdot s^2 + 4 \cdot s + 20\right)}$

if $s := -2.1$ $\quad g = \dfrac{-\left(s^2 + 7 \cdot s\right)}{\left(2 \cdot s^2 + 4 \cdot s + 20\right)} = 0.504$

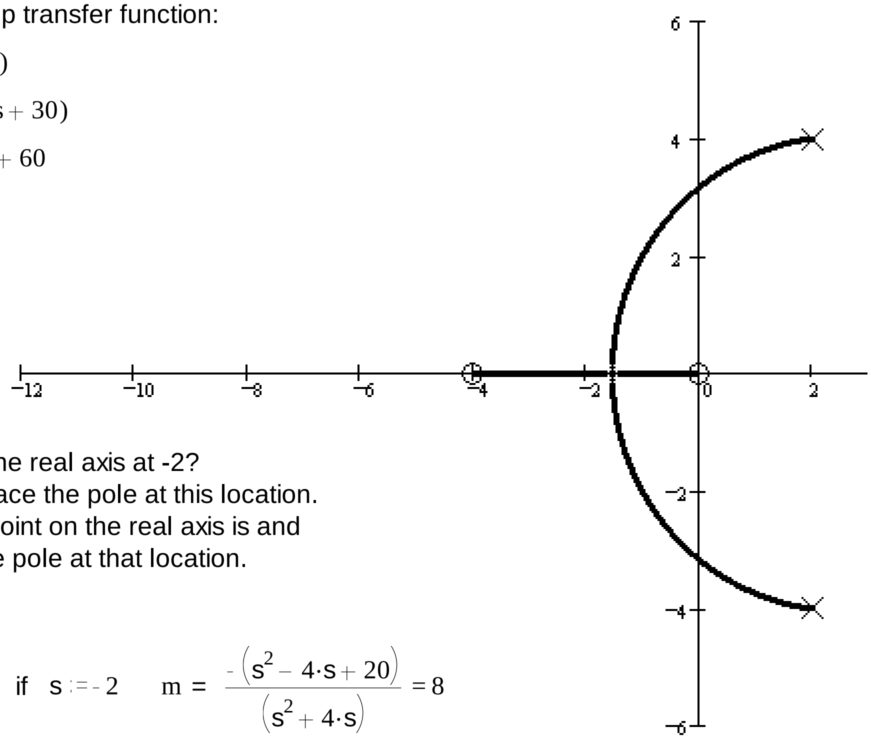
Ex.3 From E3, S12 Sketch the unconventional root-locus plot for the open-loop transfer function below. The root-locus should be plotted for an increasing m.

$$G(s) = \frac{k\cdot(s+30)}{(m\cdot s+s-10)\cdot(s+4)} \qquad k = 2 \text{ and is fixed}$$

The denominator of the closed-loop transfer function:

$$(m\cdot s+s-10)\cdot(s+4)+2\cdot(s+30)$$
$$m\cdot s^2+4\cdot m\cdot s+s^2-6\cdot s-40+2\cdot(s+30)$$
$$m\cdot s^2+4\cdot m\cdot s+s^2-6\cdot s-40+2\cdot s+60$$
$$m\cdot s^2+4\cdot m\cdot s+s^2-4\cdot s+20$$
$$\underbrace{\left(s^2-4\cdot s+20\right)}_{D'(s)}+\underbrace{m\cdot\left(s^2+4\cdot s\right)}_{m\cdot N'(s)}$$

$$a := 2$$
$$b := \sqrt{20-2^2} \qquad b = 4$$



b) Can you place a closed-loop pole on the real axis at -2?
If yes, find the value of m needed to place the pole at this location.
If no, indicate what you think the best point on the real axis is and find the value of m needed to place the pole at that location.

$$0 = \left(s^2-4\cdot s+20\right)+m\cdot\left(s^2+4\cdot s\right)$$

solve for $m = \dfrac{-\left(s^2-4\cdot s+20\right)}{\left(s^2+4\cdot s\right)}$ if $s := -2$ $m = \dfrac{-\left(s^2-4\cdot s+20\right)}{\left(s^2+4\cdot s\right)} = 8$

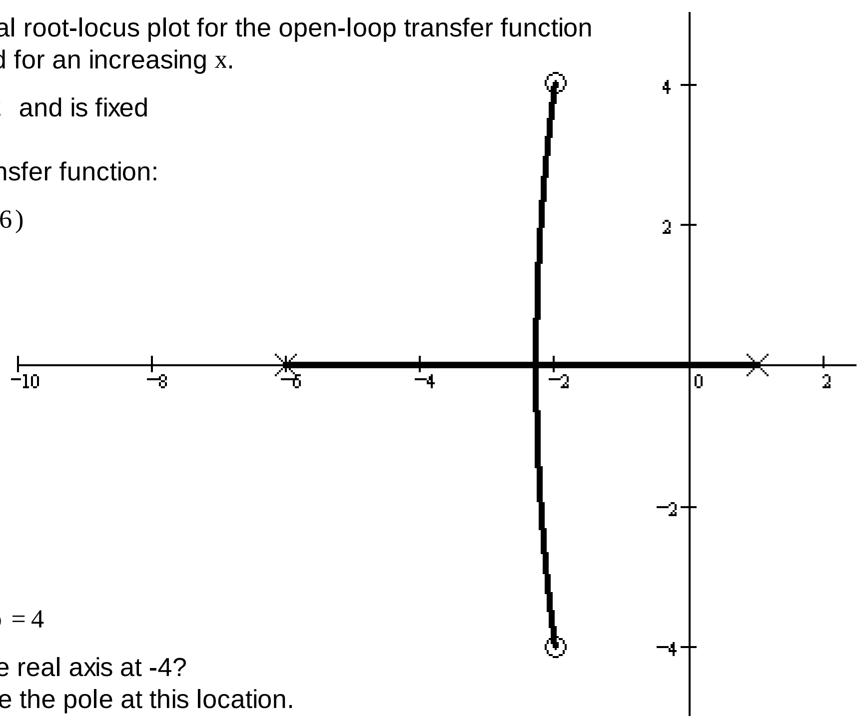
Ex.4 From E3, S13 Sketch the unconventional root-locus plot for the open-loop transfer function below. The root-locus should be plotted for an increasing x.

$$G(s) = \frac{k\cdot(5\cdot(s+2\cdot x)-6)}{s\cdot(x\cdot s+2\cdot(s+2\cdot x))} \qquad k = 2 \text{ and is fixed}$$

The denominator of the closed-loop transfer function:

$$s\cdot(x\cdot s+2\cdot(s+2\cdot x))+2\cdot(5\cdot(s+2\cdot x)-6)$$
$$x\cdot s^2+2\cdot s^2+4\cdot x\cdot s+10\cdot(s+2\cdot x)-12$$
$$x\cdot s^2+2\cdot s^2+4\cdot x\cdot s+10\cdot s+20\cdot x-12$$
$$2\cdot s^2+10\cdot s-12+x\cdot s^2+4\cdot x\cdot s+20\cdot x$$
$$\underbrace{2\cdot\left(s^2+5\cdot s-6\right)}_{D'(s)}+\underbrace{x\cdot\left(s^2+4\cdot s+20\right)}_{x\cdot N'(s)}$$
$$2\cdot(s+6)\cdot(s-1)+x\cdot\left(s^2+4\cdot s+20\right)$$

$$a := -2$$
$$b := \sqrt{20-2^2} \qquad b = 4$$



b) Can you place a closed-loop pole on the real axis at -4?
If yes, find the value of x needed to place the pole at this location.
If no, indicate what you think the best point on the real axis is and find the value of x needed to place the pole at that location.

$$0 = 2\cdot(s+6)\cdot(s-1)+x\cdot\left(s^2+4\cdot s+20\right)$$

solve for $x = \dfrac{-(2\cdot(s+6)\cdot(s-1))}{\left(s^2+4\cdot s+20\right)}$ if $s := -4$ $x = \dfrac{-(2\cdot(s+6)\cdot(s-1))}{\left(s^2+4\cdot s+20\right)} = 1$