# ECE 3510 Root Locus Design Crib Sheet

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Using 2nd-order approximation: $\dfrac{N(s)}{(s+a)^2+b^2} = \dfrac{N(s)}{s^2+2\cdot a\cdot s+a^2+b^2} = \dfrac{N(s)}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n{}^2}$

$\omega_n{}^2 = a^2+b^2$ $\omega_n$ = natural frequency

$\zeta\cdot\omega_n = a$

$\zeta = \dfrac{a}{\omega_n} = \dfrac{a}{\sqrt{a^2+b^2}}$ = damping factor $\zeta = \sin\left(\operatorname{atan}\left(\dfrac{a}{b}\right)\right)$

Overshoot: $OS = e^{-\pi\cdot\frac{a}{b}}$ $\%OS = 100\%\cdot e^{-\pi\cdot\frac{a}{b}}$ $\dfrac{a}{b} = \dfrac{\ln(OS)}{-\pi}$

angle of constant damping line: $90\cdot\deg+\operatorname{atan}\left(\dfrac{a}{b}\right)$

2% settling time: $T_s = \dfrac{4}{a} = \dfrac{4}{\zeta\cdot\omega_n}$

Time of first peak: $T_p = \dfrac{\pi}{b}$

Static error constant (position): $K_p = \lim_{s\to 0} K\cdot C(s)\cdot G(s)$ $e_{step}(\infty) = e_{step} = \dfrac{1}{1+K_p}$

Lag compensation improves $K_p$, $K_v$ and $K_a$ by $K_{pc} \simeq K_{puc}\cdot\dfrac{z_c}{p_c}$

Searching along a line of constant damping:

Try s values, choosing b: $s = -\left(\dfrac{a}{b}\cdot b\right)+b\cdot j$ Test: $\arg(G(s))$ $\pm 180^o$ or $\operatorname{Re}(G(s)) >> \operatorname{Im}(G(s))$

Linear interpolation: new b = $b_1 - \dfrac{b_2-b_1}{\operatorname{Im}(G(s_2))-\operatorname{Im}(G(s_1))}\cdot\operatorname{Im}(G(s_1))$

Can also try a values with slight modification.

Weird forms from Nise book:

$\sigma_d = a$ $\%OS = 100\%\cdot e^{\frac{-\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}$

$\omega_d = b$ $\zeta = \dfrac{-\ln(OS)}{\sqrt{\pi^2+(\ln(OS))^2}}$ $T_p = \dfrac{\pi}{\omega_n\cdot\sqrt{1-\zeta^2}}$

Static error constant (ramp): (velocity) $K_v = \lim_{s\to 0} s\cdot K\cdot C(s)\cdot G(s)$ $e_{ramp} = \dfrac{1}{K_v}$

Static error constant (parabola): (acceleration) $K_a = \lim_{s\to 0} s^2\cdot K\cdot C(s)\cdot G(s)$ $e_{parabola} = \dfrac{1}{K_a}$