ECE 3510

Implementation (Physical Realization) of Feedback System Components and Compensators

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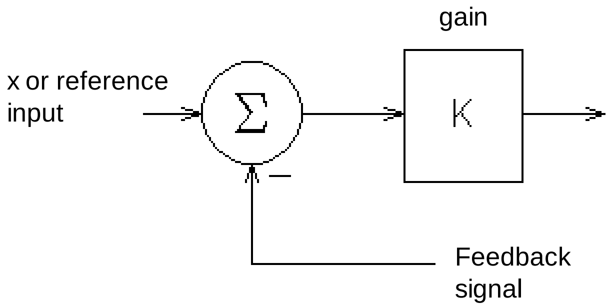
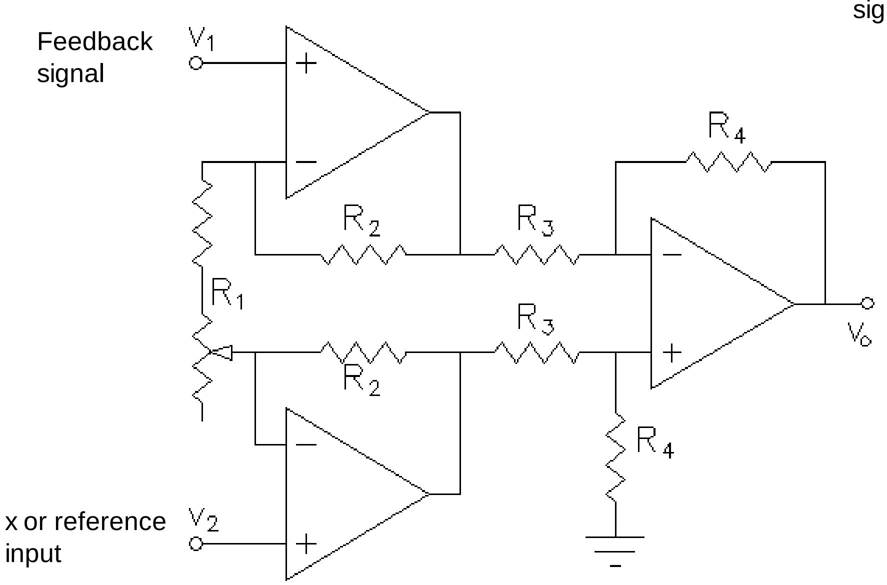
2/24/09,
rev,

One way to implement this:



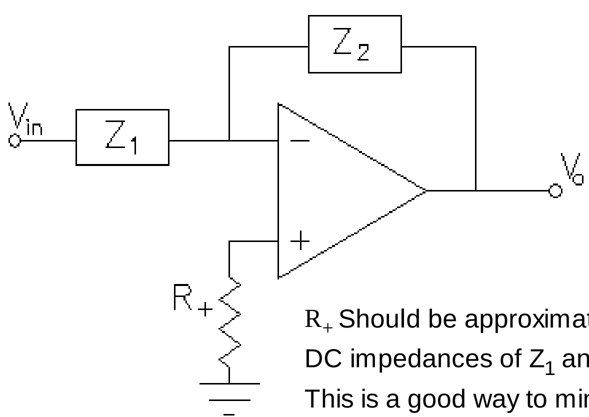
Is the **instrumentation amplifier**:



$$V_o = \left(1 + 2\cdot\frac{R_2}{R_1}\right)\cdot\frac{R_4}{R_3}\cdot\left(V_2 - V_1\right)$$

$$K = \left(1 + 2\cdot\frac{R_2}{R_1}\right)\cdot\frac{R_4}{R_3}$$

To build **active compensators**, use this basic circuit and then consult Table 9.10 (p.555 in 3rd ed. p.504 in 6th) in the Nise textbook.



R_+ Should be approximately the parallel combination of the DC impedances of Z_1 and Z_2.
This is a good way to minimize the effects of the Op-amp's bias currents.

Beware! This is an inverting circuit. You will have to follow it with another inverter.

Or... you could just swap the inputs to the instrumentation amplifier, if you are using one.

The resistors used in Op-amp circuits should be 100Ω to 1MΩ, and preferably 1kΩ to 100kΩ.

These Op-amp circuits require + and - power supplies.

To build **passive compensators**, consult Table 9.11 (p.558 in 3rd ed. p.506 in 6th) in the Nise textbook.

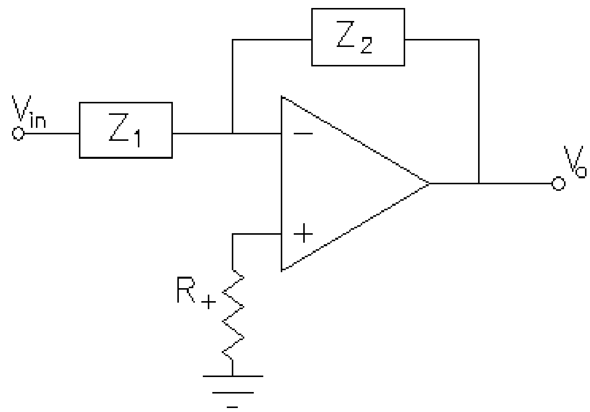
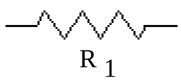
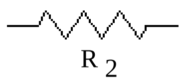
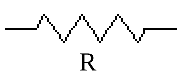
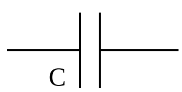
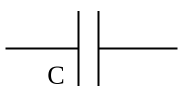
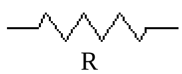
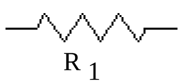
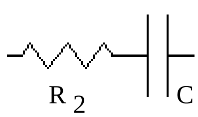
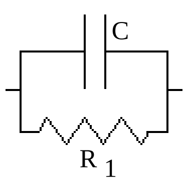
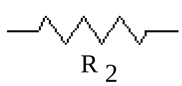
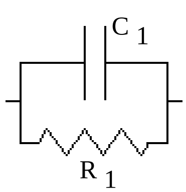
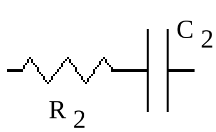
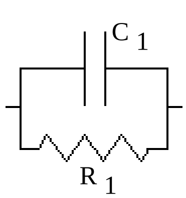
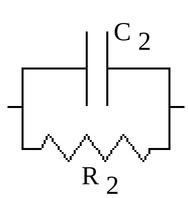


Table similar to table 9.10 (p.555 in 3rd ed. p.504 in 6th) in the Nise textbook.

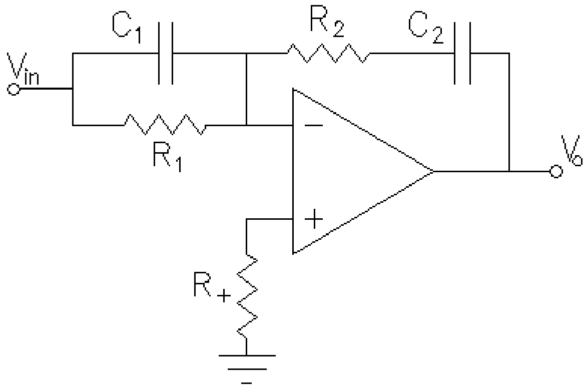
Function	Z_1	Z_2	C(s)
Amplifier (Proportional Gain)	 R_1	 R_2	$k_p = -\frac{R_1}{R_2}$
Integrator	 R	 C	$\frac{k_i}{s} = -\frac{1}{R\cdot C\cdot s} = -\frac{1}{R\cdot C}\cdot\frac{1}{s}$
Differentiator	 C	 R	$k_d\cdot s = -R\cdot C\cdot s$
PI Controller	 R_1	 R_2, C	$k_p + \frac{k_i}{s} = k_p\cdot\frac{\left(s+\frac{k_i}{k_p}\right)}{s} = -\frac{R_1}{R_2}\cdot\frac{\left(s+\frac{1}{R_2\cdot C}\right)}{s}$
PD Controller	 C, R_1	 R_2	$k_p + k_d\cdot s = k_p\cdot\left(1+\frac{k_d}{k_p}\cdot s\right) = -R_2\cdot C$
PID Controller	 C_1, R_1	 R_2, C_2	$k_p+\frac{k_i}{s}+s\cdot k_d = -\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+\frac{\left(\frac{1}{R_1\cdot C_2}\right)}{s}+R_2\cdot C_1\cdot s\right]$ $\frac{s^2\cdot k_d+s\cdot k_p+k_i}{s} = -\frac{R_2\cdot C_1\cdot s^2+\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)\cdot s+\frac{1}{R_1\cdot C_2}}{s}$
Lag or Lead Compensator	 C_1, R_1	 C_2, R_2	$-\left[\frac{C_1}{C_2}\cdot\frac{\left(s+\frac{1}{R_1\cdot C_1}\right)}{\left(s+\frac{1}{R_2\cdot C_2}\right)}\right]$ Lag: $R_1\cdot C_1 < R_2\cdot C_2$ Lead: $R_1\cdot C_1 > R_2\cdot C_2$

$$\mathbf{C}(s) = 0.418\cdot\frac{(s+0.1)\cdot(s+24.28)}{s} = 0.418\cdot\frac{s^2+24.38\cdot s+2.48}{s} = \frac{0.418\cdot s^2+10.19\cdot s+1.037}{s}$$

$$= \frac{k_d\cdot s^2+k_p\cdot s+k_i}{s}$$

$$k_d := 0.418\cdot sec \qquad k_p := 10.19 \qquad k_i := \frac{1.016}{sec}$$

Using the PID design from table 9.10 (p.555 in 3rd ed. p.504 in 6th) in Nise:
This could be implemented with:



$$C(s) = -\left[\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)+R_2\cdot C_1\cdot s+\frac{\left(\frac{1}{R_1\cdot C_2}\right)}{s}\right]$$

$$= -\frac{R_2\cdot C_1\cdot s^2+\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right)\cdot s+\frac{1}{R_1\cdot C_2}}{s}$$

If we use an instrumentation amplifier with a gain of, say 3, and invert the two inputs to "fix" the inversion above, then:

$$R_2\cdot C_1 = \frac{k_d}{3} = 0.139\cdot sec \qquad \left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right) = \frac{10.19}{3} = 3.397 \qquad \frac{1}{R_1\cdot C_2} = \frac{k_i}{3} = 0.339\cdot sec^{-1}$$

There are 4 component values to select and only 3 coefficients to match, so arbitrarily select 1 component.

Try $C_1 := 0.1\cdot\mu F$ $\quad R_2 := \frac{0.139\cdot sec}{C_1}$ $\quad R_2 = 1.39\cdot M\Omega$ too high

Try $C_1 := 10\cdot\mu F$ $\quad R_2 := \frac{0.139\cdot sec}{C_1}$ $\quad R_2 = 13.9\cdot k\Omega$ Use $R_2 := 14\cdot k\Omega$

Now $\left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right) = 3.397$ And $\frac{1}{R_1\cdot C_2} = \frac{0.339}{sec}$ So, $C_2 = \frac{sec}{0.339\cdot R_1}$

Combining: $= \frac{14\cdot k\Omega}{R_1}+\frac{10\cdot\mu F\cdot(0.339\cdot R_1)}{sec} - 3.397 = 0$

Solve: $R_1 := \frac{3.397+\sqrt{3.397^2-4\cdot\frac{C_1\cdot 0.339\cdot R_2}{sec}}}{2\cdot\frac{C_1\cdot 0.339}{sec}}$ $\quad R_1 = 997.927\cdot k\Omega$ Use $R_1 := 1\cdot M\Omega$

Let $R_+ = 1M\Omega$

$C_2 := \frac{sec}{0.339\cdot R_1}$ $\quad C_2 = 2.95\cdot\mu F$ Use $C_2 := 3\cdot\mu F$

Test: $R_2\cdot C_1 = 0.14\cdot sec$ $\quad \left(\frac{R_2}{R_1}+\frac{C_1}{C_2}\right) = 3.347$ $\quad \frac{1}{R_1\cdot C_2} = 0.333\cdot sec^{-1}$ Close enough

PID Design Example Implementation p.2

Instrumentation amp gain: $K_{inst} := 3$ $\quad k_d := 0.418 \cdot sec$ $\quad k_p := 10.19$ $\quad k_i := \frac{1.016}{sec}$

$R_2 \cdot C_1 = k'_d := \frac{k_d}{3}$ $\quad \left(\frac{R_2}{R_1} + \frac{C_1}{C_2}\right) = k'_p := \frac{10.19}{3}$ $\quad \frac{1}{R_1 \cdot C_2} = k'_i := \frac{k_i}{3}$

$k'_d = 0.139 \cdot sec$ $\quad k'_p = 3.397$ $\quad k'_i = 0.339 \cdot sec^{-1}$

For standard capacitor values from $C_{1_0} = 0.01 \cdot \mu F$ to $C_{1_{32}} = 82 \cdot \mu F$ $\quad R_{2_i} := \frac{k'_d}{C_{1_i}}$

Combining equations above

$$\left(\frac{R_2}{R_1} + \frac{C_1}{C_2}\right) = k'_p = \left[\frac{R_2}{R_1} + \frac{C_1}{\left(\frac{1}{k'_i \cdot R_1}\right)}\right] = \left[\frac{R_2}{R_1} + C_1 \cdot (k'_i \cdot R_1)\right] \quad \text{OR} \quad R_2 + C_1 \cdot k'_i \cdot R_1^2 - k_p \cdot R_1 = 0$$

Rearrange: $C_1 \cdot k'_i \cdot R_1^2 - k'_p \cdot R_1 + R_2 = 0$ $\quad$ And solve: $R_{1_i} := \frac{k'_p + \sqrt{k'_p{}^2 - 4 \cdot \left(C_{1_i} \cdot k'_i \cdot R_{2_i}\right)}}{2 \cdot \left(C_{1_i} \cdot k'_i\right)}$

Finally: $C_{2_i} := \frac{1}{k'_i \cdot R_{1_i}}$

Possible solutions

$\frac{C_{1_i}}{\mu F}$	$\frac{R_{2_i}}{k\Omega}$	$\frac{R_{1_i}}{k\Omega}$	$\frac{C_{2_i}}{\mu F}$
0.12	1161.11	83236	0.035
0.15	928.89	66589	0.044
0.18	774.07	55491	0.053
0.22	633.33	45402	0.065
0.33	422.22	30268	0.098
0.39	357.26	25611	0.115
0.47	296.45	21252	0.139
0.56	248.81	17836	0.166
0.68	204.9	14689	0.201
0.82	169.92	12181	0.242
1	139.33	9988	0.296
1.2	116.11	8324	0.355
1.5	92.89	6659	0.443
1.8	77.41	5549	0.532
2.2	63.33	4540	0.65
3.3	42.22	3027	0.976
3.9	35.73	2561	1.153
4.7	29.65	2125	1.389
5.6	24.88	1784	1.655
6.8	20.49	1469	2.01
8.2	16.99	1218	2.424
10	13.93	999	2.956
12	11.61	832	3.547
15	9.29	666	4.434
18	7.74	555	5.321
22	6.33	454	6.504
33	4.22	303	9.755
39	3.57	256	11.529
47	2.96	213	13.894
56	2.49	178	16.555
68	2.05	147	20.102
82	1.7	122	24.241