ECE 3510 Nyquist Plot Notes

A. Stolp
4/9/09,
4/3/21

A Nyquist plot is essentially a polar Bode plot. Like a Bode plot, it is plotted for the Open-Loop (OL) Transfer function and will give information about the stability of the Closed-Loop (CL) system.

Open-Loop (OL) Transfer function: $\mathbf{G}(s) = \dfrac{N_G(s)}{D_G(s)}$ m = number of zeros, n = number of poles

Basic Nyquist Rules

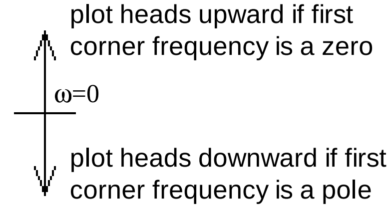
1. "Clean up" any "$-s$" terms in $\mathbf{G}(s)$ by multiplying by -1 as needed.
   If an overall "-" remains in $\mathbf{G}(s)$, then add 180^o to all the angles below. (rare)

2. Start at $\mathbf{G}(0)$, the DC gain, a point on the real axis.

   If $\mathbf{G}(s)$ has a zero at the origin: $\mathbf{G}(0) = 0$

   If $\mathbf{G}(s)$ has a pole at the origin: $\mathbf{G}(0) = \pm\infty$

   If $\mathbf{G}(s)$ has no poles or zeros in the right-half plane then:

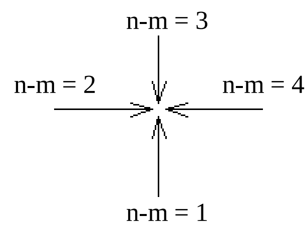


3. End at $\mathbf{G}(\infty)$.

   $n < m$ (rare) Plot ---> ∞, almost always $+\infty$

   $n = m$ Plot ---> $\mathbf{G}(\infty)$, a point on the real axis

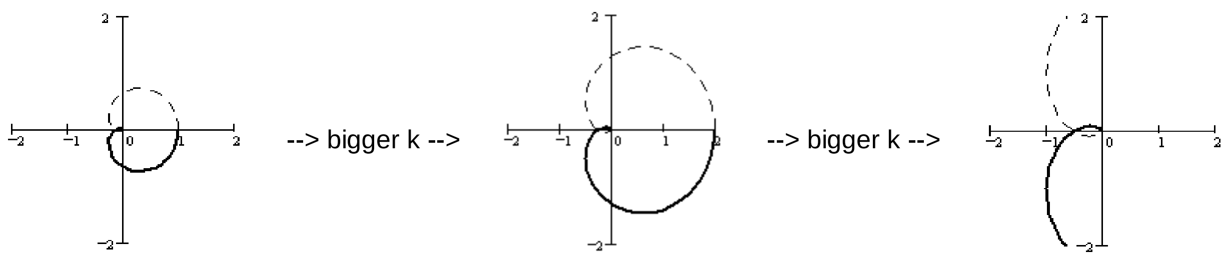
   $n > m$ (most common) Plot ---> 0 Angle of approach to origin = $(n - m)\cdot(-90\cdot deg)$



4. Plot the rest of the frequency response of $\mathbf{G}(j\omega)$. Use Bode plot to gude you.

5. Add the $\omega < 0$ curve (dashed line). It is simply the mirror image of the $\omega > 0$ curve about the real axis.
   This part of the curve is usually not necessary, it doesn't provide any more information.

6. Gain, k, makes entire plot grow in all directions (or shrink if $k<1$).



7. $Z = N + P$

   P = OL ($\mathbf{G}(s)$) poles in RHP (0 if open-loop stable). P cannot be -

   N = CW encirclements of -1, CCW encirclements are counted as negative and may make up for P.

   Z = CL poles in RHP (must be zero if closed-loop stable). Z cannot be -

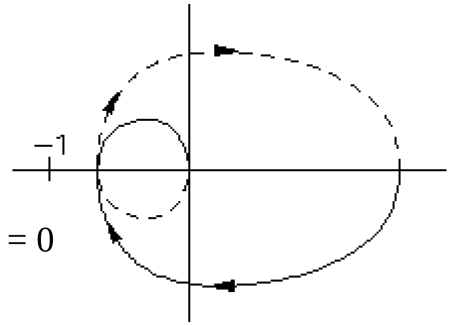
8. ANY CW encirclements means Closed-Loop system is UNSTABLE

   $\mathbf{N} > 0$ --> CL unstable (P cannot be -)

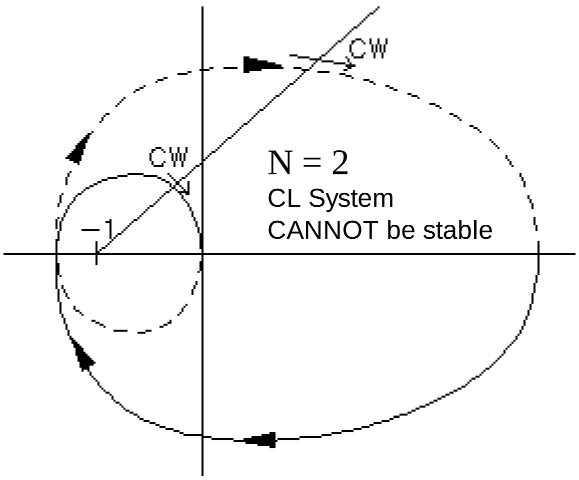
ECE 3510 Nyquist Plot Notes p.2

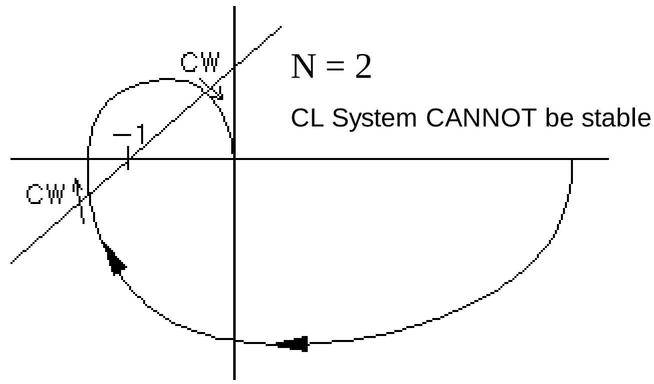
Counting Clockwise Encirclements

N = CW encirclements of -1,
CCW encirclements are counted as negative and may make up for P.



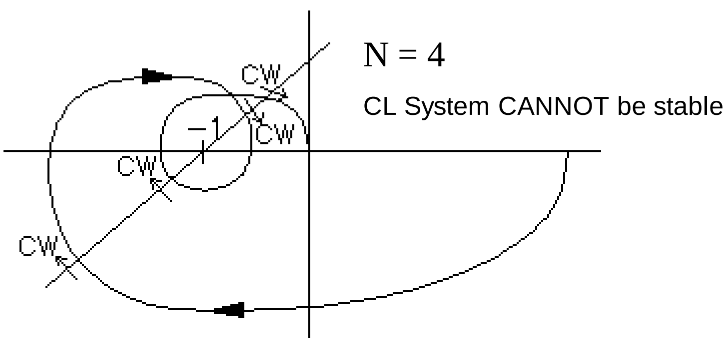
N = 0

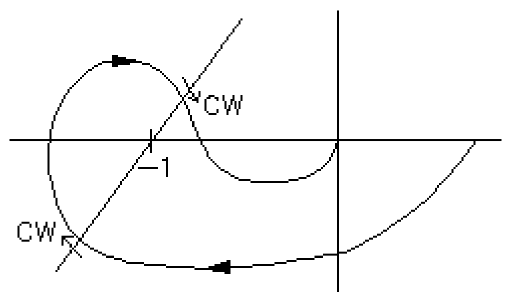




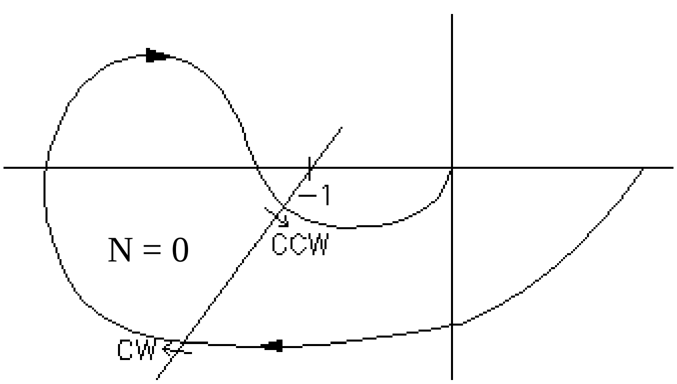
If you have the $\omega < 0$ curve (dashed line), then you can use any single-ended line that starts at -1 to help you count encirclements.

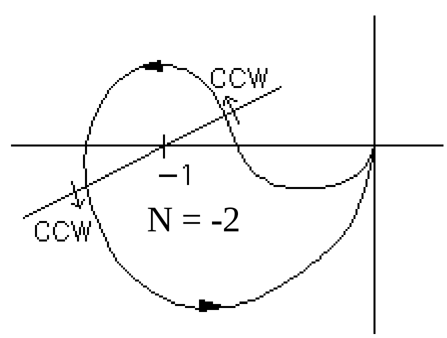
If you don't have the $\omega < 0$ curve (dashed line), then make your line extend both directions from -1.





N = 2 CL System CANNOT be stable





CCW encirclements are counted as negative.

CL System CAN be stable, if $P \le 2$
-N can make up for +P. and stabilize an OL unstable system

$$Z = N + P$$

P = OL poles in RHP (0 if open-loop stable)

N = CW encirclements of -1. CL System CANNOT be stable if $\mathbf{N > 0}$

Z = CL poles in RHP (must be zero if closed-loop stable)

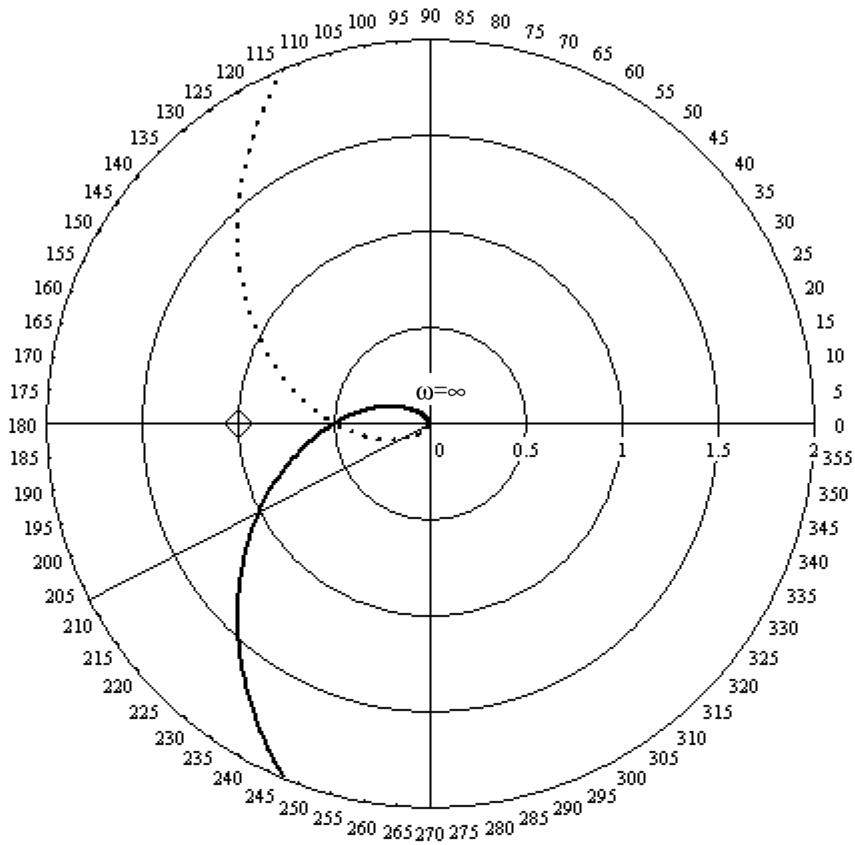
Gain Margin (GM) and Phase Margin (PM)

To find the Phase Margin (PM):

1. Find where the Nyquist plot crosses the unit circle. These crossings separate the unit circle into regions.
2. Decide which of these regions have unacceptable CW encirclements.
3. Determine what phase change would cause the -1 point to be an unacceptable region, usually 180^o - <u>/ crossing</u>

$$GM = \frac{1}{\left(\frac{1}{2}\right)} = 2$$

$$PM = 207 - 180 = 27 \text{ deg}$$



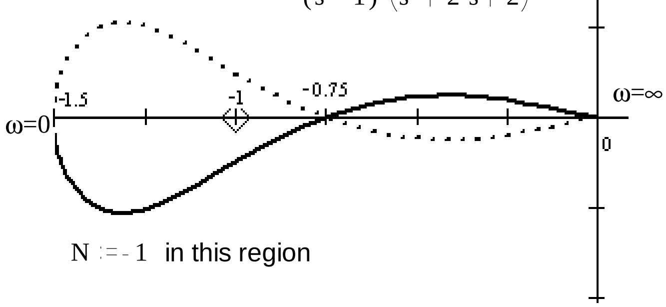
To find the Gain Margin (GM):

1. Find where the Nyquist plot crosses the negative real axis. These crossings separate the negative real axis into regions.
2. Decide which of these regions have unacceptable CW encirclements.
3. Determine what gain would cause the -1 point to be an unacceptable region, usually $\frac{1}{-\text{crossing}}$ into the unacceptable region.
4. Usually there is just one upper limit of gain-- in that case report that as the Gain Margin.
5. If there is a lower limit of gain, report the Gain Margin as: $GM = \left[\text{Lower limit , upper limit}\right]$

If there is no upper limit, then report it as ∞

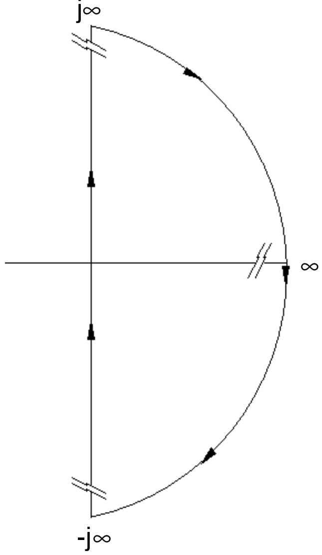
$$\mathbf{G}(s) := \frac{1.5\cdot(s+2)}{(s-1)\cdot\left(s^2+2\cdot s+2\right)}$$

$P := 1$ For CL stability, $N := -1$ or more

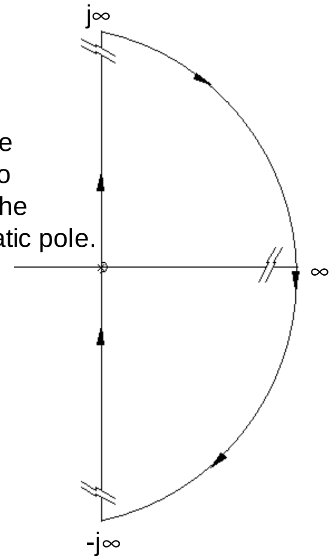


$$GM = \left[\frac{1}{1.5}, \frac{1}{0.75}\right] = \left[0.667, 1.333\right]$$

The normal contour



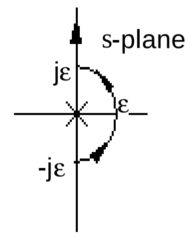
A pole on the imaginary axis causes a problem.
Is it inside or outside of the contour?



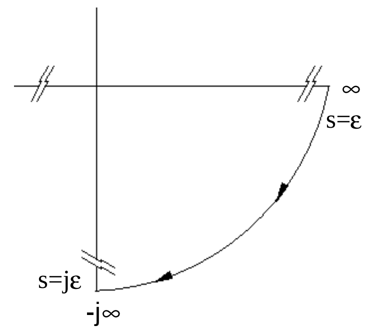
Modify the contour to exclude the problematic pole.

A **single pole** at the origin

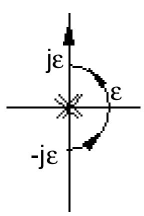
A closer look



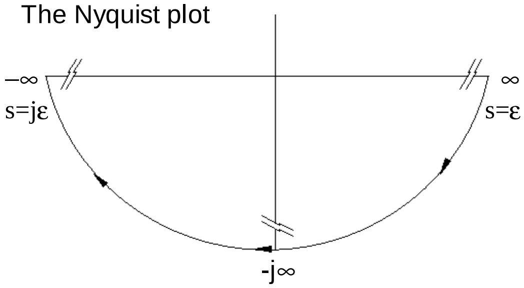
The Nyquist plot of this



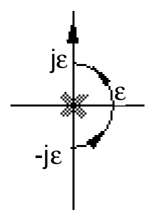
A **double pole** at the origin



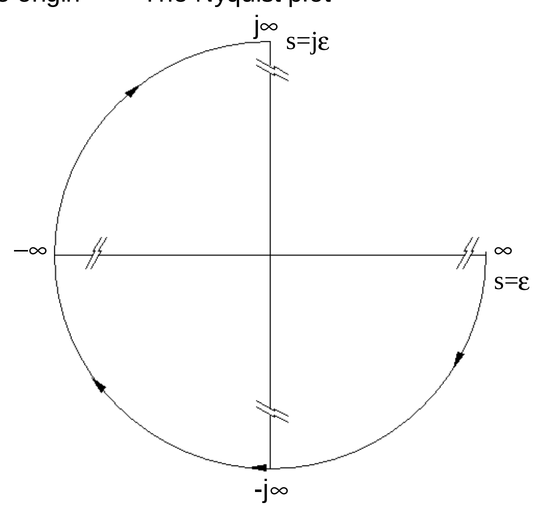
The Nyquist plot



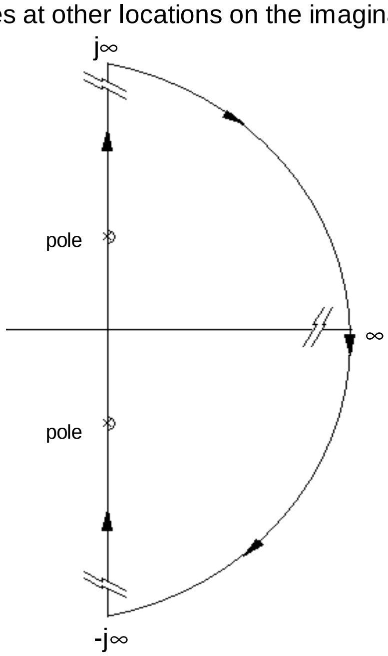
A **triple pole** at the origin



The Nyquist plot



Poles at other locations on the imaginary axis



Possible Nyquist plots