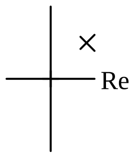
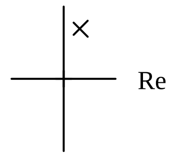
ECE 3510 How Nyquist Plots Work see pages 615 and 616 in Nise

For any point on the s-plane

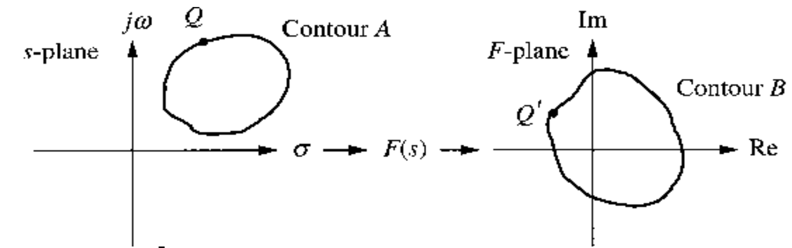


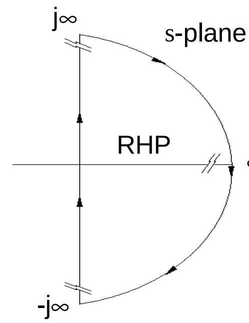
You can find a corresponding point on the **F**(s) plane



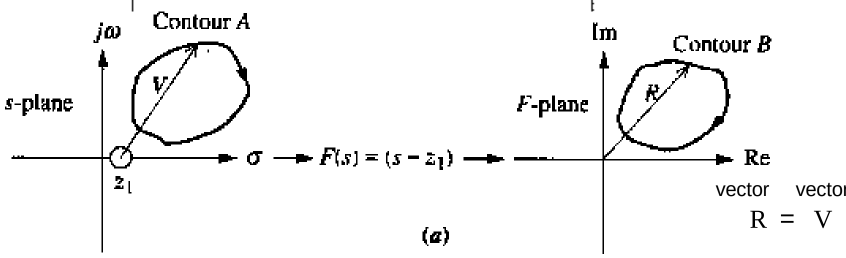
For any contour on the s-plane

There is a corresponding contour on the **F**(s) plane





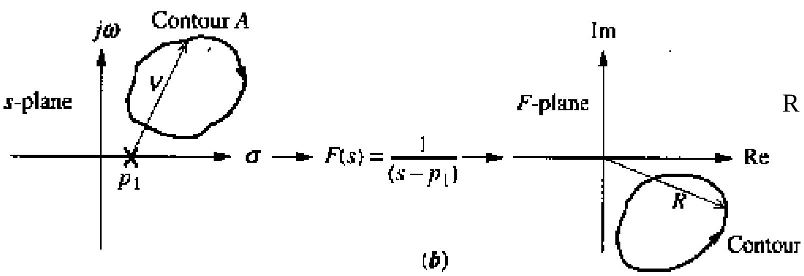
When we make a Nyquist plot for frequencies from o to ∞, and then from -∞ to 0 again, we are essentially making a contour that circles the entire RHP



vector vector

R = V

This gives information about poles and zeros in the RHP. This information reveals itself in the encirclements of 0, OR if we want the same information about 1 + **G**(s), in the encirclements of -1.



$R = \frac{1}{V}$

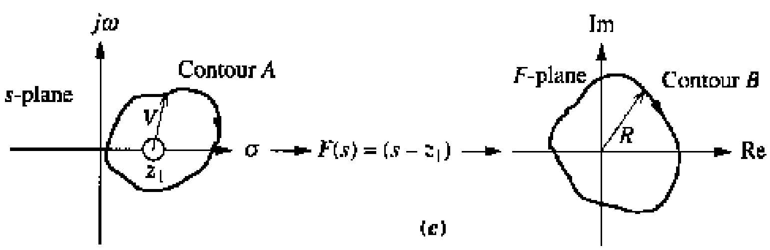
Closed-loop transfer function

(**G**(s) includes gain, k)

$$\frac{\mathbf{G}(s)}{1+\mathbf{G}(s)} = \frac{\mathbf{G}(s)}{\left(\frac{D_G + N_G}{D_G}\right)}$$

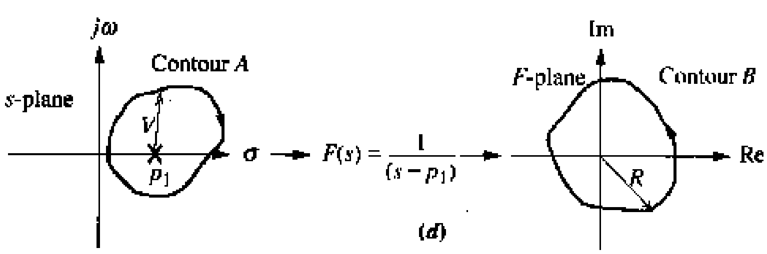
poles of CL = zeros of $1+\mathbf{G}(s)$ $= Z$ (RHP)

poles of $1+\mathbf{G}(s)$ = poles of $\mathbf{G}(s)$ $= P$ (RHP)



R = V

We plot **G**(s), but count CW encirclements of -1, rather than 0 to get the CW encirclements that 1 + **G**(s) would have of 0, Thus.



$R = \frac{1}{V}$

N = CW encirclements of -1

= Z - P

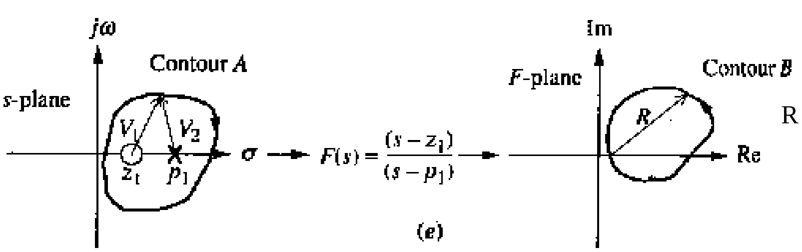
rearrange

$$Z = N + P$$

P = OL poles in RHP

(0 if open-loop stable)

(**P** cannot be negative)



$R = \frac{V_1}{V_2}$

N = CW encirclements of -1

CL System CANNOT be stable if **N** > **0**

Z = CL poles in RHP

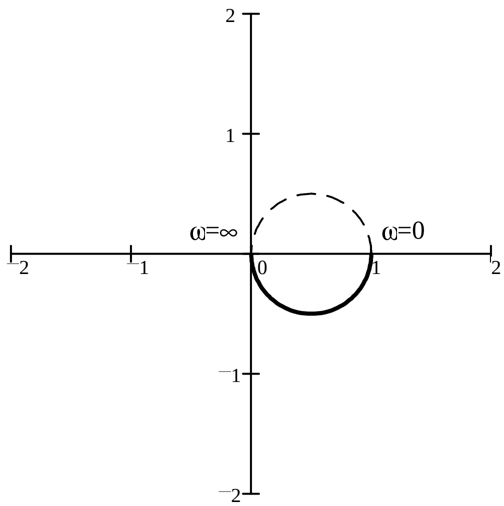
(must be zero (or ≤ 0) if closed-loop stable)

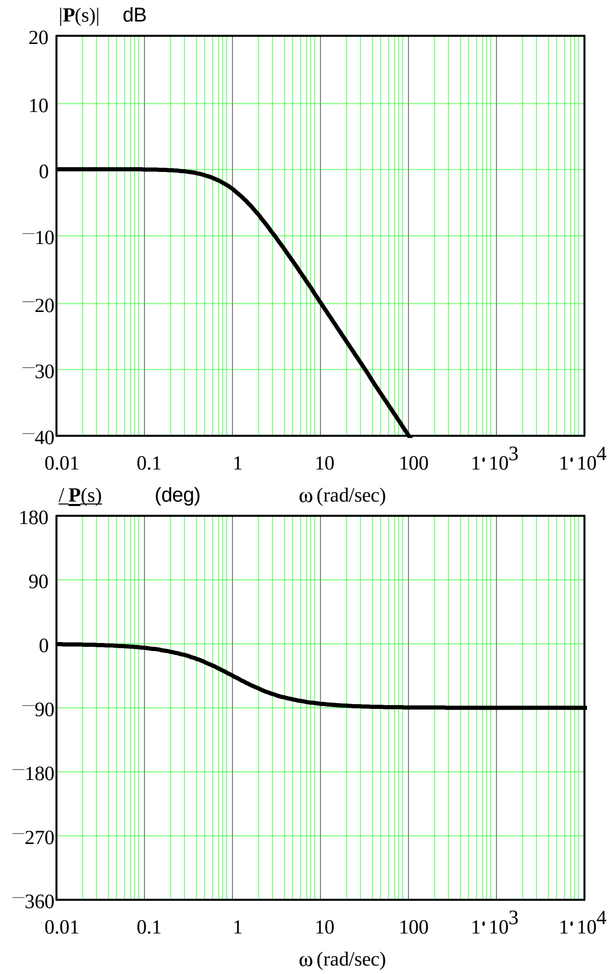
ECE 3510 Nyquist Examples

Example 1, Bodson, section 5.2 $k := 1$

$$\mathbf{P}(s) := \frac{1}{(s+1)}$$

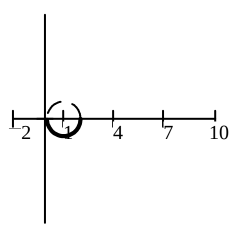
$$\frac{1}{j\cdot\omega + 1} = \frac{1}{\sqrt{\omega^2 + 1^2}} \quad -\operatorname{atan}\left(\frac{\omega}{1}\right)$$



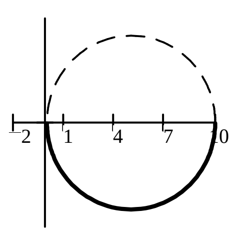


$$\mathbf{G}(s) = k\cdot\mathbf{P}(s) = k\cdot\frac{1}{(s+1)}$$

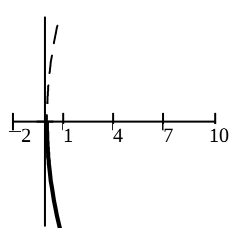
$k := 2$



$k := 10$



$k := 50$



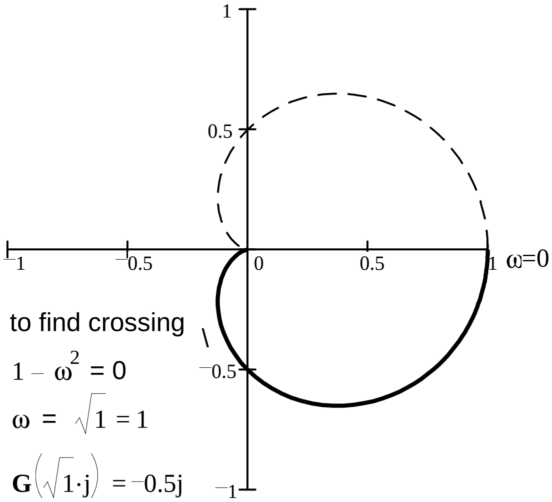
This plot never circles -1, no matter what the gain, so N = 0, no matter what k is.

ECE 3510 Nyquist Examples p.2

$k := 1$

$$\mathbf{G}(s) := \frac{k}{(s+1)^2} = \frac{1}{s^2+2\cdot s+1} = \frac{1}{(j\cdot\omega)^2+2\cdot(j\cdot\omega)+1} = \frac{1}{\left(1-\omega^2\right)+2\cdot j\cdot\omega}\cdot\left[\frac{\left(1-\omega^2\right)-2\cdot j\cdot\omega}{\left(1-\omega^2\right)-2\cdot j\cdot\omega}\right] = \frac{\left(1-\omega^2\right)-2\cdot j\cdot\omega}{\left(1-\omega^2\right)^2+(2\cdot j\cdot\omega)^2}$$

$$\frac{1}{(j\cdot\omega+1)^2} = \frac{1}{\omega^2+1^2} \qquad -2\cdot\text{atan}\left(\frac{\omega}{1}\right)$$

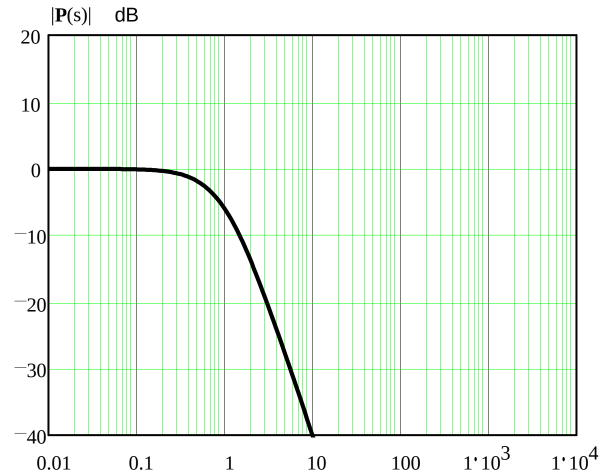


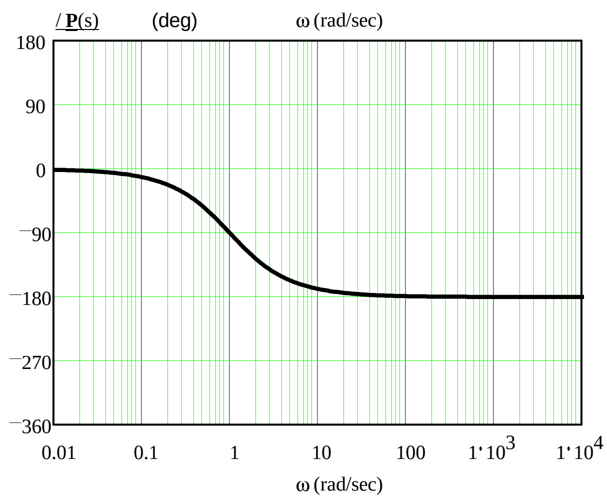
to find crossing

$1-\omega^2 = 0$

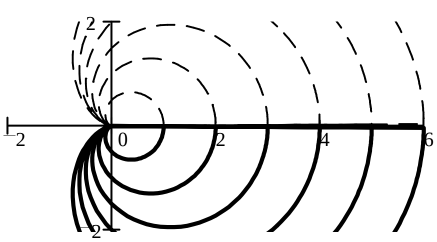
$\omega = \sqrt{1} = 1$

$\mathbf{G}\left(\sqrt{1}\cdot j\right) = -0.5j$



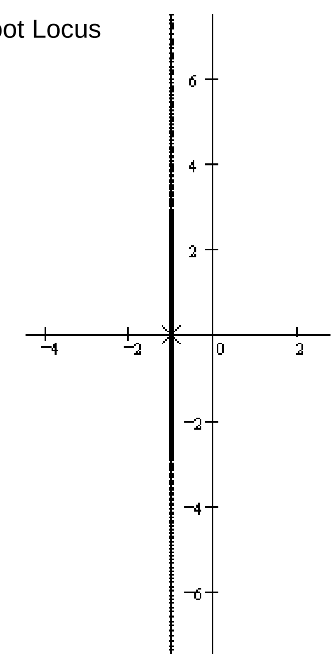


$$\mathbf{G}(s) = k\cdot\mathbf{P}(s) = k\cdot\frac{1}{(s+1)^2}$$

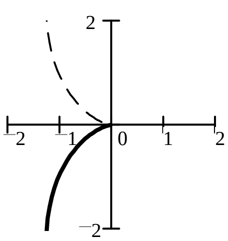


bigger gain -->

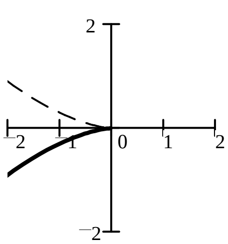
Root Locus



$k := 10$



$k := 50$



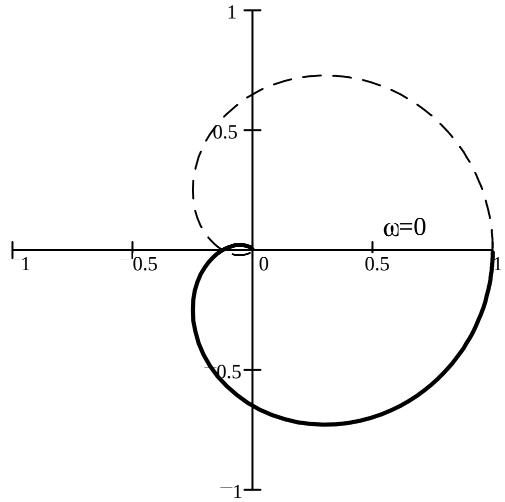
N = 0, no matter what k is

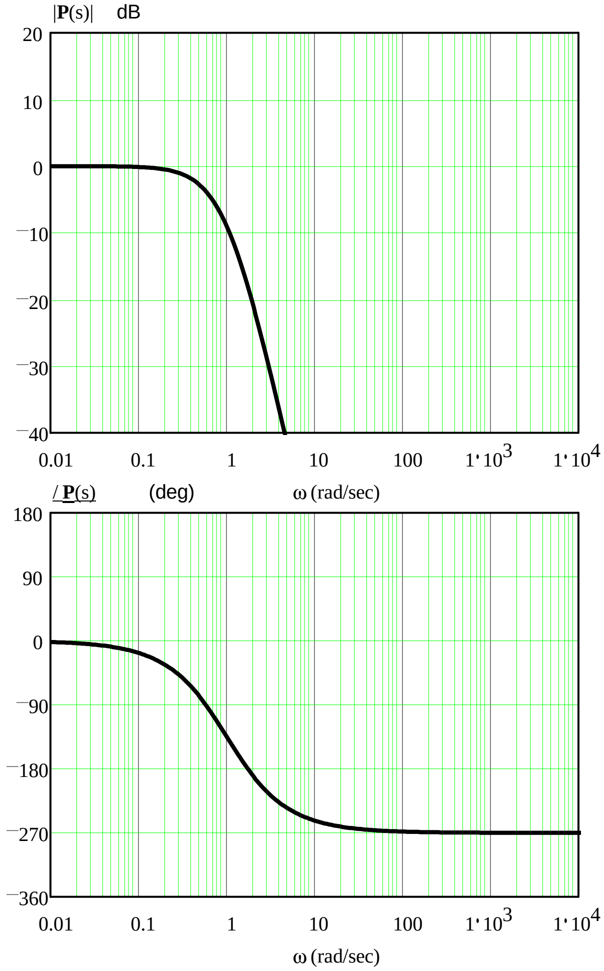
But ringing gets worse with k

Example 2, Bodson, section 5.2.

$$\mathbf{P}(s) := \frac{1}{(s+1)^3}$$

$$\frac{1}{(j\cdot\omega+1)^3} = \frac{1}{\left(\sqrt{\omega^2+1^2}\right)^3} \qquad -3\cdot \mathrm{atan}\left(\frac{\omega}{1}\right)$$





Find real-axis crossing (in left half plane)

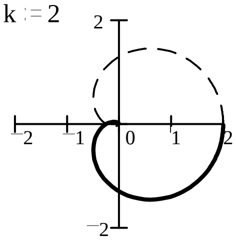
$$\frac{180}{3} = 60 \ \mathrm{deg} \qquad \tan(60\cdot \mathrm{deg}) = \sqrt{3} = \frac{\omega}{1} = \omega$$

$$\frac{1}{\left(\sqrt{\omega^2+1^2}\right)^3} = \frac{1}{\left[\sqrt{\left(\sqrt{3}\right)^2+1^2}\right]^3} = \frac{1}{8}$$

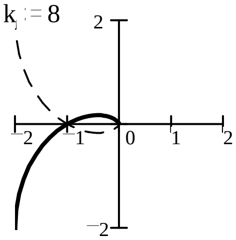
$$\mathbf{G}(s) = k\cdot\mathbf{P}(s)$$

$$= k\cdot\frac{1}{(s+1)^3}$$

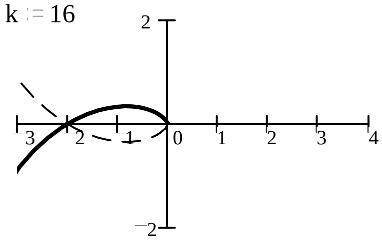
$k := 2$



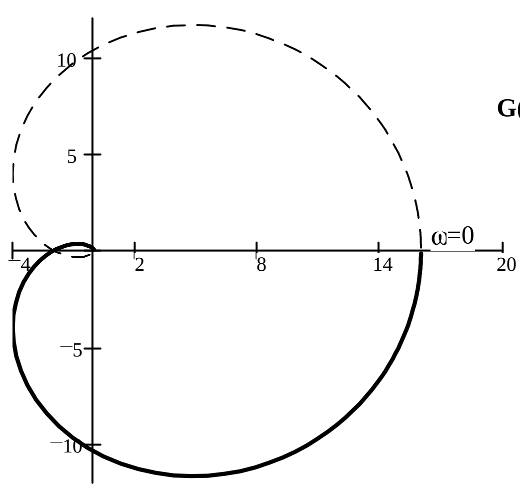
$k := 8$



$k := 16$



$k := 16$



$$\mathbf{G}(s) = k\cdot\frac{1}{(s+1)^3}$$

$P := 0$

$N := 2$

$Z := N + P \qquad Z = 2$ 2 closed-loop poles in RHP

Closed-loop UNSTABLE

Gain Margins

For: $\mathbf{G}(s) = \frac{1}{(s+1)^3}$ Real-axis crossing (in left half plane) is at $1/8$

Gain can be 8 times bigger before $N = 2$ $GM := 8$

For: $\mathbf{G}(s) = \frac{2}{(s+1)^3}$ $GM := 4$

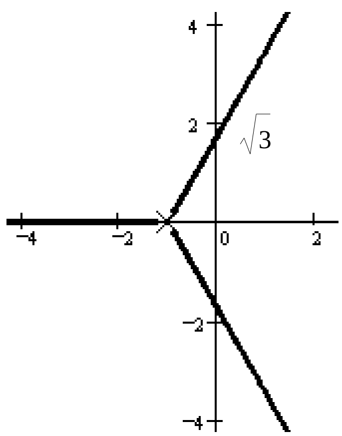
For: $\mathbf{G}(s) = \frac{4}{(s+1)^3}$ $GM := 2$

For: $\mathbf{G}(s) = \frac{20}{(s+1)^3}$ $GM := \frac{8}{20} = \frac{2}{5}$

Other ways to find the same maximum gain

$G(s) = \frac{1}{(s+1)^3}$

Root Locus



By Routh-Hurwitz:

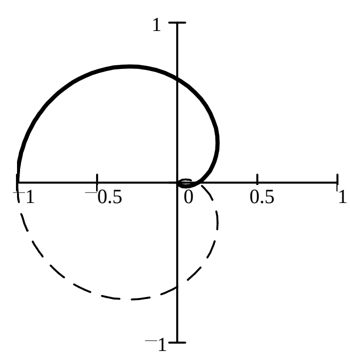
Closed-loop denominator

$(s+1)^3 + k = s^3 + 3 \cdot s^2 + 3 \cdot s + 1 + k$

s^3	1	3	
s^2	3	$1+k$	
s^1	$\frac{9 - 1 \cdot (1+k)}{3}$	$k < 8$	same result as previous page
s^0	$1+k$	$k > -1$	

Nyquist plot for negative k

(plot rotates 180^o around the origin)



$N = 2$ for any k more negative than -1

Example 10.4, Nise 3 p.620.

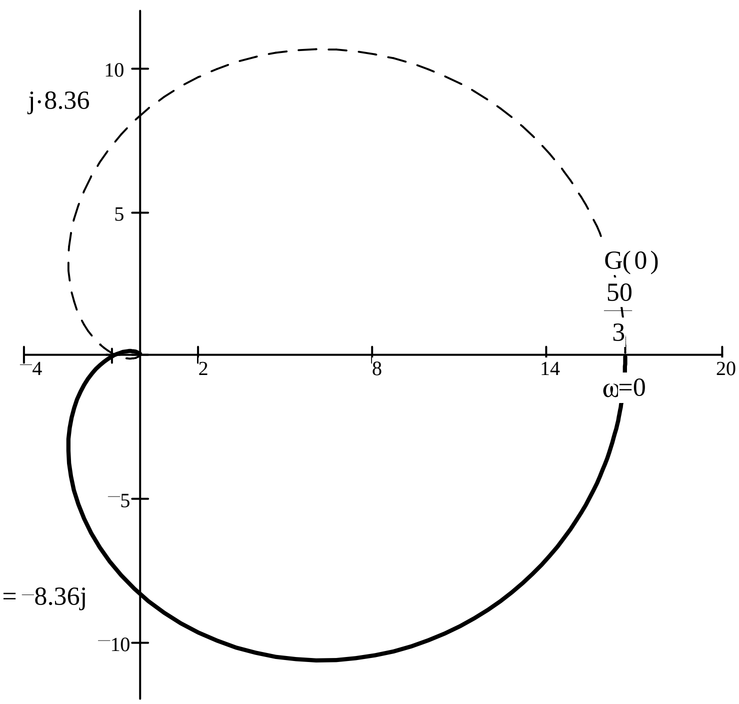
$$\mathbf{G}(s) := \frac{100}{(s+10)} \cdot \frac{1}{(s+3)} \cdot \frac{5}{(s+1)} = \frac{500}{(s+10)\cdot(s+3)\cdot(s+1)}$$

$$\mathbf{G}(0) = \frac{500}{10\cdot3\cdot1} = \frac{50}{3}$$

$$\mathbf{G}(\omega) = \frac{500}{(j\cdot\omega+10)\cdot(j\cdot\omega+3)\cdot(j\cdot\omega+1)} = \frac{500}{\left[(-1)\cdot\omega^2+13\cdot j\cdot\omega+30\right]\cdot(j\cdot\omega+1)} = \frac{500}{-\omega^3\cdot j-\omega^2+13\cdot(-1)\cdot\omega^2+43\cdot j\cdot\omega+30}$$

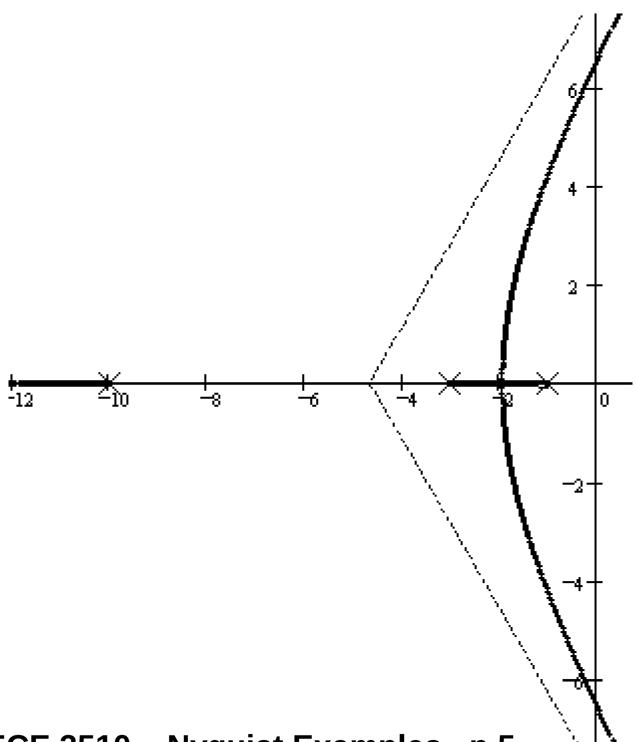
$$= \frac{500}{\left(-14\cdot\omega^2+30\right)+j\cdot\left(43\cdot\omega-\omega^3\right)} \cdot \frac{\left(-14\cdot\omega^2+30\right)-j\cdot\left(43\cdot\omega-\omega^3\right)}{\left(-14\cdot\omega^2+30\right)-j\cdot\left(43\cdot\omega-\omega^3\right)}$$

$$= \frac{\left(-14\cdot\omega^2+30\right)-j\cdot\left(43\cdot\omega-\omega^3\right)}{\left(-14\cdot\omega^2+30\right)^2+\left(43\cdot\omega-\omega^3\right)^2}$$



Real part goes to zero

$-14\cdot\omega^2+30 = 0$ $\quad \boldsymbol{\omega} := \sqrt{\frac{30}{14}}$ $\quad \mathbf{G}(j\cdot\boldsymbol{\omega}) = -8.36j$

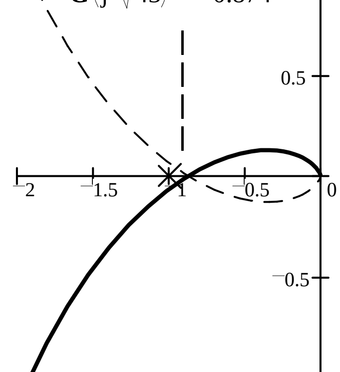


Imaginary part goes to zero

$43\cdot\boldsymbol{\omega}-\boldsymbol{\omega}^3 = 0$

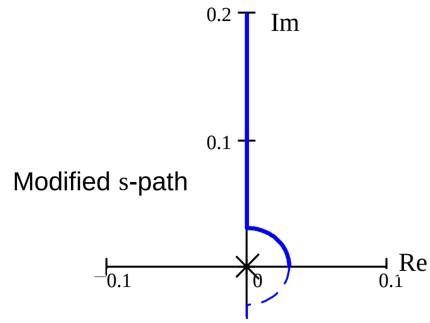
$\boldsymbol{\omega} := \sqrt{43}$

$\mathbf{G}\left(j\cdot\sqrt{43}\right) = -0.874$



$$GM = \frac{1}{0.874} = 1.144$$

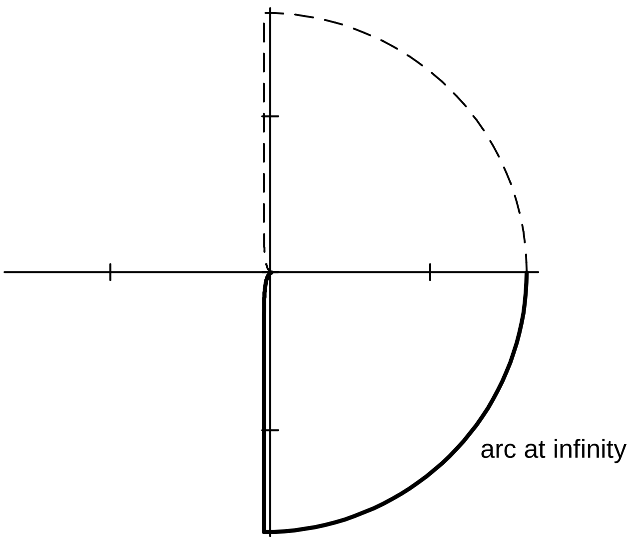
Examples of Poles on the imaginary (jω) axis

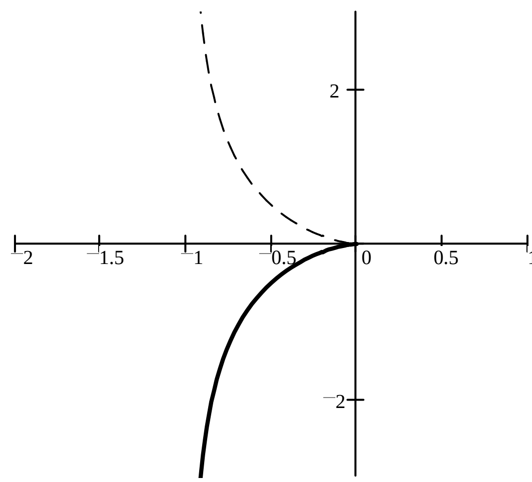


Single Pole at the origin

$$\mathbf{G}(s) := \frac{1}{s\cdot(s+1)}$$

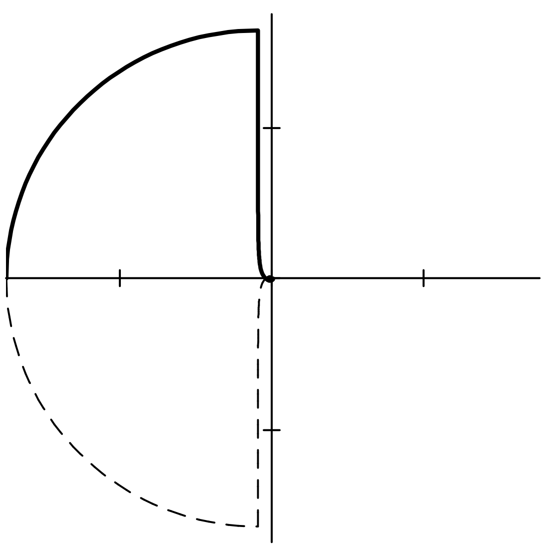
$\text{siz} := \mathbf{G}(\text{sz}) + 1.5$
(for plotting)

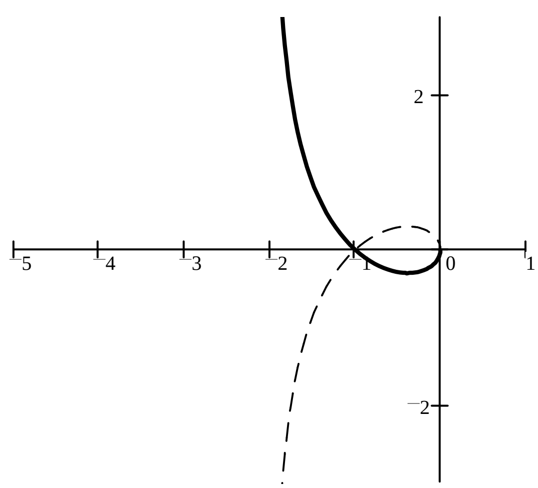




$$\mathbf{G}(s) := \frac{s+1}{s\cdot(s-1)}$$

$\text{siz} := \left|\mathbf{G}(\text{sz})\right|$

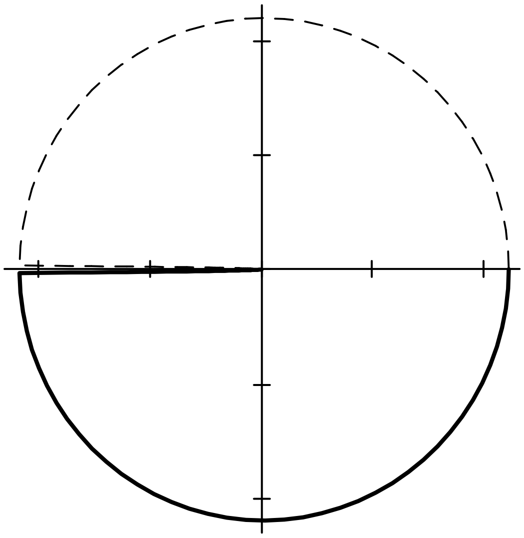


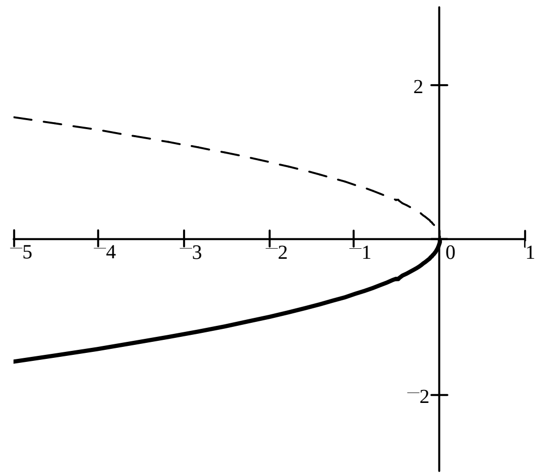


Double Pole at the origin

$$G(s) := \frac{(s+2)}{s^2}$$

$$siz := G(sz) + 100$$

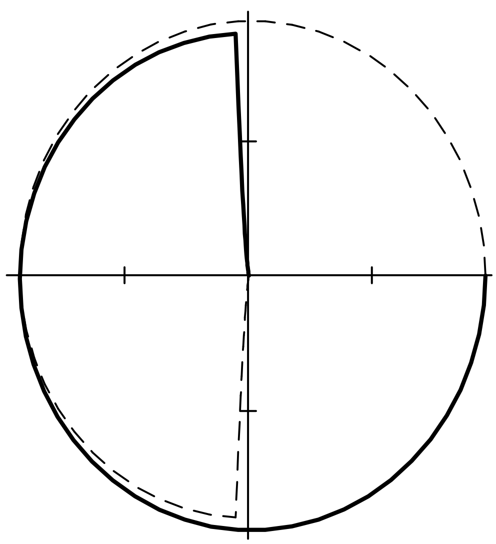


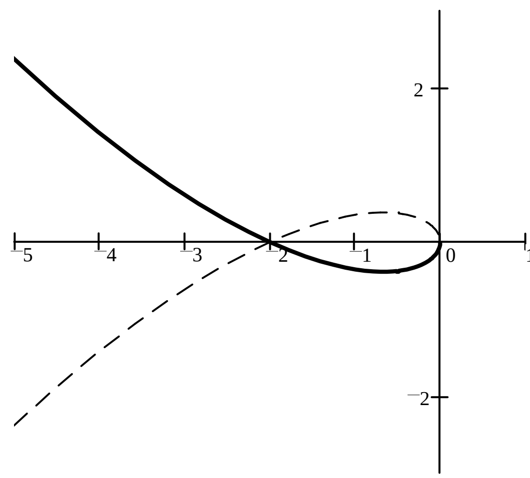


Triple Pole at the origin

$$G(s) := \frac{(s+1)^2}{s^3}$$

$$siz := G(sz) + 1000$$

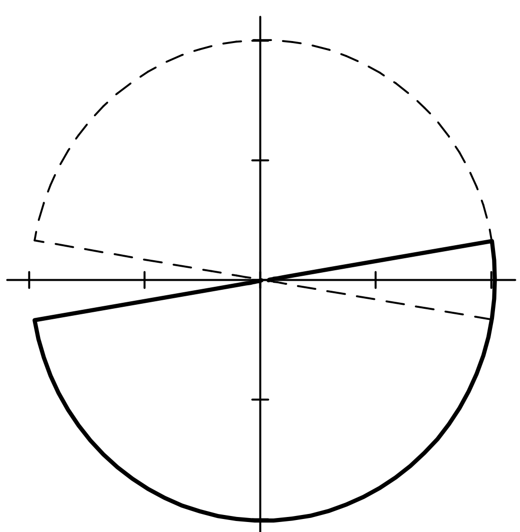


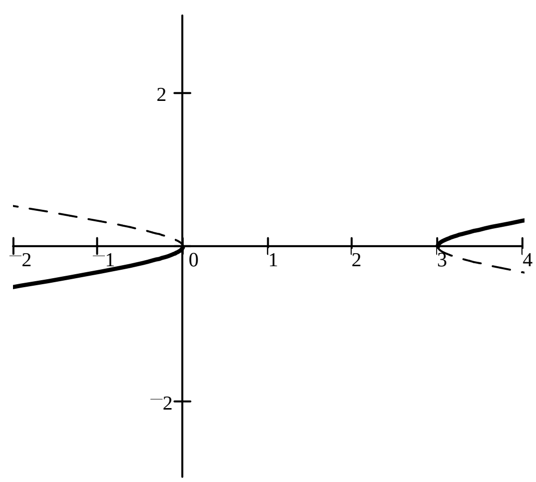


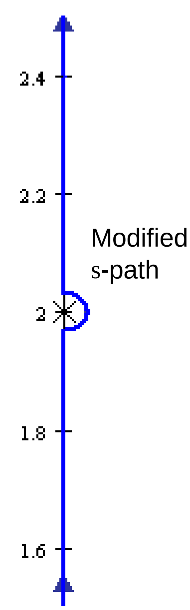
Poles at the ± 2j

$$G(s) := \frac{s+12}{\left(s^2+4\right)}$$

$$siz := 110$$

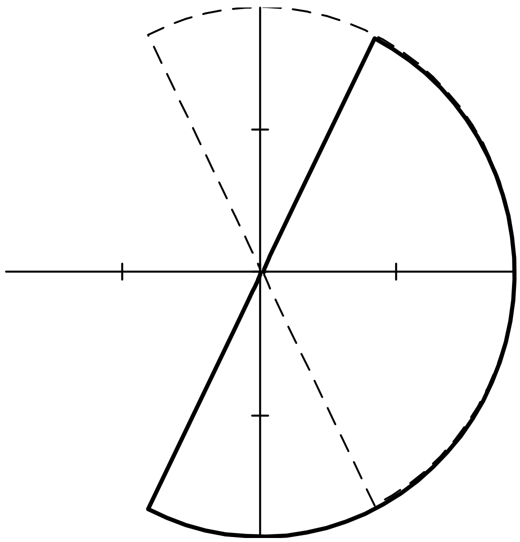


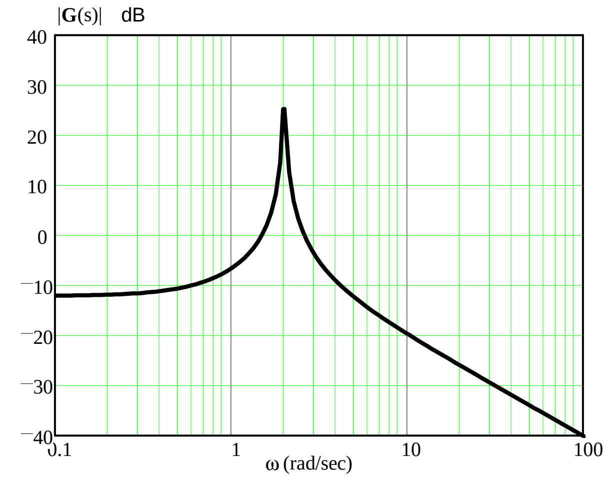


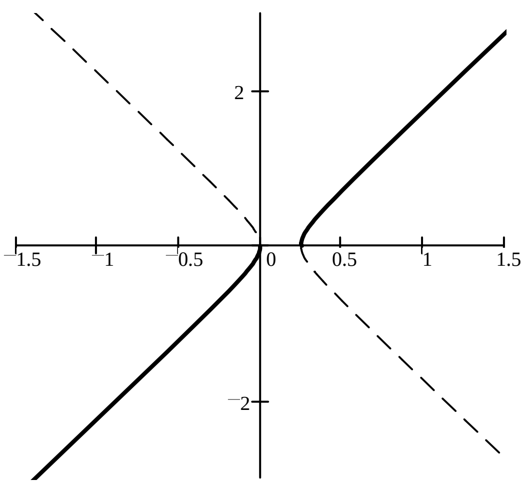


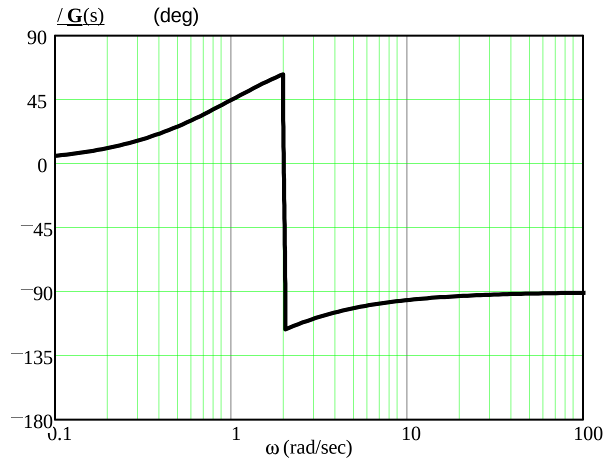
$\mathbf{G}(s) := \frac{s+1}{\left(s^2+4\right)}$

siz := 18.5









$\mathbf{G}(s) := \frac{s-12}{\left(s^2+4\right)\cdot(s+6)}$ $\quad \mathbf{G}(0) = -0.5$

$\text{siz} := \left|\frac{\mathbf{G}(sz)}{sz}\right|$