ECE 3510
Bode Plot Examples

A. Stolp
3/27/14,
4/4/17 (E3)

Ex. 1 $P(s) = \dfrac{2\cdot(s+10)}{s+100}$

$\omega < 10$

$\dfrac{2\cdot(\_+10)}{\_+100} = 0.2$ $\quad 20\cdot\log(0.2) = -14\ \text{dB}$

$\angle\ 0^o$

$10 < \omega < 100$

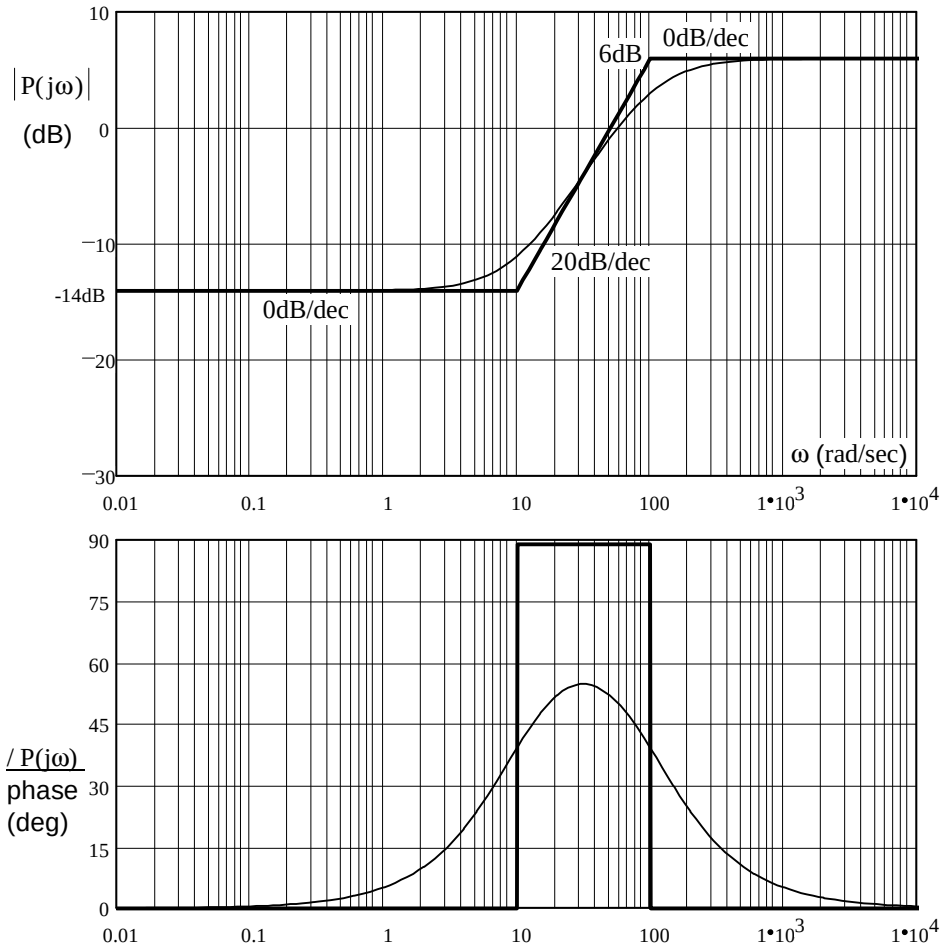
$\dfrac{2\cdot(j\omega+0)}{\_+100}$ Slopes up at 20dB/dec

$\angle\ 90^o$

$100 < \omega$

$\dfrac{2\cdot(j\omega+0)}{j\omega+0} = 2$ $\quad 20\cdot\log(2) = 6\ \text{dB}$

$\angle\ 0^o$



Ex. 2 $P(s) = \dfrac{s+20}{4\cdot(s+1)^2}$

$\omega < 1$

$\dfrac{\_+20}{4\cdot(\_+1)^2} = 5$ $\quad 20\cdot\log(5) = 14\ \text{dB}$

$\angle\ 0^o$

$1 < \omega < 20$

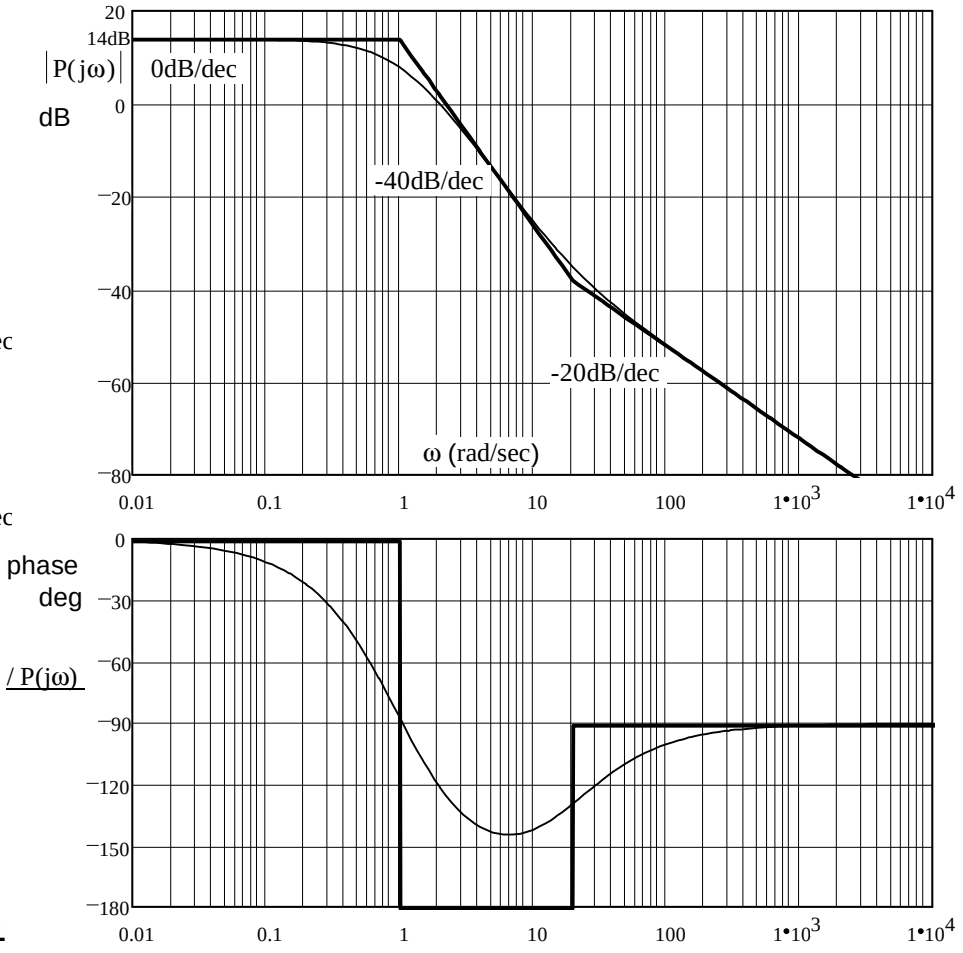
$\dfrac{\_+20}{4\cdot(j\omega+0)^2}$ Slopes down at -40dB/dec

$\angle\ -180^o$

$20 < \omega$

$\dfrac{j\omega+0}{4\cdot(j\omega+0)^2}$ Slopes down at -20dB/dec

$\angle\ -90^o$



Ex. 3 $P_3(s) := \dfrac{20000\cdot(-s+0.1)}{(s+4)\cdot(s+1000)}$

clear the - in front of the s $= \dfrac{-20000\cdot(s-0.1)}{(s+4)\cdot(s+1000)}$

$\omega < 0.1$

$180^o+180^o=360^o$

$\dfrac{-20000\cdot(\_-0.1)}{(\_+4)\cdot(\_+1000)} = 0.5$ $\quad -6\cdot dB$

$\underline{/\ 360^o}$

$0.1 < \omega < 4$

$180^o+90^o=270^o$

$\dfrac{-20000\cdot(j\omega-0)}{(\_+4)\cdot(\_+1000)}$ $\quad$ 20dB/dec

$\underline{/\ 270^o}$

$4 < \omega < 1000$

270^o

$\dfrac{-20000\cdot(j\omega-0)}{(j\omega+0)\cdot(\_-1000)} = 20$ $\quad 16\cdot dB$

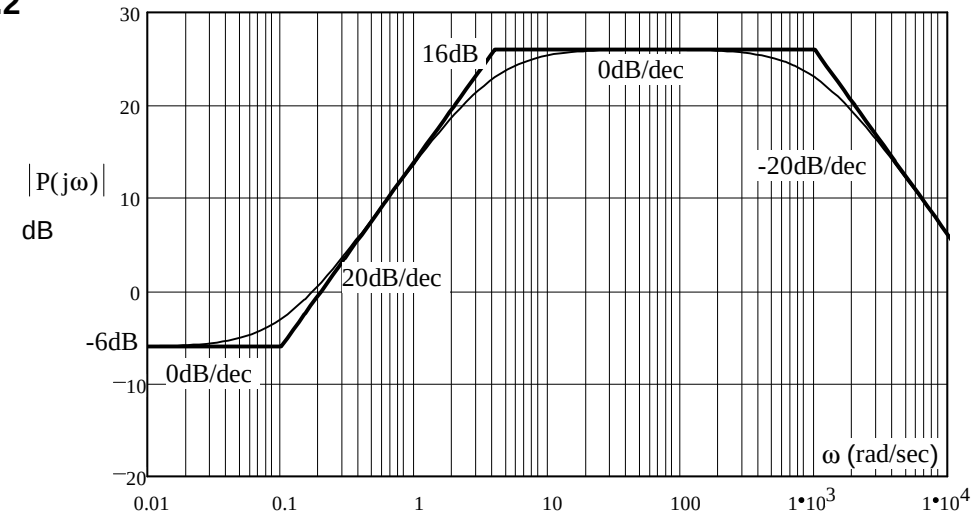
-90^o $\quad \underline{/\ 180^o}$

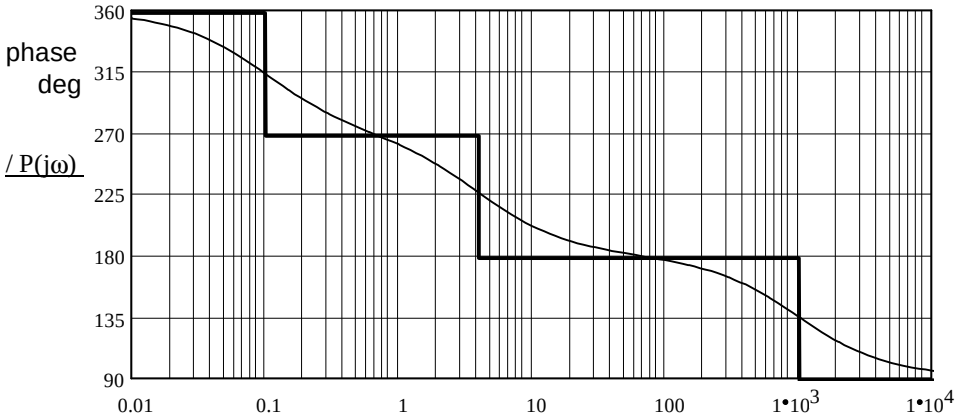
$1000 < \omega$

270^o

$\dfrac{-20000\cdot(j\omega+0)}{(j\omega+0)\cdot(j\omega+0)}$ $\quad$ -20dB/dec

$270^o\text{-}180^o=90^o$ $\quad \underline{/\ 90^o}$





Ex. 4 $P_4(s) := \dfrac{0.5\cdot(s+1)\cdot(s-20)}{s\cdot(s+100)}$

$\omega < 1$

$\dfrac{0.5\cdot(\_+1)\cdot(\_-20)}{j\omega\cdot(\_+100)}$ $\quad$ -20dB/dec

$\underline{/180 - 90^o} = \underline{/\ 90^o}$

Start: $\omega = 0.01$, $\quad 20\cdot\log\left(\dfrac{0.5\cdot1\cdot20}{0.01\cdot100}\right) = 20\ \bullet dB$

$1 < \omega < 20$

$\dfrac{0.5\cdot(j\omega+0)\cdot(\_-20)}{j\omega\cdot(\_+100)}$ $\quad 20\cdot\log\left(\dfrac{.5\cdot20}{100}\right) = -20\ \bullet dB$

$\underline{/\ 180^o}$

$20 < \omega < 100$

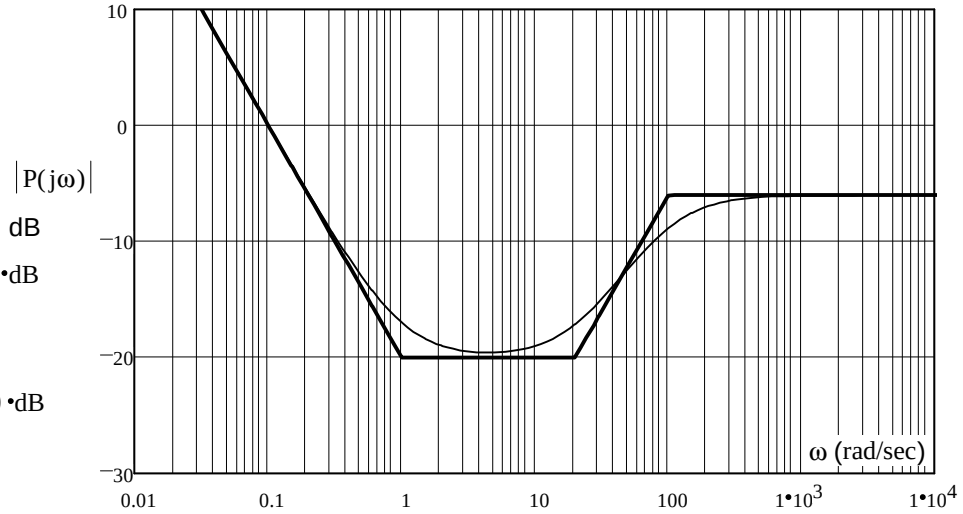
$\dfrac{0.5\cdot(j\omega+0)\cdot(j\omega-0)}{j\omega\cdot(\_+100)}$ $\quad$ 20dB/dec

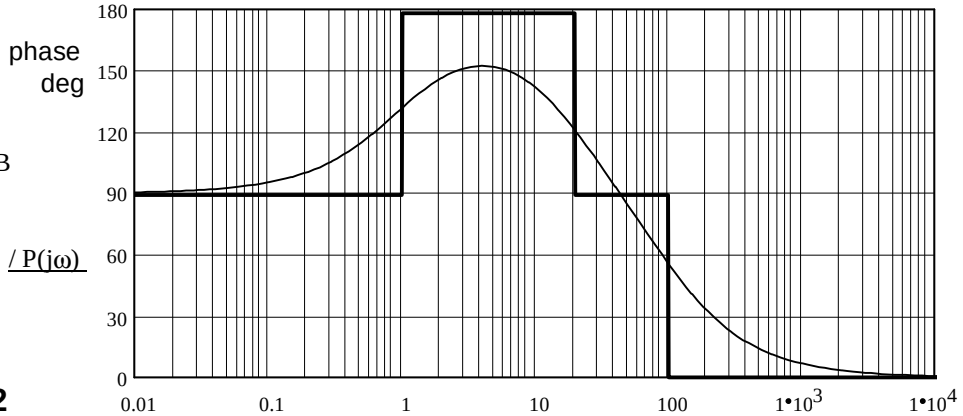
$\underline{/\ 90^o}$

$100 < \omega$

$\dfrac{0.5\cdot(j\omega+0)\cdot(j\omega-0)}{j\omega\cdot(j\omega+0)}$ $\quad 20\cdot\log(.5) = -6\ \bullet dB$

$\underline{/\ 0^o}$





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Ex. 5 $P_5(s) := \dfrac{5000 \cdot s \cdot (s-4)}{(s+0.2)\cdot(s+20)\cdot(s+1000)}$

$\omega < 0.2$

$\dfrac{5000 \cdot j\omega \cdot (\_ - 4)}{(\_ + 0.2)\cdot(\_ + 20)\cdot(\_ + 1000)}$ 20dB/dec

$\angle 180 + 90^o = \angle 270^o$

Start: $\omega = 0.01$, $20 \cdot \log\left(\dfrac{5000 \cdot 0.01 \cdot 4}{0.2 \cdot 20 \cdot 1000}\right) = -26$ dB

$0.2 < \omega < 4$

$\dfrac{5000 \cdot (j\omega) \cdot (\_ - 4)}{(j\omega + 0)\cdot(\_ + 20)\cdot(\_ + 1000)}$

$20 \cdot \log\left(\dfrac{5000 \cdot 4}{20 \cdot 1000}\right) = 0$ dB $\quad \angle 180^o$

$4 < \omega < 20$

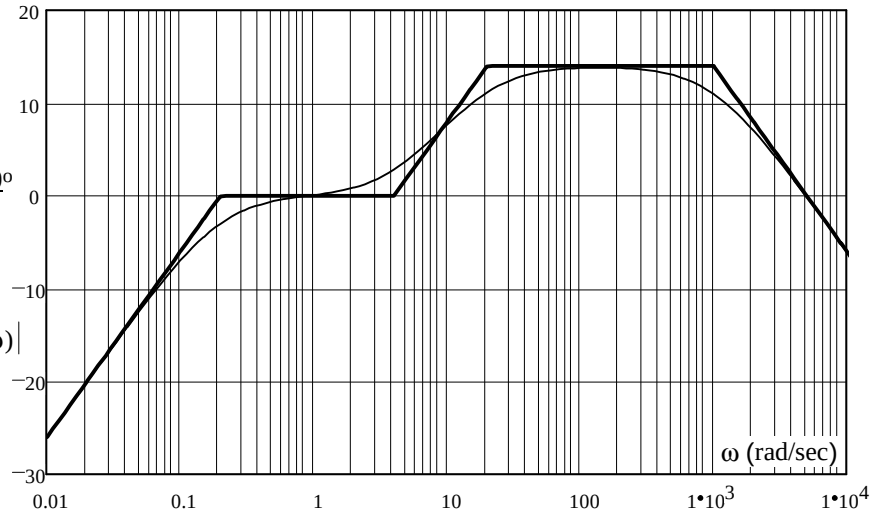
$\dfrac{5000 \cdot (j\omega) \cdot (j\omega - 0)}{(j\omega + 0)\cdot(\_ + 20)\cdot(\_ + 1000)}$ 20dB/dec $\quad \angle 90^o$

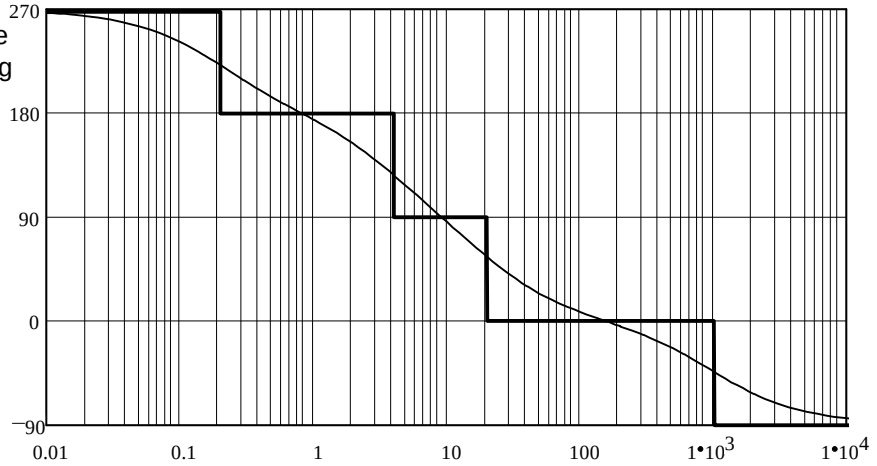
$20 < \omega < 1000$

$\dfrac{5000 \cdot (j\omega) \cdot (j\omega - 0)}{(j\omega + 0)\cdot(j\omega + 0)\cdot(\_ + 1000)}$ $20 \cdot \log(5) = 14$ dB $\quad \angle 0^o$

$1000 < \omega$

$\dfrac{5000 \cdot (j\omega) \cdot (j\omega - 0)}{(j\omega + 0)\cdot(j\omega + 0)\cdot(j\omega + 0)}$ -20dB/dec $\quad \angle -90^o$





Ex. 6

$P_6(s) := \dfrac{20000 \cdot (s + 0.1)}{(s+2)\cdot\left(s^2 + 10 \cdot s + 10000\right)}$

$s^2 + 2 \cdot \zeta \cdot \omega_n \cdot s + \omega_n^2$

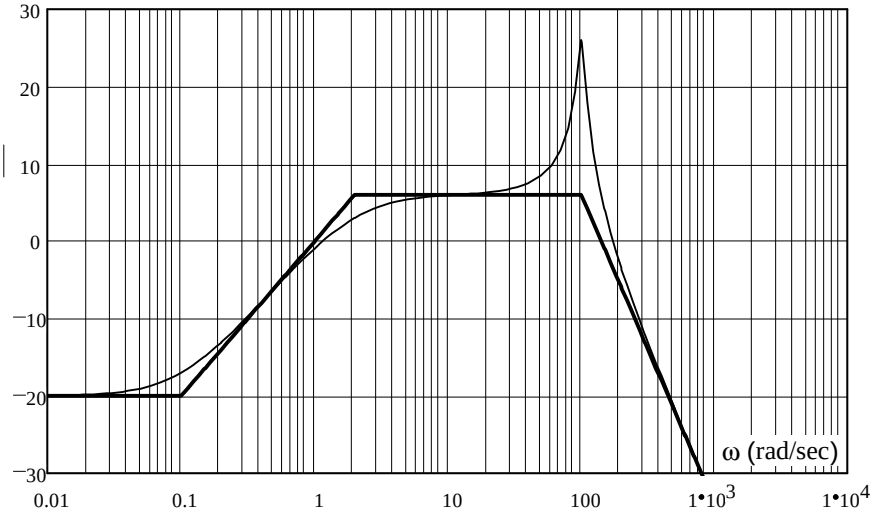
to get straight lines:

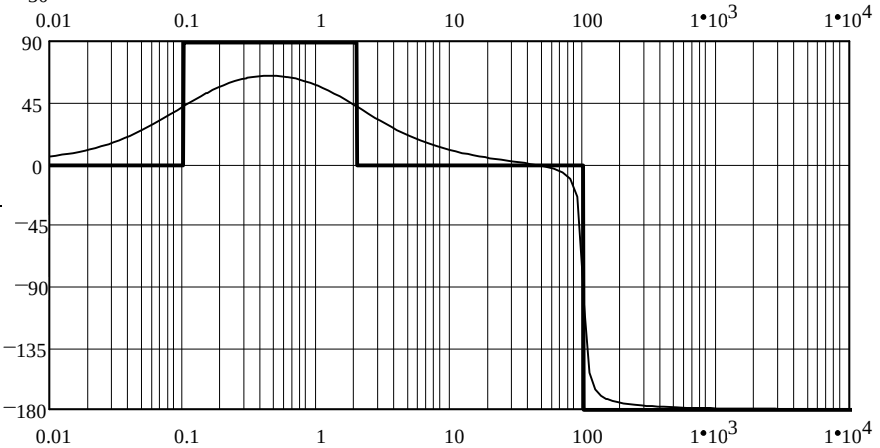
$\dfrac{20000 \cdot (\_ + 0.1)}{(\_ + 2)\cdot(\_ + 100)\cdot(\_ + 100)}$

natural frequency $\omega_n = \sqrt{\omega_n^2} = \sqrt{10000} = 100$

damping factor: $\zeta = \dfrac{2 \cdot \zeta \cdot \omega_n}{2 \cdot \omega_n} = \dfrac{10}{2 \cdot 100} = 0.05$

$20 \cdot \log\left(\dfrac{1}{2 \cdot .05}\right) = 20$ dB peak





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Ex. 7

$$P_7(s) := \frac{400\cdot(s+0.1)\cdot(s+100)}{\left[(s+0.4)^2+15.84\right]\cdot(s+1000)}$$

natural freq. $(s+a)^2+b^2$

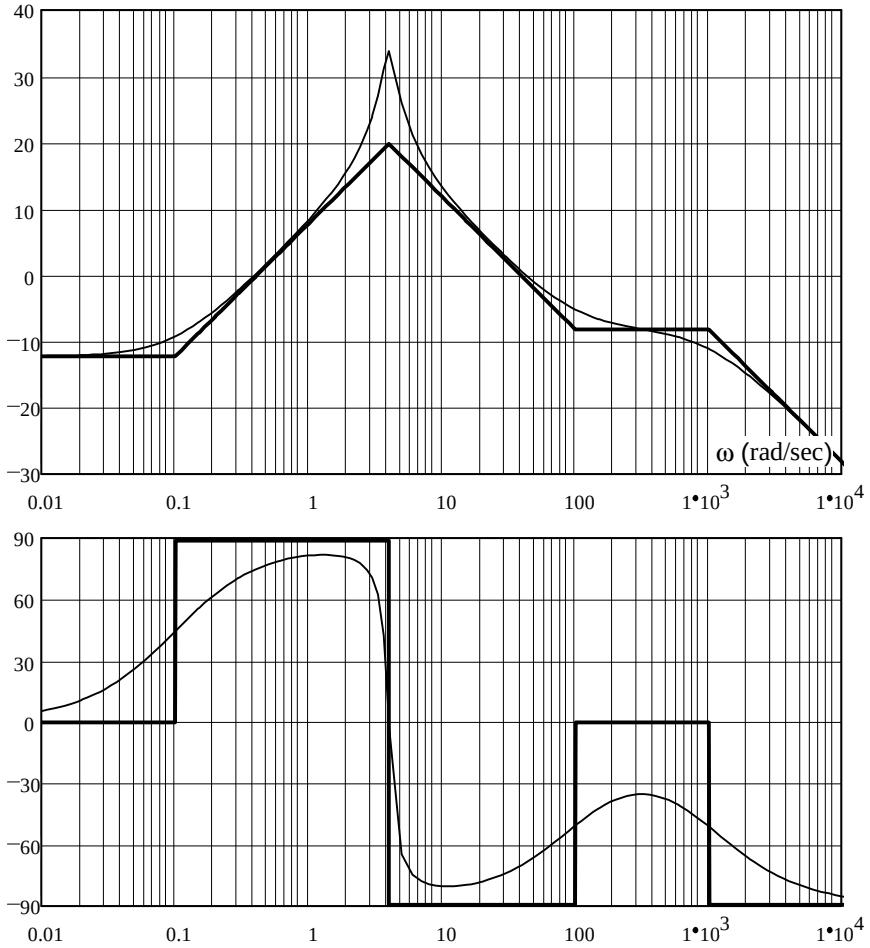
$\omega_n = \sqrt{a^2+b^2} = \sqrt{0.4^2+15.84} = 4$

damping factor: $\zeta = \frac{a}{\omega_n} = \frac{0.4}{4} = 0.1$

$20\cdot\log\left(\frac{1}{2\cdot.1}\right) = 14$ dB peak

to get straight lines:

$$\frac{400\cdot(\_+0.1)\cdot(\_+100)}{(\_+4)\cdot(\_+4)\cdot(\_+1000)}$$



Ex. 8 $(s+a)^2+b^2$

$$P_8(s) := \frac{25\cdot\left[(s+10)^2+9900\right]}{\left(s^2+s+4\right)\cdot(s+2000)}$$

$s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^2$

natural frequency $\omega_{n1} = \sqrt{\omega_{n1}^2} = \sqrt{4} = 2$

damping factor: $\zeta = \frac{2\cdot\zeta\cdot\omega_{n1}}{2\cdot\omega_{n1}} = \frac{1}{2\cdot2} = 0.25$

$20\cdot\log\left(\frac{1}{2\cdot.25}\right) = 6$ dB peak

natural freq. $\omega_{n2} = \sqrt{a^2+b^2} = \sqrt{10^2+9900} = 100$

damping factor: $\zeta = \frac{10}{100} = 0.1$

to get straight lines:

$$\frac{25\cdot(\_+100)\cdot(\_+100)}{(\_+2)\cdot(\_+2)\cdot(s+2000)}$$