# ECE 3510 Finish Ch 2

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## Non-strictly-proper transforms

section 2.2.5, p.16 in Bodson text

What if the order of the numerator is equal to or even greater than the order of the denominator? $m \geq n$ ?

Example: $F(s) = \dfrac{2 \cdot s^2 + 100}{s^2 + 8 \cdot s + 41}$ $m := 2$ $n := 2$ $\overset{?}{=} \dfrac{A}{s + 4 + 5 \cdot j} + \dfrac{B}{s + 4 - 5 \cdot j} = \dfrac{A \cdot (s + 4 + 5 \cdot j) + B \cdot (s + 4 - 5 \cdot j)}{s^2 + 8 \cdot s + 41}$

can't work ! no $s^2$ term in numerator !

First divide, before partial fraction expansion

$$\begin{array}{r|l} & 2 \\ s^2 + 8 \cdot s + 41 & 2 \cdot s^2 + 0 \cdot s + 100 \\ & -2 \cdot (s^2 + 8 \cdot s + 41) \\ \hline \text{"remainder"} & 0 \cdot s^2 - 16 \cdot s + 18 \end{array}$$

$$F(s) = \frac{2 \cdot s^2 + 100}{s^2 + 8 \cdot s + 41} = 2 + \frac{-16 \cdot s + 18}{s^2 + 8 \cdot s + 41}$$

$$f(t) = 2 \cdot \delta(t) + \left( \frac{82}{5} \cdot e^{-4 \cdot t} \cdot \sin(5 \cdot t) - 16 \cdot e^{-4 \cdot t} \cdot \cos(5 \cdot t) \right) \cdot u(t)$$

Delta functions are not very common in real life.

Non-strictly-proper transforms are just as common.

## Properties of Signals

Can you tell what f(t) must be just by looking at F(s)? YES, somewhat...

$$\frac{s + 5}{s \cdot (s^2 + 4 \cdot s + 13) \cdot (s - 10)}$$

$s$ means a pole at origin, so output has some DC of unknown $\pm$ value, and may converge to a non-zero (DC) value

$(s^2 + 4 \cdot s + 13) = (s + 2 + 3 \cdot j) \cdot (s + 2 - 3 \cdot j)$ Decaying cosine wave of unknown phase or amplitude

$e^{-2t}$ decay rate, time constant = 1/2 sec, frequency = 3 rad/sec or $3/2\pi$ Hz

$(s - 10)$ means signal is unbounded and (of course) doesn't converge

$$\frac{s + 5}{s \cdot (s^2 + 64) \cdot (s + 10)}$$

$s + 10$ means an $e^{-10t}$ decaying exponential is part of the signal

DC

$(s^2 + 64) = (s + 8 \cdot j) \cdot (s - 8 \cdot j)$ cosine wave of unknown phase or amplitude, frequency = 8 rad/sec or $4/\pi$ Hz
means signal is bounded but doesn't converge

$$\frac{s + 5}{s \cdot (s^2 - 4 \cdot s + 13) \cdot (s + 10)}$$

$s^2 - 4 \cdot s + 13 = (s - 2 + 3 \cdot j) \cdot (s - 2 - 3 \cdot j)$ Cosine wave of unknown phase and exponentially increasing amplitude

$$\frac{s + 5}{s \cdot (s^2 + 4 \cdot s + 13)^2 \cdot (s + 10)} = \frac{A}{s} + \frac{B \cdot (s + 2) + C \cdot 3}{(s^2 + 4 \cdot s + 13)} + \frac{D \cdot \left[ (s + 2)^2 - 3^2 \right] + E \cdot (6 \cdot (s + 2))}{(s^2 + 4 \cdot s + 13)^2} + \frac{F}{(s + 10)}$$

$$\left[ A + B \cdot e^{2 \cdot t} \cdot \cos(3 \cdot t) + C \cdot \left( e^{2 \cdot t} \cdot \sin(3 \cdot t) \right) + D \cdot t \cdot e^{2 \cdot t} \cdot \cos(3 \cdot t) + E \cdot t \cdot e^{2 \cdot t} \cdot \sin(3 \cdot t) \right] \cdot u(t)$$

A and F cannot be zero, neither can both D and E ($D^2 + E^2 > 0$)

$$\frac{s + 5}{s^3 \cdot (s^2 + 4 \cdot s + 13)^2 \cdot (s + 10)^2}$$

$s^3$ term results in a rising $t^2$ in time domain as well as possible DC and/or ramp