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ECE 3510 Lecture 4 notes Inverse Laplace transforms by Partial Fraction Expansion

Ex. 1 by Clearing Fractions

Like Example 1-b from page 13, but with more interesting numbers

$$F(s) = \frac{12\cdot s+64}{(s+4)^2\cdot(s+6)} = \frac{A}{s+4} + \frac{B}{(s+4)^2} + \frac{C}{s+6}$$

Multiply both sides by: $(s+4)^2\cdot(s+6)$

$$12\cdot s+64 = A\cdot(s+4)\cdot(s+6) + B\cdot(s+6) + C\cdot(s+4)^2$$

$$12\cdot s+64 = A\cdot s^2 + A\cdot 10\cdot s + A\cdot 24 + B\cdot s + B\cdot 6 + C\cdot s^2 + C\cdot 8\cdot s + C\cdot 16$$

$$0\cdot s^2 = A\cdot s^2 + 0\cdot s^2 + C\cdot s^2 \qquad A := -C$$

no s^2 term on the left

$$12\cdot s = A\cdot 10\cdot s + B\cdot s + C\cdot 8\cdot s$$

$$12 = A\cdot 10 + B + -A\cdot 8 \qquad B := 12 - 2\cdot A$$

$$64 = A\cdot 24 + B\cdot 6 + C\cdot 16$$

$$64 = A\cdot 24 + (12-2\cdot A)\cdot 6 + -A\cdot 16$$

$$64 - 72 = -8 = -4\cdot A \qquad A := 2$$

$$C := -2 \qquad B := 8$$

$$F(s) = \frac{12\cdot s+64}{(s+4)^2\cdot(s+6)} = \frac{2}{s+4} + \frac{8}{(s+4)^2} + \frac{-2}{s+6}$$

$$f(t) = 2\cdot e^{-4\cdot t} + 8\cdot t\cdot e^{-4\cdot t} + -2\cdot e^{-6\cdot t}$$

Ex. 1 by Residue Method

Like Example 1-a from page 12, but with more interesting numbers

$$12\cdot s+64 = A\cdot(s+4)\cdot(s+6) + B\cdot(s+6) + C\cdot(s+4)^2$$

Set $s := -4$

$$\underset{16}{12\cdot(-4)+64} = 0 + \underset{2}{B\cdot(-4+6)} + 0 \qquad B := (s+4)^2\cdot\overbrace{\frac{12\cdot s+64}{(s+4)^2\cdot(s+6)}}^{F(s)}$$

$$B := 8$$

Set $s := -6$

$$\underset{-8}{12\cdot(-6)+64} = 0 + 0 + \underset{(-2)^2}{C\cdot(-6+4)^2} \qquad C := -2$$

See Eq. 2.9, page 22 of Bodson Text

$$A = \frac{d}{ds}\left[(s+4)^2\cdot\frac{12\cdot s+64}{(s+4)^2\cdot(s+6)}\right]\Bigg|_{s:=-4} = \frac{d}{ds}\frac{12\cdot s+64}{(s+6)}\Bigg|_{s:=-4}$$

Recall: $$\frac{d\cdot\frac{h}{g}}{ds} = \frac{h\cdot\frac{dg}{ds} - g\cdot\frac{dh}{ds}}{g^2}$$

$$\frac{d}{ds}\frac{12\cdot s+64}{(s+6)} = \frac{(s+6)\cdot\left[\frac{d}{ds}(12\cdot s+64)\right] - (12\cdot s+64)\cdot\left[\frac{d}{ds}(s+6)\right]}{(s+6)^2} = \frac{(s+6)\cdot 12 - (12\cdot s+64)\cdot 1}{(s+6)^2}$$

$$= \frac{(12\cdot s+72) - 12\cdot s - 64}{(s+6)^2} = \frac{8}{(s+6)^2} \qquad \frac{8}{(s+6)^2}\Bigg|_{s:=-4} = \frac{8}{2^2} = \frac{8}{4} = 2 = A$$

Same results as above

Ex. 1 by the Mixed Method

$$F(s) = \frac{12\cdot s+64}{(s+4)^2\cdot(s+6)} = \frac{A}{s+4} + \frac{B}{(s+4)^2} + \frac{C}{s+6}$$

Multiply both sides by: $(s+4)^2\cdot(s+6)$

$$12\cdot s+64 = A\cdot(s+4)\cdot(s+6) + B\cdot(s+6) + C\cdot(s+4)^2$$

Set $s := -4$

$$\underset{16}{12\cdot(-4)+64} = 0 + B\cdot\underset{2}{(-4+6)} + 0 \qquad B := 8$$

Set $s := -6$

$$\underset{-8}{12\cdot(-6)+64} = 0 + 0 + C\cdot\underset{(-2)^2}{(-6+4)^2} \qquad C := -2$$

Back to equation above

$$12\cdot s+64 = A\cdot(s+4)\cdot(s+6) + B\cdot(s+6) + C\cdot(s+4)^2$$

$$12\cdot s+64 = A\cdot s^2 + A\cdot 10\cdot s + A\cdot 24 + 8\cdot s + 8\cdot 6 + C\cdot s^2 + C\cdot 8\cdot s + C\cdot 16$$

$$0\cdot s^2 = A\cdot s^2 + 0\cdot s^2 + C\cdot s^2 \qquad A := -C$$

no s^2 term on the left $\qquad A = 2$

And the rule is: Get as many easy answers as possible before clearing fractions!

$$F(s) = \frac{12\cdot s+64}{(s+4)^2\cdot(s+6)} = \frac{2}{s+4} + \frac{8}{(s+4)^2} + \frac{-2}{s+6}$$

$$f(t) = 2\cdot e^{-4\cdot t} + 8\cdot t\cdot e^{-4\cdot t} + -2\cdot e^{-6\cdot t}$$

$$f(t) = 2\cdot e^{-4\cdot t} + 8\cdot t\cdot e^{-4\cdot t} - 2\cdot e^{-6\cdot t}$$

Same results again