ECE 3510 Laplace Transforms (Unilateral)

f(t)		F(s)	
	$f(t) = \frac{1}{2\cdot\pi\cdot j}\cdot\int_{c-j\infty}^{c+j\infty} F(s)\cdot e^{s\cdot t}\,ds$	$F(s) = \int_{0}^{\infty} f(t)\cdot e^{-s\cdot t}\,dt$	
	c is a constant within the region of convergence		
1	$\delta(t)$	1	
2	$u(t)$	$\frac{1}{s}$	
3	$t\cdot u(t)$	$\frac{1}{s^2}$	
4	$t^n\cdot u(t)$	$\frac{n!}{s^{n+1}}$	
5a	$e^{a\cdot t}\cdot u(t)$	$\frac{1}{s-a}$	
5b	$e^{-\frac{t}{\tau}}\cdot u(t)$	$\frac{1}{s+\frac{1}{\tau}}$	$\tau = -\frac{1}{a}$ = time constant
6	$t\cdot e^{a\cdot t}\cdot u(t)$	$\frac{1}{(s-a)^2}$	
7	$t^n\cdot e^{a\cdot t}\cdot u(t)$	$\frac{n!}{(s-a)^{n+1}}$	
8a	$\cos(b\cdot t)\cdot u(t)$	$\frac{s}{s^2+b^2}$	$b = \omega$ = radian frequency
8b	$\sin(b\cdot t)\cdot u(t)$	$\frac{b}{s^2+b^2}$	
9a	$e^{a\cdot t}\cdot\cos(b\cdot t)\cdot u(t)$	$\frac{s-a}{(s-(a+bj))\cdot(s-(a-bj))}$	$= \frac{s-a}{s^2-2\cdot a\cdot s+\left(a^2+b^2\right)}$
		$\frac{s-a}{(s-a)^2+b^2}$	$= \frac{s-a}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}}$
9b	$e^{a\cdot t}\cdot\sin(b\cdot t)\cdot u(t)$	$\frac{b}{(s-(a+bj))\cdot(s-(a-bj))}$	$= \frac{b}{s^2-2\cdot a\cdot s+\left(a^2+b^2\right)}$
		$\frac{b}{(s-a)^2+b^2}$	$= \frac{b}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}}$
11a	$t\cdot e^{a\cdot t}\cdot\cos(b\cdot t)\cdot u(t)$	$\frac{(s-a)^2-b^2}{\left[(s-a)^2+b^2\right]^2}$	$= \frac{(s-a)^2-b^2}{\left[s^2-2\cdot a\cdot s+\left(a^2+b^2\right)\right]^2}$
11b	$t\cdot e^{a\cdot t}\cdot\sin(b\cdot t)\cdot u(t)$	$\frac{2\cdot b\cdot(s-a)}{\left[(s-a)^2+b^2\right]^2}$	$= \frac{2\cdot b\cdot(s-a)}{\left[s^2-2\cdot a\cdot s+\left(a^2+b^2\right)\right]^2}$

Euler's equations $\cos(\omega\cdot t) = \frac{e^{j\cdot\omega\cdot t}+e^{-j\cdot\omega\cdot t}}{2}$ $\sin(\omega\cdot t) = \frac{e^{j\cdot\omega\cdot t}-e^{-j\cdot\omega\cdot t}}{2\cdot j}$

ECE 3510 Laplace Transforms

ECE 3510 Laplace Properties

Operation	f(t)	F(s)
Addition	$f(t) + g(t)$	$F(s) + G(s)$
Scalar multiplication	$k \cdot f(t)$	$k \cdot F(s)$
Linearity	$k \cdot f(t) + n \cdot g(t)$	$k \cdot F(s) + n \cdot G(s)$
Time differentiation	$\frac{d}{dt} f(t)$	$s \cdot F(s) - f(0^-)$
	$\frac{d^2}{dt^2} f(t)$	$s^2 \cdot F(s) - s \cdot f(0^-) - \frac{d}{dt} f(0^-)$ initial slope
	$\frac{d^3}{dt^3} f(t)$	$s^3 \cdot F(s) - s^2 \cdot f(0^-) - s \cdot \left(\frac{d}{dt} f(0^-)\right) - \frac{d^2}{dt^2} f(0^-)$ initial slope, initial curvature
Time integration	$\int_{0^-}^{t} f(\tau)\, d\tau$	$\frac{1}{s} \cdot F(s)$
	$\int_{-\infty}^{t} f(\tau)\, d\tau$	$\frac{1}{s} \cdot F(s) + \frac{1}{s} \cdot \int_{-\infty}^{0^-} f(t)\, dt$
Time shift	$f(t - t_0) \cdot u(t - t_0)$	$F(s) \cdot e^{-s \cdot t_0}$ $\quad t_0 \geq 0$
Frequency shift	$f(t) \cdot e^{s_0 \cdot t}$	$F(s - s_0)$
Frequency differentiation	$-t \cdot f(t)$	$\frac{d}{ds} F(s)$
Frequency integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(s')\, ds'$
Scaling	$f(a \cdot t)$ $\quad a \geq 0$	$\frac{1}{a} \cdot F\left(\frac{s}{a}\right)$
Time convolution	$f(t) * g(t)$	$F(s) \cdot G(s)$
Frequency convolution	$f(t) \cdot g(t)$	$\frac{1}{2 \cdot \pi \cdot j} \cdot F(s) * G(s)$
Initial value	$f(0^+)$	$\lim_{s \to \infty} s \cdot F(s)$ $\quad n > m$ # of poles > # of zeroes
Final value	$f(\infty)$	$\lim_{s \to 0} s \cdot F(s)$ $\quad$ (all poles of $sF(s)$ in LHP)

ECE 3510 Laplace Properties