ECE 3510 Evaluating Laplace transforms

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rev,

$$F(s) = \int_0^{\infty} f(t)\cdot e^{-s\cdot t}\,dt$$ Unilateral Laplace transform

Ex. 1 $f(t) = \delta(t)$ The Impulse or "Dirac" function

$$F(s) = \int_0^{\infty} \delta(t)\cdot e^{-s\cdot t}\,dt$$ but: $\underbrace{\delta(t)\cdot g(t)}_{\text{any function}} = \delta(t)\cdot g(0)$ so:

$$= \int_0^{\infty} \delta(t)\cdot e^{-s\cdot 0}\,dt = \int_0^{\infty} \delta(t)\cdot 1\,dt = 1$$

Ex. 2 $f(t) = u(t)\cdot e^{at}$

$$F(s) = \int_0^{\infty} e^{at}\cdot e^{-s\cdot t}\,dt = \int_0^{\infty} e^{(a-s)\cdot t}\,dt = \frac{1}{(a-s)}\cdot e^{(a-s)\cdot t}\Bigg|_0^{\infty}$$

$$= \frac{1}{(a-s)}\cdot \overset{0}{e^{(a-s)\cdot\infty}} - \frac{1}{(a-s)}\cdot \overset{1}{e^{(a-s)\cdot 0}} = 0 - \frac{1}{(a-s)}\cdot(1) = \frac{1}{s-a}$$

if $s > a$

Ex. 3 $f(t) = u(t)\cdot\cos(\omega\cdot t)$

$$F(s) = \int_0^{\infty} \cos(\omega\cdot t)\cdot e^{-s\cdot t}\,dt = \int_0^{\infty} \left(\frac{e^{j\cdot\omega\cdot t} + e^{-j\cdot\omega\cdot t}}{2}\right)\cdot e^{-s\cdot t}\,dt = \int_0^{\infty} \frac{e^{(j\cdot\omega - s)\cdot t} + e^{-(j\cdot\omega + s)\cdot t}}{2}\,dt$$

$$= \frac{1}{2}\cdot\int_0^{\infty} e^{(j\cdot\omega - s)\cdot t}\,dt + \frac{1}{2}\cdot\int_0^{\infty} e^{-(j\cdot\omega + s)\cdot t}\,dt$$

$$= \frac{1}{2}\cdot\left(\frac{1}{j\cdot\omega - s}\right)\cdot e^{(j\cdot\omega - s)\cdot t}\Bigg|_0^{\infty} + \frac{1}{2}\cdot\left[\frac{1}{-(j\cdot\omega + s)}\right]\cdot e^{-(j\cdot\omega + s)\cdot t}\Bigg|_0^{\infty}$$

$$= \frac{1}{2}\cdot\left(\frac{1}{j\cdot\omega - s}\right)\cdot(-1) + \frac{1}{2}\cdot\left[\frac{1}{-(j\cdot\omega + s)}\right]\cdot(-1) = \frac{-1}{-2\cdot j\cdot\omega - 2\cdot s} + \frac{-1}{2\cdot j\cdot\omega - 2\cdot s}$$

$$= \frac{1}{2\cdot j\cdot\omega + 2\cdot s} + \frac{-1}{2\cdot\omega - 2\cdot s} = \frac{(2\cdot j\cdot\omega - 2\cdot s) - (2\cdot j\cdot\omega + 2\cdot s)}{(2\cdot j\cdot\omega + 2\cdot s)\cdot(2\cdot j\cdot\omega - 2\cdot s)}$$

$$= \frac{-4\cdot s}{(2\cdot j\cdot\omega + 2\cdot s)\cdot(2\cdot j\cdot\omega - 2\cdot s)} = \frac{-4\cdot s}{4\cdot j^2\cdot\omega^2 - 4\cdot s^2} = \frac{-s}{-\omega^2 - s^2}$$

$$= \frac{s}{\omega^2 + s^2}$$