ECE 3510 Second_Order lab, Position Control Appendix

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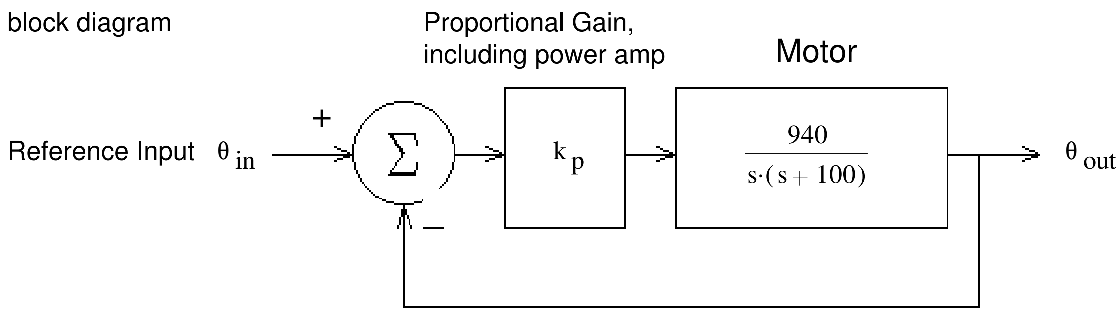
rev

Simplified motor transfer function (Without inductance, L_a): $\frac{\theta(s)}{V_a(s)} = \frac{k}{s \cdot (s+a)}$

From the first lab you should have found the constants k and a to be approximately: $k := 940$ $a := 100$

Now this motor is part of a control loop:

The system block diagram



Overall transfer function: $$H(s) = \frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{k_p \cdot \left[\frac{940}{s \cdot (s+100)}\right]}{1 + k_p \cdot \left[\frac{940}{s \cdot (s+100)}\right] \cdot 1} = \frac{k_p \cdot 940}{s \cdot (s+100) + k_p \cdot 940}$$

$$= \frac{k_p \cdot 940}{s^2 + 100 \cdot s + k_p \cdot 940}$$

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For critical damping: $100^2 - 4\cdot\left(k_p\cdot 940\right) = 0$

$$10000 = 3760\cdot k_p \qquad k_p := \frac{10000}{3760} \qquad k_p = 2.66$$

Input step: $90\cdot deg = 1.571\cdot rad \qquad X(s) = \frac{1.571}{s}$

Step response: $Y(s) = X(s)\cdot H(s) = \frac{1.571}{s}\cdot\frac{k_p\cdot 940}{s\cdot(s+100)+k_p\cdot 940} = \frac{1.571}{s}\cdot\frac{2.66\cdot 940}{s^2+100\cdot s+2.66\cdot 940}$

$$= \frac{1.571}{s}\cdot\frac{2.66\cdot 940}{(s+50)^2} = \frac{3928}{s\cdot(s+50)^2} \qquad H(0) = 1.571$$

$$= \frac{1.571}{s}+\frac{B}{(s+50)}+\frac{C}{(s+50)^2}$$

$$3928 = 1.571\cdot(s+50)^2+B\cdot s\cdot(s+50)+C\cdot s$$

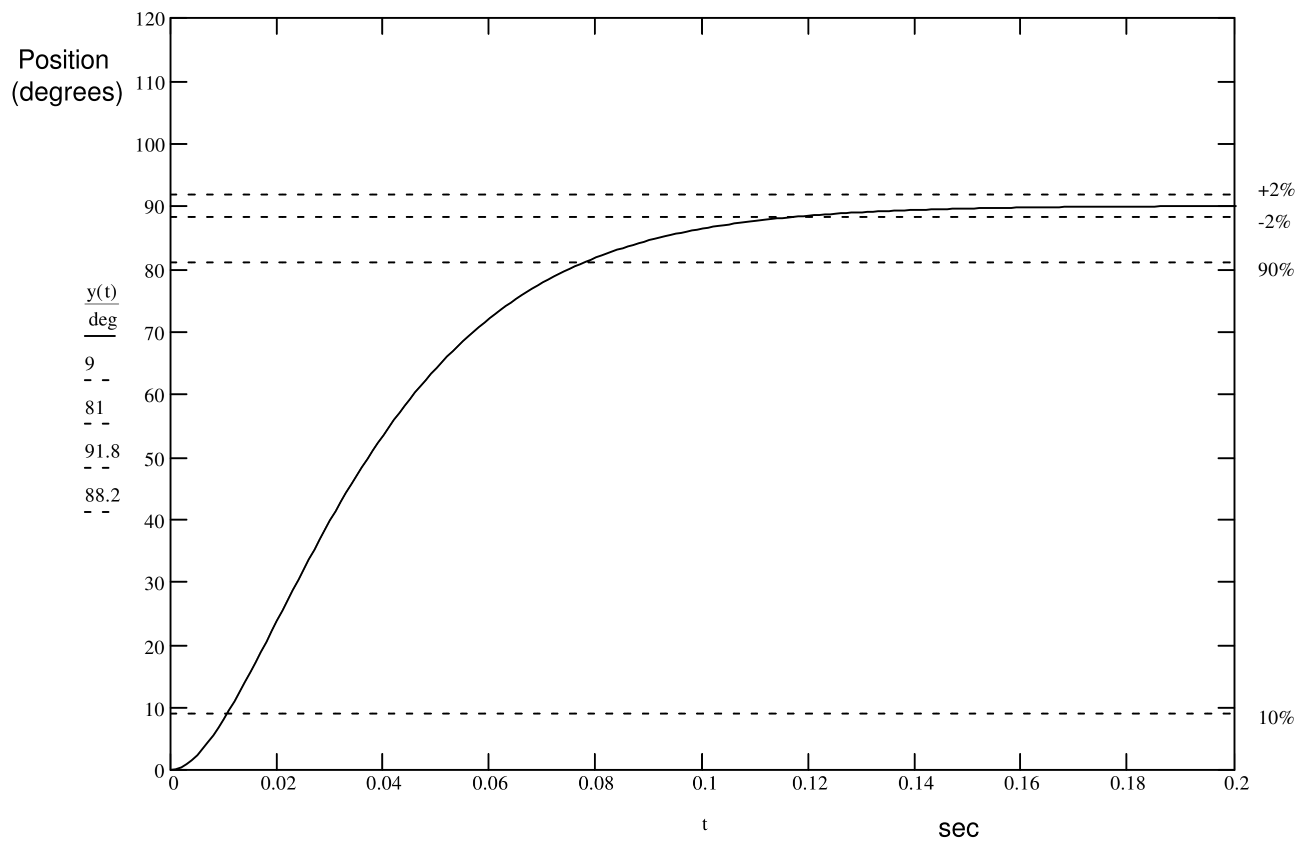
let $s := -50$

$$3928 = 0+0+C\cdot(-50) \qquad C = \frac{3928}{-50} = -78.56$$

$$3928 = 1.571\cdot s^2+157.1\cdot s+3928+B\cdot s^2+B\cdot 50\cdot s-75.56\cdot s$$

$$B := -1.571$$

$$y(t) := 1.571 - 1.571\cdot e^{-50.\cdot t} - \frac{3928}{50}\cdot t\cdot e^{-50.\cdot t}$$



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$$Y(s) = X(s)\cdot H(s) \quad = \frac{1.571}{s}\cdot\frac{k_p\cdot 940}{s^2+100\cdot s+k_p\cdot 940}$$

For under damping: $s^2+100\cdot s+k_p\cdot 940 \quad = (s+a)^2+b^2 \quad = s^2+2\cdot a\cdot s+a^2+b^2$

$$a := 50$$

Choose imaginary part at twice the real part: $\frac{a}{b} = \frac{1}{2}$ in eq. 3.37, p.33 in text $\quad b := 100$

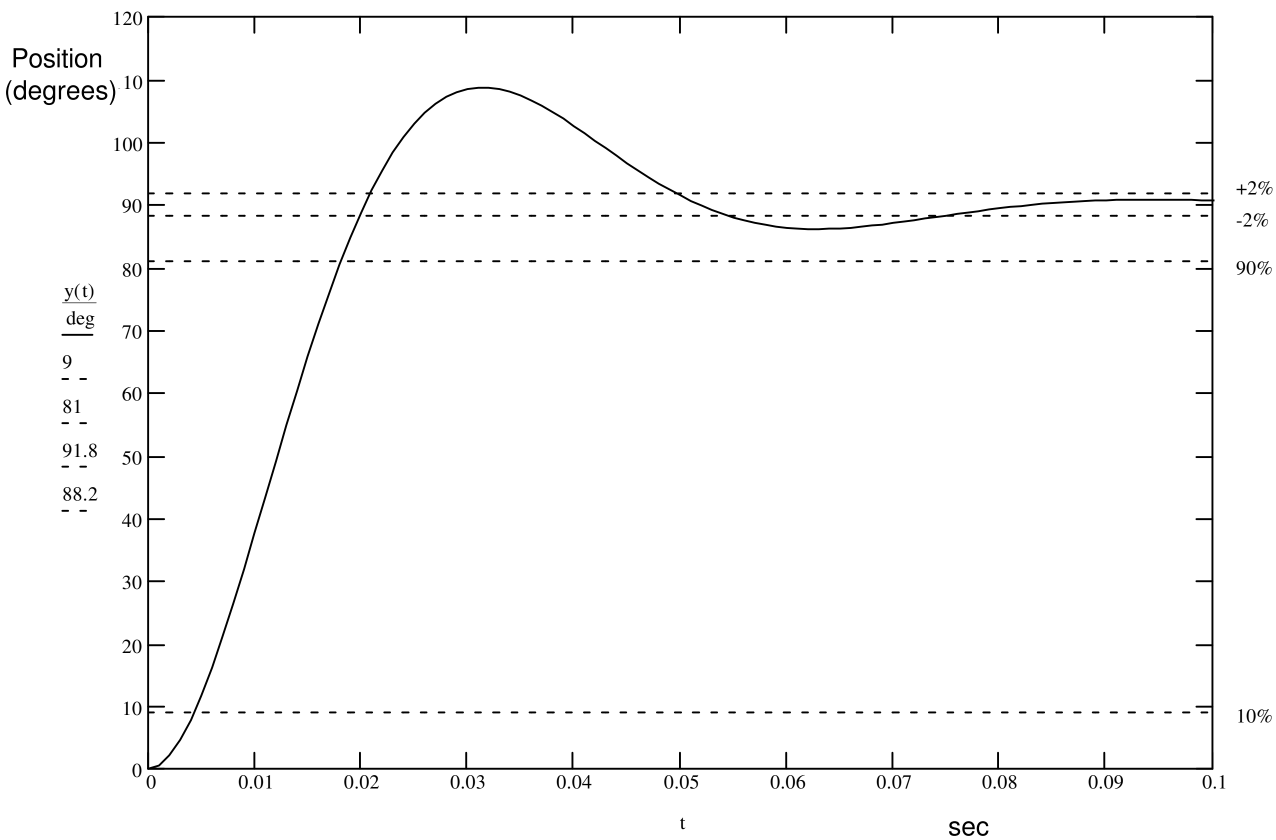
$$a^2+b^2 \quad = \quad 50^2+100^2 = 1.25\cdot 10^4 \quad = k_p\cdot 940$$

$$k_p := \frac{1.25\cdot 10^4}{940} \qquad k_p = 13.298$$

Input step: $90\cdot deg = 1.571\cdot rad \qquad X(s) = \frac{1.571}{s}$

Step response: $Y(s) = X(s)\cdot H(s) \quad = \frac{1.571}{s}\cdot\frac{13.3\cdot 940}{(s+50)^2+100^2} \qquad x_m\cdot H(0) = \frac{1.571\cdot 13.3\cdot 940}{(0+50)^2+100^2} = 1.571$

$$y(t) := 1.571 - 1.571\cdot e^{-50\cdot t}\cdot\cos(100\cdot t) - 1.571\cdot\frac{50}{100}\cdot e^{-50\cdot t}\cdot\sin(100\cdot t)$$ From eq. 3.37, p.33



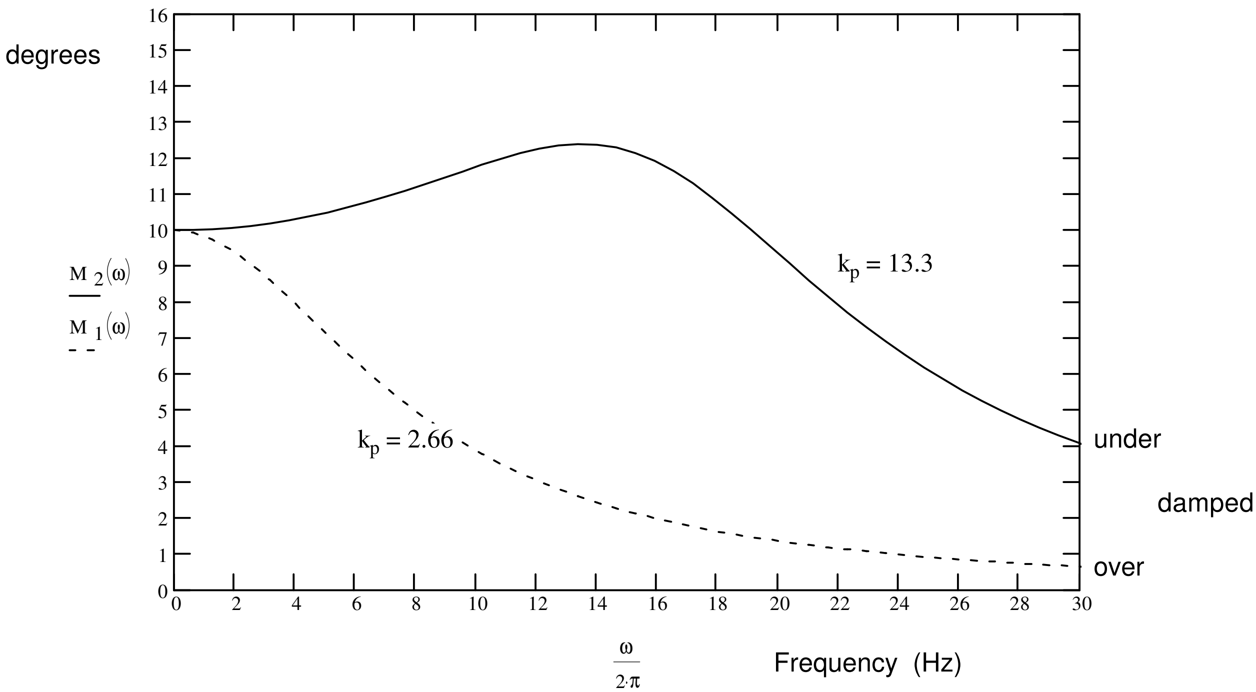
Frequency Response Curves

$k_p := 2.66 \qquad M_1(\omega) := \left|\dfrac{10\cdot k_p\cdot 940}{-\omega^2 + 100\cdot j\cdot\omega + k_p\cdot 940}\right| \qquad \text{Phase}_1(\omega) := \arg\left(\dfrac{k_p\cdot 940}{-\omega^2 + 100\cdot j\cdot\omega + k_p\cdot 940}\right)$

$k_p := 13 \qquad M_2(\omega) := \left|\dfrac{10\cdot k_p\cdot 940}{-\omega^2 + 100\cdot j\cdot\omega + k_p\cdot 940}\right| \qquad \text{Phase}_2(\omega) := \arg\left(\dfrac{k_p\cdot 940}{-\omega^2 + 100\cdot j\cdot\omega + k_p\cdot 940}\right)$

(Note that I used 10 rather than 1.571 / 9 to get degrees)

Magnitude of the Transfer function



Phase of the Transfer function