

**University of Utah**  
**Electrical & Computer Engineering Department**  
ECE 3510 Lab 4  
**DC Motor Characteristics**

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rev,2/1/07, 2/5/08, 2/9/09

**Note:** Bring the lab-2 handout to use as a dSpace tutorial.

### **Objectives**

- Perform a number of tests to determine the individual parameters which determine the DC motor's characteristics.
- Create a full model of the motor and compare the theoretical results from this full model to a measured angular velocity step response.
- Explore the sensitivity of the model to the individual motor parameters.
- Tweak one of the parameters to improve the model.
- Learn how the position encoder works.

### **Equipment and materials from stockroom:**

- DC Permanent-magnet Motor (If you can, get the same-number motor you used last time. This is the motor you will characterize)
- Small DC motor (Only used to drive the other one as a generator)
- Motor rack & BOB (Bucket of Bolts)
- Dual Power Amp
- DSpace kit

### **Experiment**

If you page through this lab assignment, you'll see that it looks quite different from what you're used to, it's much less wordy. At each step along the way, you will get only a short description of what to do. That's followed by the results that I got when I did that part of the lab and the calculations that I made with that data to get the parameters that I am interested in. Your job will be to roughly reproduce what I have done and to get similar (but not exactly the same) results.

Hook up the computer, dSpace box, power amp, but not the motor as you did in lab 2 Refer to the lab 2 handout if necessary. Leave the power amp switched off for now. Download the experiment files for lab 2 from one of the two sites listed below (If they are not still on the computer). You've used these files before. You can continue to setup the software and data saving now, but you won't need to use that for a while yet.

<http://www.ece.utah.edu/~ece3510/> ----> Labs

## DC Motor Basics

The DC permanent-magnet motor is modeled as a resistor ( $R_a$ ) in series with an inductance ( $L_a$ ) and a voltage source that depends on the angular velocity of the motor.

$$\text{Voltage generated inside the armature} = K_V \cdot \omega \quad (\omega \text{ is angular velocity}) \quad K_V = \text{Voltage constant} \left[ \frac{\text{V}}{\left( \frac{\text{rad}}{\text{sec}} \right)} \right]$$

When current flows through the armature, the magnetic fields create a torque.

$$\text{Torque} = T = K_T \cdot i_a \quad K_T = \text{Torque constant} \left( \frac{\text{N}\cdot\text{m}}{\text{A}} \right)$$

$$\text{Theoretically, } K_T = K_V$$

This torque goes to overcoming friction and accelerating the motor (and attachments).

$$\text{Torque: } T = K_T \cdot I_a(s) = J \cdot \frac{d^2}{dt^2} \theta + B_m \cdot \frac{d}{dt} \theta + C_m$$

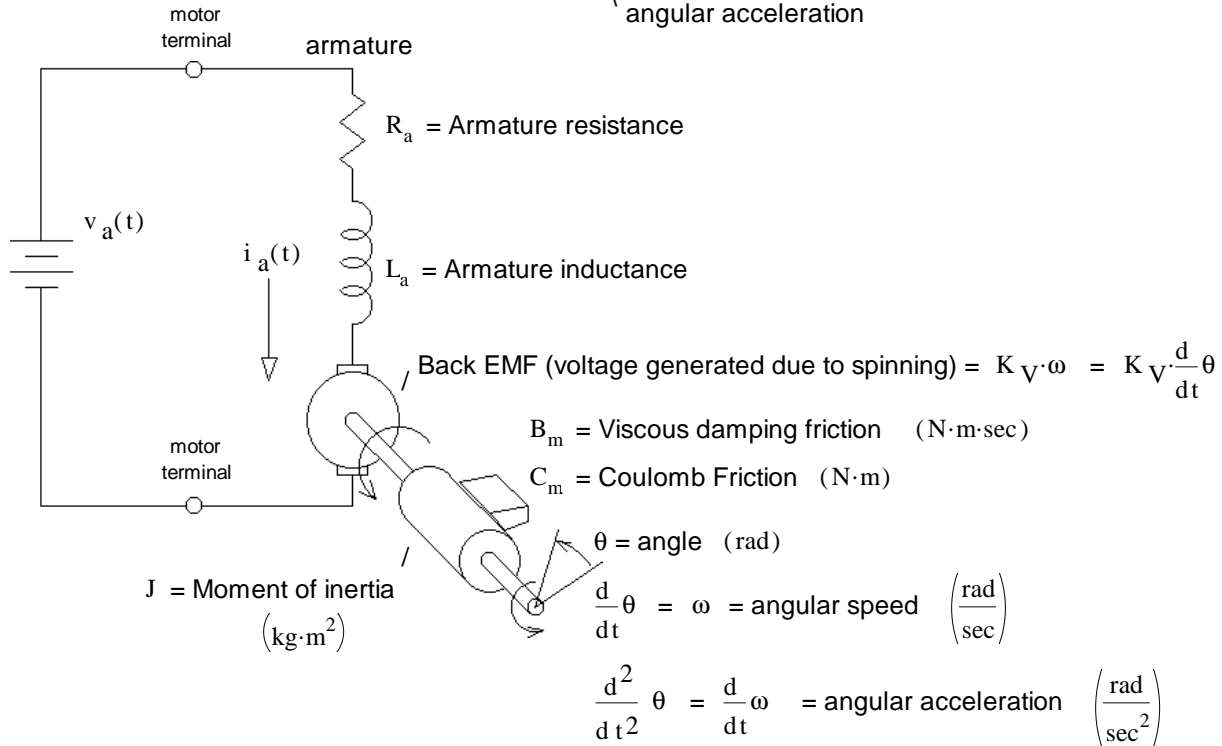
The friction comes in two varieties, one that depends on angular velocity:  $B_m = \text{Viscous damping friction}$   $\left[ \frac{\text{N}\cdot\text{m}}{\left( \frac{\text{rad}}{\text{sec}} \right)} \right]$   
and one that doesn't  $C_m = \text{Coulomb Friction}$  (N·m) This one is just a constant.

The moment of inertia plays the same roll in rotational systems that mass does in linear systems.

$$J = \text{Moment of inertia of the spinning part of the motor and what it's attached to. } (\text{kg}\cdot\text{m}^2)$$

$$\text{what's left after friction losses} \quad \text{Torque} = J \cdot \left( \frac{d}{dt} \omega \right) \quad \text{Just like: } F = M \cdot a$$

\ angular acceleration



## Procedures

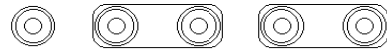
### $R_a$ & $L_a$

Use the HP 34401A DVM to measure the armature resistance at 5 to 10 different shaft positions and take the average:

$$R_a := \frac{2.43 + 2.95 + 3.35 + 2.75 + 3.315 + 2.684 + 2.84 + 2.69 + 2.48 + 2.38}{10} \cdot \Omega \quad R_a = 2.787 \cdot \Omega$$

Use the HP LCR meter in lab (set to 120 Hz) to measure the armature inductance at 5 to 10 different shaft positions and take the average:

Note if you remove the test fixture be sure to bind these posts together:



$$L_a := \frac{3.68 + 3.74 + 3.65 + 4.01 + 3.84 + 3.72 + 4.17 + 4.12 + 3.70 + 3.71}{10} \cdot \text{mH}$$

$$L_a = 3.834 \cdot \text{mH}$$

## $K_V$ & $K_T$

Using the bucket O' bolts and the motor rack, couple the small motor to the big one that you are characterizing and secure them both to the rack. Use a rubber coupler between the two motors and don't push them tightly against each other. Leave plenty of slop and play. Hook the power amp to the small motor (red and black terminals). Hook up the encoder on the big motor so you can see the angular velocity on the computer. Hook a voltmeter up to the big motor terminals. Turn on the power amp.

Make the small motor turn the big one and measure the generated voltage at several different speeds. When you read the angular velocity on the computer you may notice that it bounces between two values that are significantly different. This is an artifact of how the speed is computed and I will try to get it changed. In the meantime make a mental average of the two readings and record that value.

My measurements:

$$\omega_g := \begin{bmatrix} 0 \\ 50.5 \\ 217 \\ 277 \\ 385 \end{bmatrix} \cdot \frac{\text{rad}}{\text{sec}}$$

$$V_g := \begin{bmatrix} 0 \\ 5.36 \\ 22.5 \\ 29.14 \\ 40.5 \end{bmatrix} \cdot \text{V}$$

Enter your data into matlab by hand and plot it.

Linear regression:

You can use "polyfit (x,y,1)" to find the slope and y-intercept of the best-fit straight line through your data and "polyval" to plot it on top of your measured data. The functions below do the same for me in Mathcad.

$$\text{slp} := \text{slope}(\omega_g, V_g) \quad \text{Mathcad function}$$

$$\text{slp} = 0.105 \cdot \frac{\text{V}}{\left(\frac{\text{rad}}{\text{sec}}\right)}$$

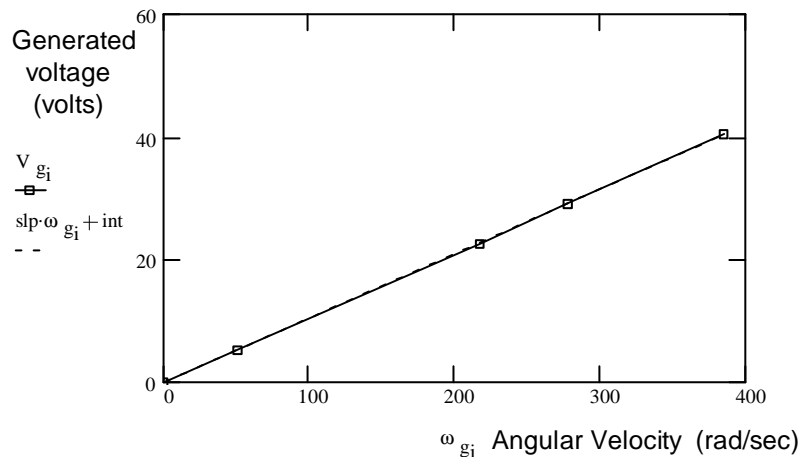
$$\text{int} := \text{intercept}(\omega_g, V_g) \quad \text{int} = -0.026 \cdot \text{V}$$

Intercept is close to zero, as it should be.

Voltage generation constant

$$K_V := \text{slp} \quad \text{slope of line}$$

$$K_V = 0.105 \cdot \frac{\text{V}}{\left(\frac{\text{rad}}{\text{sec}}\right)}$$



We don't have a good way to measure torque so we'll assume that the torque constant is the equal to the generation constant, as it theoretically should be.

$$K_T := K_V \quad K_T = 0.105 \cdot \frac{\text{N}\cdot\text{m}}{\text{A}}$$

Compare to the published specs for this motor:

$$\text{Specified } K_T = K_{Ts} := \frac{14.8 \cdot \text{ozf}\cdot\text{in}}{\text{amp}} = K_{Ts} = 0.105 \cdot \frac{\text{N}\cdot\text{m}}{\text{A}}$$

note: ozf = oz force

$$\text{Specified } K_V = K_{Vs} := 11 \cdot \frac{\text{V}}{1000\text{-rpm}} = K_{Vs} = 0.105 \cdot \frac{\text{V}}{\left(\frac{\text{rad}}{\text{sec}}\right)}$$

## B<sub>m</sub> & C<sub>m</sub>

The next test will consist of free-running the motor at several different speeds. When the motor shaft is disconnected from all loads, any torque it develops is dissipated as friction, so this is a good way to find that friction. Turn off the power amp and decouple the small motor from the big motor. Setup the HP 34401A DVM to read current and hook the +25V terminals of the HP E3631A power supply through the ammeter to the big motor.

Let the motor free-run (fr) with various input voltages

Measurements

$$V_{in} := \begin{bmatrix} 5 \\ 8 \\ 10 \\ 15 \\ 20 \\ 25 \end{bmatrix} \cdot V \quad I_{fr} := \begin{bmatrix} 0.14 \\ 0.146 \\ 0.155 \\ 0.173 \\ 0.182 \\ 0.186 \end{bmatrix} \cdot A \quad \omega_{fr} := \begin{bmatrix} 43 \\ 71 \\ 90 \\ 137 \\ 184 \\ 231 \end{bmatrix} \cdot \frac{\text{rad}}{\text{sec}}$$

Calculate torques:

$$T_{fr_i} := I_{fr_i} \cdot K_T = T_{fr} = \begin{bmatrix} 0.0147 \\ 0.0153 \\ 0.0163 \\ 0.0182 \\ 0.0191 \\ 0.0195 \end{bmatrix} \cdot N \cdot m$$

All this torque must be lost to friction

V<sub>in</sub> is unneeded

Linear regression

$$\text{slp} := \text{slope}(\omega_{fr}, T_{fr}) \quad \text{slp} = 2.76 \cdot 10^{-5} \cdot \frac{N \cdot m}{\left(\frac{\text{rad}}{\text{sec}}\right)} \quad \text{int} := \text{intercept}(\omega_{fr}, T_{fr}) \quad \text{int} = 1.371 \cdot 10^{-2} \cdot N \cdot m$$

Viscous damping

$$B_m := \text{slp} \quad \text{slope of line}$$

$$B_m = 2.76 \cdot 10^{-5} \cdot \frac{N \cdot m}{\frac{\text{rad}}{\text{sec}}}$$

Constant (Coulomb) friction

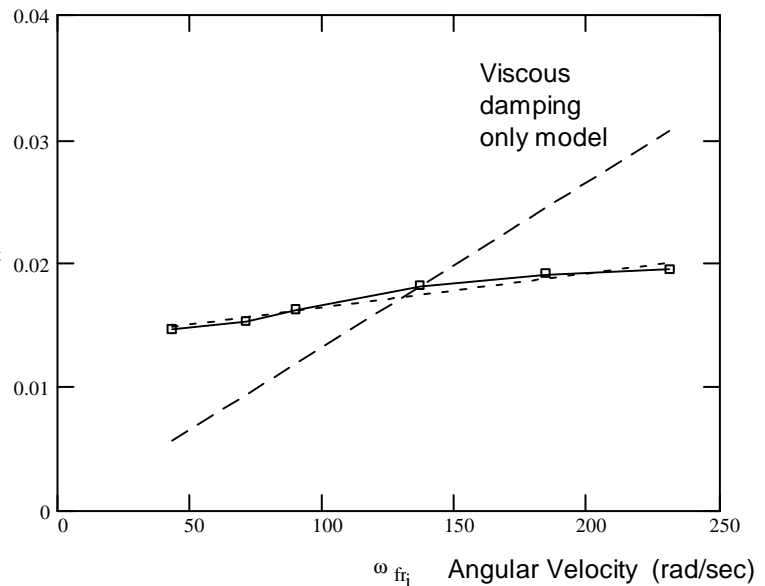
$$C_m := \text{int}$$

$$C_m = 0.014 \cdot N \cdot m$$

If I try to force the friction to all be viscous damping and use measurements near the center:

$$B_{m2} := \frac{0.0182 \cdot N \cdot m}{137 \cdot \frac{\text{rad}}{\text{sec}}}$$

T<sub>fr<sub>i</sub></sub>  
 slp · ω<sub>fr<sub>i</sub></sub> + int  
 B<sub>m2</sub> · ω<sub>fr<sub>i</sub></sub>



Now turn down the voltage to about 0.7V. Then slowly continue turning down the voltage until the motor stops turning. At this voltage and current the motor torque just balances the coulomb friction.

Measurements:  $I_{stall} := 130 \cdot \text{mA}$

$V_{stall} := 0.4 \cdot V$

Calculated from C<sub>m</sub> found above:

$$I_{stall} := \frac{C_m}{K_V}$$

$I_{stall} = 130.6 \cdot \text{mA}$

$V_{stall} := I_{stall} \cdot R_a$

$V_{stall} = 0.364 \cdot V$

Comparing these numbers, it looks like our C<sub>m</sub> checks out pretty well.

Compare to the steady-state error from lab 3, critically damped curve:

gain:  $k_p := 2.66 \cdot \frac{V}{\text{rad}}$

$\frac{V_{stall}}{k_p} = 0.137 \cdot \text{rad}$

$\frac{V_{stall}}{k_p} = 7.837 \cdot \text{deg}$

## J, the Moment of Inertia

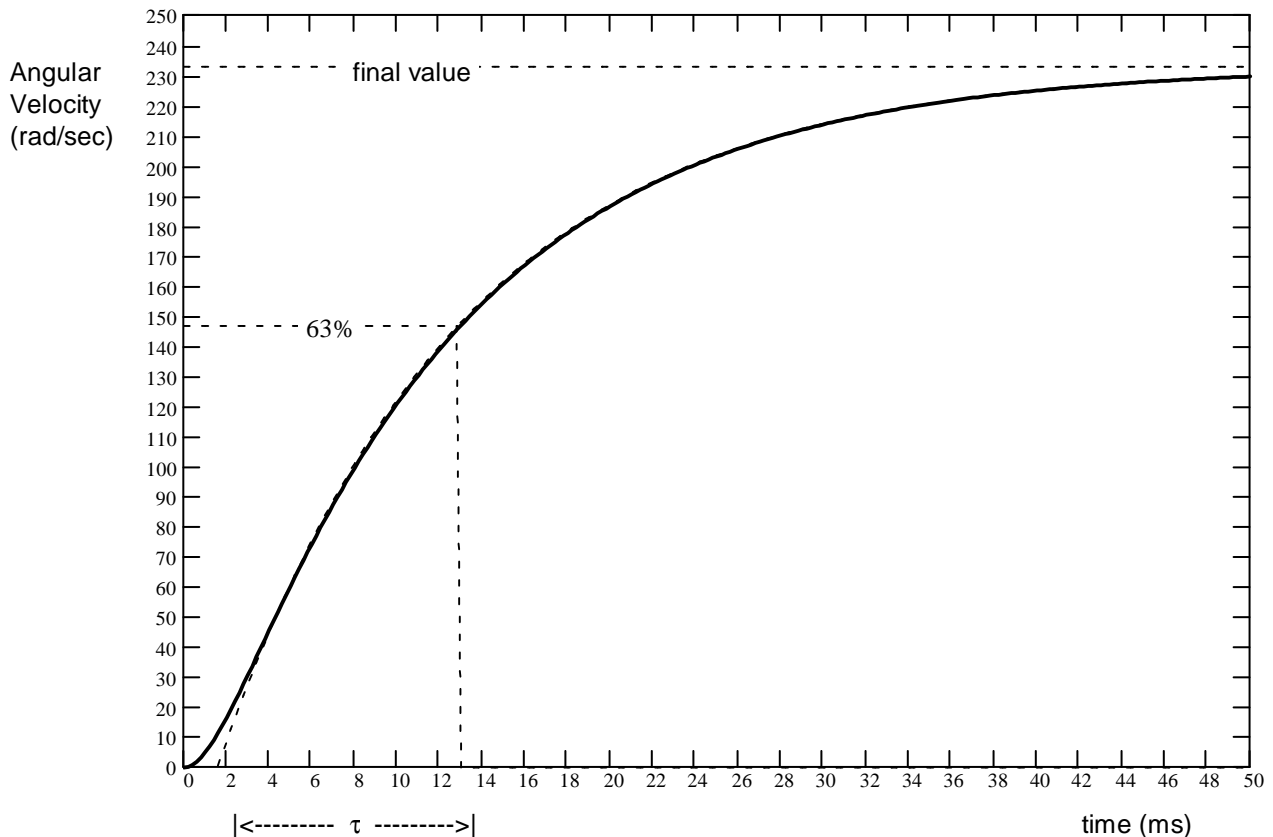
To get the Motor's moment of inertia (with coupler) we'll go back to the very simple motor transfer function used in the first lab:

$$\frac{\omega(s)}{V_a(s)} = \frac{K_T}{J \cdot R_a \cdot s + (B_m \cdot R_a + K_T \cdot K_V)} = \frac{\frac{K_T}{J \cdot R_a}}{s + \frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot R_a}} = \frac{k}{s + a} \quad a = \frac{1}{\tau}$$

$$\tau = \frac{J \cdot R_a}{B_m \cdot R_a + K_T \cdot K_V}$$

Also set up the equipment as you did in the first lab.

Use a 25V step input and take data to get curve similar to what you did in the first lab. Accurately measure and record the step voltage (the 25V), don't just depend on what the slider says. Move this data into Matlab and make a plot like the one below. You'll need to get the time constant of the big curve from this plot. If you can do that from the computer screen then you don't need to print the plot. Don't include the first part of the curve in the time constant, see below.



Measurement:

$$\tau := 0.0115 \cdot \text{sec} \quad a := \frac{1}{\tau} \quad a = 86.957 \cdot \text{sec}^{-1} \quad J := \frac{\tau \cdot (B_m \cdot R_a + K_T \cdot K_V)}{R_a} \quad J = 4.584 \cdot 10^{-5} \cdot \text{kg} \cdot \text{m}^2$$

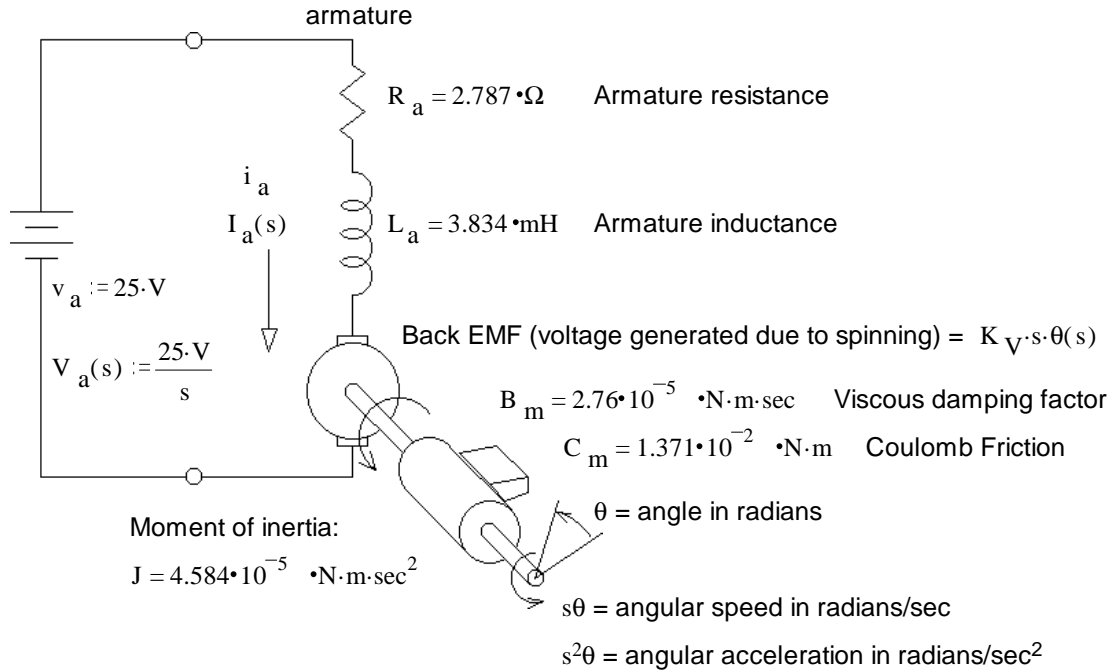
Compare to the published specifications for this motor:

$$J_s := 8.8 \cdot 10^{-3} \cdot \text{ozf} \cdot \text{in} \cdot \text{sec}^2 = J_s = 6.214 \cdot 10^{-5} \cdot \text{kg} \cdot \text{m}^2$$

Not too close on this one, I wonder about this measurement and about the spec. You'll get a chance to tweak this value later.

## Full model of DC permanent-magnet motor

It's time to develop the full model of DC permanent-magnet motor, including all the parameters that we've just found. You don't have to enter this in your notebook, just try to follow along. I added in the  $C_m$  as a constant in one direction because in the step response the motor is always running in only one direction. In general  $C_m$  is much more complex to include.



$$\text{Torque: } T(s) = J \cdot (s^2 \cdot \theta(s)) + B_m \cdot (s \cdot \theta(s)) + \frac{C_m}{s} = K_T \cdot I_a(s) = K_T \cdot \frac{V_a(s) - K_V \cdot s \cdot \theta(s)}{R_a + L_a \cdot s}$$

$$J \cdot (s^2 \cdot \theta(s)) + B_m \cdot (s \cdot \theta(s)) + \frac{C_m}{s} = K_T \cdot \frac{V_a(s) - K_V \cdot s \cdot \theta(s)}{R_a + L_a \cdot s}$$

$$\left[ J \cdot (s^2 \cdot \theta(s)) + B_m \cdot (s \cdot \theta(s)) + \frac{C_m}{s} \right] \cdot (R_a + L_a \cdot s) = K_T \cdot (V_a(s) - K_V \cdot s \cdot \theta(s))$$

$$J \cdot s^2 \cdot \theta(s) \cdot R_a + J \cdot s^3 \cdot \theta(s) \cdot L_a + B_m \cdot s \cdot \theta(s) \cdot R_a + B_m \cdot s^2 \cdot \theta(s) \cdot L_a + C_m \cdot L_a + \frac{C_m}{s} \cdot R_a = K_T \cdot V_a(s) - K_T \cdot K_V \cdot s \cdot \theta(s)$$

$$J \cdot s^2 \cdot \theta(s) \cdot R_a + J \cdot s^3 \cdot \theta(s) \cdot L_a + B_m \cdot s \cdot \theta(s) \cdot R_a + B_m \cdot s^2 \cdot \theta(s) \cdot L_a + C_m \cdot L_a + \frac{C_m}{s} \cdot R_a + K_T \cdot K_V \cdot s \cdot \theta(s) = K_T \cdot V_a(s)$$

$$\left( J \cdot s^2 \cdot R_a + J \cdot s^3 \cdot L_a + B_m \cdot s \cdot R_a + B_m \cdot s^2 \cdot L_a + K_T \cdot K_V \cdot s \right) \cdot \theta(s) = V_a(s) \cdot K_T - \left( C_m \cdot L_a + \frac{C_m}{s} \cdot R_a \right)$$

$$\theta(s) = V_a(s) \cdot \frac{K_T}{J \cdot L_a \cdot s^3 + (J \cdot R_a + B_m \cdot L_a) \cdot s^2 + (B_m \cdot R_a + K_T \cdot K_V) \cdot s} - \frac{C_m \cdot L_a + \frac{C_m}{s} \cdot R_a}{J \cdot L_a \cdot s^3 + (J \cdot R_a + B_m \cdot L_a) \cdot s^2 + (B_m \cdot R_a + K_T \cdot K_V) \cdot s}$$

$$\omega(s) = V_a(s) \cdot \frac{K_T}{J \cdot L_a \cdot s^2 + (J \cdot R_a + B_m \cdot L_a) \cdot s + (B_m \cdot R_a + K_T \cdot K_V)} - \frac{C_m \cdot L_a + \frac{C_m}{s} \cdot R_a}{J \cdot L_a \cdot s^2 + (J \cdot R_a + B_m \cdot L_a) \cdot s + (B_m \cdot R_a + K_T \cdot K_V)}$$

$$\omega(s) = V_a(s) \cdot \frac{\frac{K_T}{J \cdot L_a}}{s^2 + \left(\frac{R_a}{L_a} + \frac{B_m}{J}\right) \cdot s + \left(\frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot L_a}\right)} - \frac{\frac{C_m}{J} + \frac{C_m \cdot R_a}{s \cdot J \cdot L_a}}{s^2 + \left(\frac{R_a}{L_a} + \frac{B_m}{J}\right) \cdot s + \left(\frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot L_a}\right)}$$

Find the poles:  $s^2 + \left(\frac{R_a}{L_a} + \frac{B_m}{J}\right) \cdot s + \left(\frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot L_a}\right) = 0$

$$a_1 := \frac{1}{2} \cdot \left(\frac{R_a}{L_a} + \frac{B_m}{J}\right) + \sqrt{\frac{1}{4} \cdot \left(\frac{R_a}{L_a} + \frac{B_m}{J}\right)^2 - \left(\frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot L_a}\right)} \quad a_1 = 626.622 \cdot \text{sec}^{-1}$$

$$a_2 := \frac{1}{2} \cdot \left(\frac{R_a}{L_a} + \frac{B_m}{J}\right) - \sqrt{\frac{1}{4} \cdot \left(\frac{R_a}{L_a} + \frac{B_m}{J}\right)^2 - \left(\frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot L_a}\right)} \quad a_2 = 100.871 \cdot \text{sec}^{-1}$$

If I define:  $k_1 := \frac{K_T}{J \cdot L_a}$  and neglect the nonlinear, constant (Coulomb) friction then the transfer function becomes

$$\frac{\omega(s)}{V_a(s)} = \frac{k_1}{(s + a_1) \cdot (s + a_2)}$$

$$k_1 = 5.976 \cdot 10^5 \cdot \frac{\text{rad}}{\text{V} \cdot \text{sec}^3}$$

And the response to a step of  $v_a$  volts becomes:

$$\begin{aligned} \frac{v_a \cdot k_1}{s \cdot (s + a_1) \cdot (s + a_2)} &= \frac{A}{s} + \frac{B}{s + a_1} + \frac{C}{s + a_2} \\ &= \frac{v_a \cdot k_1}{(a_1 \cdot a_2)} \cdot \frac{1}{s} + \frac{v_a \cdot k_1}{(a_1 - a_2) \cdot a_1} \cdot \frac{1}{(s + a_1)} + \frac{v_a \cdot k_1}{(a_2 - a_1) \cdot a_2} \cdot \frac{1}{(s + a_2)} \end{aligned}$$

Deal with the part due to Coulomb friction:

$$\begin{aligned} -\frac{\left(\frac{C_m}{J} + \frac{C_m \cdot R_a}{J \cdot L_a} \cdot \frac{1}{s}\right)}{(s + a_1) \cdot (s + a_2)} &= \frac{-\left(\frac{C_m}{J}\right)}{(s + a_1) \cdot (s + a_2)} + \frac{-\left(\frac{C_m \cdot R_a}{J \cdot L_a}\right)}{s \cdot (s + a_1) \cdot (s + a_2)} \quad k_2 := -\left(\frac{C_m}{J}\right) \\ &= \frac{k_2}{(s + a_1) \cdot (s + a_2)} + \frac{k_3}{s \cdot (s + a_1) \cdot (s + a_2)} \quad k_3 := -\left(\frac{C_m \cdot R_a}{J \cdot L_a}\right) \\ \frac{k_2}{(s + a_1) \cdot (s + a_2)} &= \frac{-k_2}{(a_1 - a_2)} \cdot \frac{1}{(s + a_1)} + \frac{-k_2}{(a_2 - a_1)} \cdot \frac{1}{(s + a_2)} \quad k_2 = -299.141 \cdot \frac{1}{\text{sec}^2} \\ \frac{k_3}{s \cdot (s + a_1) \cdot (s + a_2)} &= \frac{k_3}{(a_1 \cdot a_2)} \cdot \frac{1}{s} + \frac{k_3}{a_1 \cdot (a_1 - a_2)} \cdot \frac{1}{(s + a_1)} + \frac{k_3}{a_2 \cdot (a_2 - a_1)} \cdot \frac{1}{(s + a_2)} \quad k_3 = -2.174 \cdot 10^5 \cdot \frac{1}{\text{sec}^3} \end{aligned}$$

Only good for one direction of rotation, otherwise signs have to be reversed

All together now in the time domain:

$$\omega(t) := \frac{v_a \cdot k_1 + k_3}{a_1 \cdot a_2} + \frac{1}{(a_1 - a_2)} \cdot \left(\frac{v_a \cdot k_1}{a_1} - k_2 + \frac{k_3}{a_1}\right) \cdot e^{-a_1 \cdot t} + \frac{1}{(a_2 - a_1)} \cdot \left(\frac{v_a \cdot k_1}{a_2} - k_2 + \frac{k_3}{a_2}\right) \cdot e^{-a_2 \cdot t}$$

## Plot and Compare

Now, you have to do the following to plot your theoretical curve on the same plot as your measured data.

Enter your parameters:  $R_a = 2.787 \cdot \Omega$      $L_a = 3.834 \cdot 10^{-3} \cdot \text{henry}$      $K_V = 0.105 \cdot \text{V} \cdot \text{sec}$      $K_T := K_V$   
 $B_m = 2.76 \cdot 10^{-5} \cdot \text{N} \cdot \text{m} \cdot \text{sec}$      $C_m = 0.014 \cdot \text{N} \cdot \text{m}$      $J = 4.584 \cdot 10^{-5} \cdot \text{kg} \cdot \text{m}^2$

Calculate the following (and make it automatic, so you can play with the numbers above and see the effects on the plot):

$$a_1 := \frac{1}{2} \cdot \left( \frac{R_a}{L_a} + \frac{B_m}{J} \right) + \sqrt{\frac{1}{4} \cdot \left( \frac{R_a}{L_a} + \frac{B_m}{J} \right)^2 - \left( \frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot L_a} \right)} \quad a_1 = 626.622 \cdot \text{sec}^{-1}$$

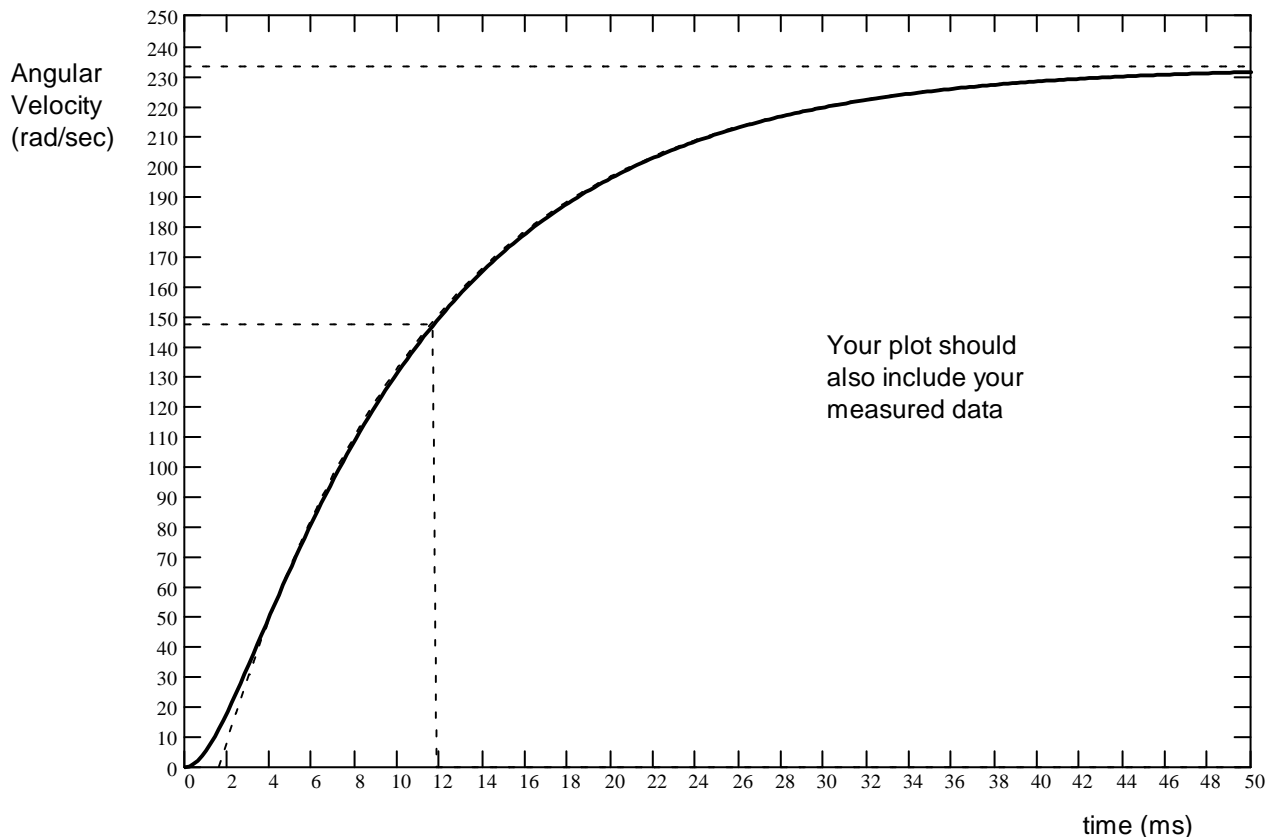
$$a_2 := \frac{1}{2} \cdot \left( \frac{R_a}{L_a} + \frac{B_m}{J} \right) - \sqrt{\frac{1}{4} \cdot \left( \frac{R_a}{L_a} + \frac{B_m}{J} \right)^2 - \left( \frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot L_a} \right)} \quad a_2 = 100.871 \cdot \text{sec}^{-1}$$

$$k_1 := \frac{K_T}{J \cdot L_a} \quad k_2 := -\left( \frac{C_m}{J} \right) \quad k_3 := -\left( \frac{C_m \cdot R_a}{J \cdot L_a} \right)$$

$$\omega(t) := \frac{v_a \cdot k_1 + k_3}{a_1 \cdot a_2} + \frac{1}{(a_1 - a_2)} \cdot \left( \frac{v_a \cdot k_1}{a_1} - k_2 + \frac{k_3}{a_1} \right) \cdot e^{-a_1 \cdot t} + \frac{1}{(a_2 - a_1)} \cdot \left( \frac{v_a \cdot k_1}{a_2} - k_2 + \frac{k_3}{a_2} \right) \cdot e^{-a_2 \cdot t}$$

If:  $v_a = 25 \cdot \text{V}$     then     $\omega_{\text{fin}} := \frac{v_a \cdot k_1 + k_3}{a_1 \cdot a_2}$      $\omega_{\text{fin}} = 233 \cdot \frac{\text{rad}}{\text{sec}}$

To find the time constant:     $0.632 \cdot \omega_{\text{fin}} = 147.214 \cdot \frac{\text{rad}}{\text{sec}}$





## Sensitivities

Next you'll artificially change each of the parameters in turn and see how much they affect the theoretical plot. You may want to add a third curve to your plot that will be the one that you change. That way the original can be a reference. Otherwise your measured data can be your reference.

Change each parameter by a factor of 2 (either bigger or smaller), see how the curve is affected, and note the type and relative magnitude of the affect in your notebook. Then change it back to what it was and try a different parameter. In the end I want you to rank the parameters from those that have the most affect on the curve to those that have the least. You are finding out how sensitive the output is to each parameter.

A few interesting things to try:

You probably found that  $L_a$  has very little effect on the curve. Try setting  $R_a$  to 20% of what it was and then messing with  $L_a$ . It has a bit more effect now, doesn't it?

Set  $R_a$  back to normal and change  $L_a$  to 1% of what it was. Note that the first curve goes away, as expected, but also note where the old and new curves cross, almost right at the 63% line, makes me wonder about eliminating the first curve from the time constant measurement.

Set  $L_a$  back to normal,  $C_m$  to 0 and  $B_m$  to  $B_{m2}$ . This eliminates the nonlinear friction and replaces it with viscous damping only. Curve still looks pretty good, doesn't it? However, there are cases where the  $C_m$  makes a big difference, like the steady-state error in the last lab. Set all the parameters back to normal.

Tweak one or two of your parameters (probably just J) to make your theoretical curve match your measured curve better. Consider the "tweaked" parameters as the true values for future use.

## Encoder

One last thing that really doesn't have much to do with the rest of this lab. The little black plastic box on the back of your motor is called an encoder. It contains a little disk with slots in it and two light sensors. Your TA has a broken one that you can look at and a little circuit board which can plug into a good one to illustrate how it works. It is probably set up somewhere in the lab, ask to see it. Look at the disk with a magnifying glass and play with the working one to see if you can figure out the pattern. Record that pattern in your notebook either as waveforms or as 1's and 0's. How can a digital circuit work out direction, speed, and location from these two signals? These discs have 500 slots. How many transitions is that per revolution? What is the smallest angular change that can be measured? If the rotational velocity is calculated every 2ms, what is the smallest increment of speed that can be measured (1 transitions in 2ms). This is why our speed display is so bouncy (unless it's been fixed to average many readings).

There is actually a third light sensor in the encoder that picks up only one slot in the whole revolution. It is connected to the extra wire that is not used on our encoder and it can give a "zero" or start location.

**Conclusion** Check - off and conclude as always.