DC Motor Basics

The DC permanent-magnet motor is modeled as a resistor (R_a) in series with an inductance (L_a) and a voltage source that depends on the angular velocity of the motor.

Voltage generated inside the armature = $K_V \cdot \omega$ (ω is angular velocity) K_V = Voltage constant $\left[\frac{V}{\left(\frac{rad}{sec}\right)}\right]$

When current flows through the armature, the magnetic fields create a torque.

Torque = $T = K_T \cdot i_a$ K_T = Torque constant $\left(\frac{N \cdot m}{A}\right)$

Theoretically, $K_T = K_V$

This torque goes to overcoming friction and accelerating the motor (and attachments).

$$\text{Torque:} \quad T = K_T \cdot I_a(s) \quad = \quad J \cdot \frac{d^2}{dt^2}\theta + B_m \cdot \frac{d}{dt}\theta + C_m$$

The friction comes in two varieties, one that defends on angular velocity: B_m = Viscous damping friction $\left[\frac{N \cdot m}{\left(\frac{rad}{sec}\right)}\right]$

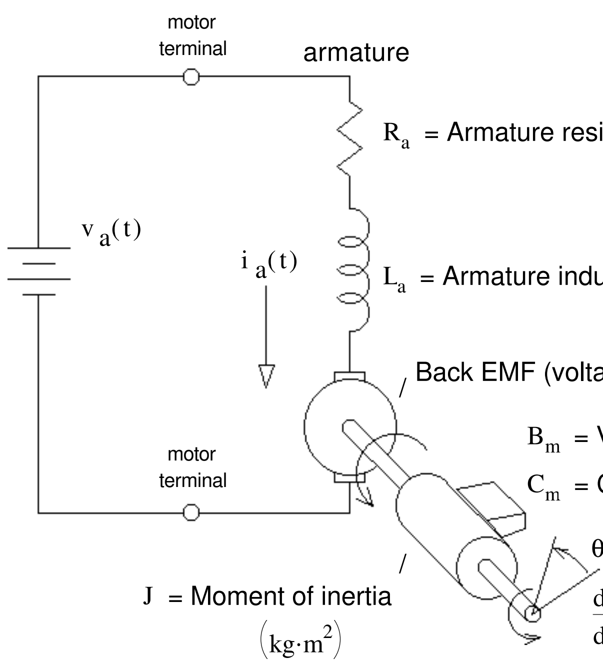
and one that doesn't C_m = Coulomb Friction $(N \cdot m)$ This one is just a constant.

The moment of inertia plays the same roll in rotational systems that mass does in linear systems.

J = Moment of inertia of the spinning part of the motor and what it's attached to. $\left(kg \cdot m^2\right)$

what's left after friction losses Torque = $J \cdot \left(\frac{d}{dt}\omega\right)$ Just like: $F = M \cdot a$

\ angular acceleration



R_a = Armature resistance

L_a = Armature inductance

/ Back EMF (voltage generated due to spinning) = $K_V \cdot \omega = K_V \cdot \frac{d}{dt}\theta$

B_m = Viscous damping friction $(N \cdot m \cdot sec)$

C_m = Coulomb Friction $(N \cdot m)$

θ = angle (rad)

/ J = Moment of inertia $\left(kg \cdot m^2\right)$

$\frac{d}{dt}\theta = \omega$ = angular speed $\left(\frac{rad}{sec}\right)$

$\frac{d^2}{dt^2}\theta = \frac{d}{dt}\omega$ = angular acceleration $\left(\frac{rad}{sec^2}\right)$

Procedures

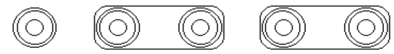
R_a & L_a

Use the HP 34401A DVM to measure the armature resistance at 5 to 10 different shaft positions and take the average:

$$R_a := \frac{2.43 + 2.95 + 3.35 + 2.75 + 3.315 + 2.684 + 2.84 + 2.69 + 2.48 + 2.38}{10} \cdot \Omega \qquad R_a = 2.787 \cdot \Omega$$

Note: let each reading settle before you record it.

Use the HP LCR meter in lab (set to 120 Hz) to measure the armature inductance at 5 to 10 different shaft positions and take the average:

Note if you remove the test fixture be sure to bind these posts together: 

$$L_a := \frac{3.68 + 3.74 + 3.65 + 4.01 + 3.84 + 3.72 + 4.17 + 4.12 + 3.70 + 3.71}{10} \cdot mH \qquad L_a = 3.834 \cdot mH$$

K_V & K_T

Using the bucket O' bolts and the motor rack, couple the small motor to the big one that you are characterizing and secure them both to the rack. Use a rubber coupler between the two motors and don' t push them tightly against each other. Leave plenty of slop and play. Hook the power amp to the small motor (red and black terminals). Hook up the encoder on the big motor so you can see the angular velocity on the computer. Hook a voltmeter up to the big motor terminals. Turn on the power amp.

Make the small motor turn the big one and measure the generated voltage at several different speeds. When you read the angular velocity on the computer you may notice that it bounces between two values that are significantly different. this is an artifact of how the speed is computed and I will try to get it changed. In the meantime make a mental average of the two readings and record that value.

My measurements:

$$\omega_g := \begin{bmatrix} 0 \\ 50.5 \\ 217 \\ 277 \\ 385 \end{bmatrix} \cdot \frac{rad}{sec} \qquad V_g := \begin{bmatrix} 0 \\ 5.36 \\ 22.5 \\ 29.14 \\ 40.5 \end{bmatrix} \cdot V$$

Enter your data into matlab by hand and plot it.

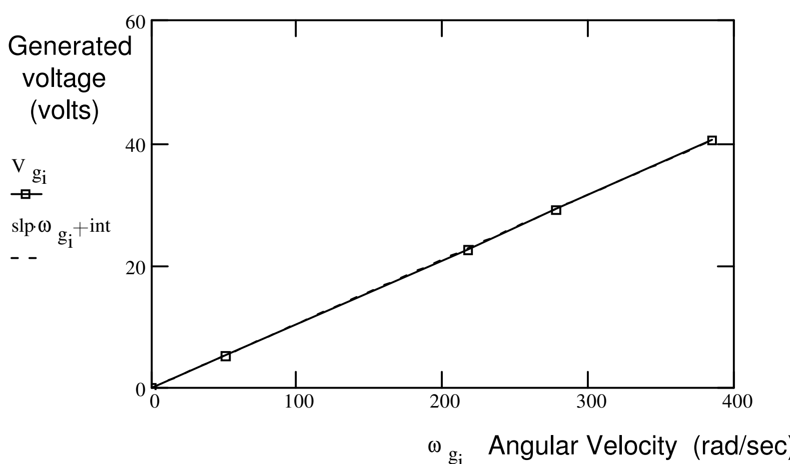
Linear regression:

You can use "polyfit (x,y,1)" to find the slope and y-intercept of the best-fit straight line through your data and "polyval" to plot it on top of your measured data. The functions below do the same for me in Mathcad.

$$slp := slope(\omega_g, V_g) \qquad slp = 0.105 \cdot \frac{V}{\left(\frac{rad}{sec}\right)}$$

$$int := intercept(\omega_g, V_g) \qquad int = -0.026 \cdot V$$

Intercept is close to zero, as it should be.



Voltage generation constant

$$K_V := slp$$

$$K_V = 0.105 \cdot \frac{V}{\left(\frac{rad}{sec}\right)}$$

We don' t have a good way to measure torque so we' ll assume that the torque constant is the equal to the generation constant, as it theoretically should be.

$$K_T := K_V \qquad K_T = 0.105 \cdot \frac{N \cdot m}{A}$$

Compare to the specs for this motor:

Specified K_T = $K_{Ts} := \frac{14.8 \cdot ozf \cdot in}{amp}$ = $K_{Ts} = 0.105 \cdot \frac{N \cdot m}{A}$

note: ozf = oz force

Specified K_V = $K_{Vs} := 11 \cdot \frac{V}{1000 \cdot rpm}$ = $K_{Vs} = 0.105 \cdot \frac{V}{\left(\frac{rad}{sec}\right)}$

Agreement is perfect

B_m & C_m

The next test will consist of free-running the motor at several different speeds. When the motor shaft is disconnected from all loads, any torque it develops is dissipated as friction, so this is a good way to find that friction. Turn off the power amp and decouple the small motor from the big motor. Setup the HP 34401A DVM to read current and hook the +25V terminals of the HP E3631A power supply through the ammeter to the big motor.

Let the motor free-run (fr) with various input voltages

Measurements

$$V_{in} := \begin{bmatrix} 5 \\ 8 \\ 10 \\ 15 \\ 20 \\ 25 \end{bmatrix} \cdot V \qquad I_{fr} := \begin{bmatrix} 0.14 \\ 0.146 \\ 0.155 \\ 0.173 \\ 0.182 \\ 0.186 \end{bmatrix} \cdot A \qquad \omega_{fr} := \begin{bmatrix} 43 \\ 71 \\ 90 \\ 137 \\ 184 \\ 231 \end{bmatrix} \cdot \frac{rad}{sec}$$

V_{in} is unneeded

Calculate torques:

$$i := 0..5$$

$$T_{fr_i} := I_{fr_i} \cdot K_T \qquad T_{fr} = \begin{bmatrix} 0.0147 \\ 0.0153 \\ 0.0163 \\ 0.0182 \\ 0.0191 \\ 0.0195 \end{bmatrix} \cdot N \cdot m$$

All this torque must be lost to friction

Linear regression

$$slp := slope(\omega_{fr}, T_{fr}) \qquad slp = 2.76 \cdot 10^{-5} \cdot \frac{N \cdot m}{\left(\frac{rad}{sec}\right)} \qquad int := intercept(\omega_{fr}, T_{fr}) \qquad int = 1.371 \cdot 10^{-2} \cdot N \cdot m$$

Viscous damping

$$B_m := slp$$

$$B_m = 2.76 \cdot 10^{-5} \cdot \frac{N \cdot m}{\frac{rad}{sec}}$$

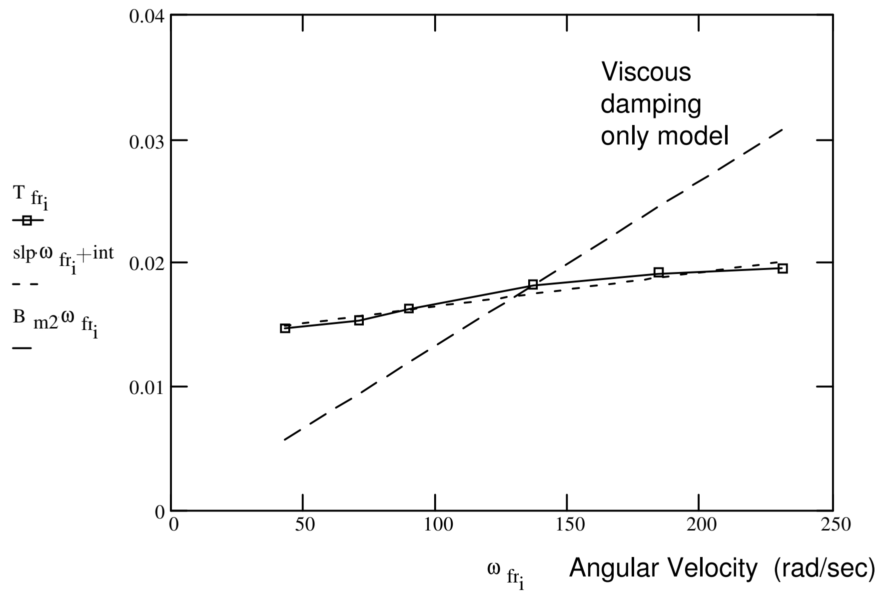
Constant (Coulomb) friction

$$C_m := int$$

$$C_m = 0.014 \cdot N \cdot m$$

If I try to force the friction to all be viscous damping and use measurements near the center:

$$B_{m2} := \frac{0.0182 \cdot N \cdot m}{137 \cdot \frac{rad}{sec}}$$



Now turn down the voltage to about 0.7V, then slowly continue turning down the voltage until the motor stops turning. At this voltage and current the motor torque just balances the coulomb friction.

Measurements: $V_{stall} := 0.4 \cdot V$ $\qquad I_{stall} := 0.13 \cdot A$

Calculated from C_m:

$$I_{stall} := \frac{C_m}{K_V} \qquad I_{stall} = 130.6 \cdot mA \qquad V_{stall} := I_{stall} \cdot R_a \qquad V_{stall} = 0.364 \cdot V$$

Compare to the steady-state error from lab 3, critically damped curve:

gain: $k_p := 2.66 \cdot \frac{V}{rad}$ $\qquad \frac{V_{stall}}{k_p} = 0.137 \cdot rad \qquad \frac{V_{stall}}{k_p} = 7.837 \cdot deg$

Actual steady-state error in the last lab: $.15 \cdot rad$ compares well

J, the Moment of Inertia

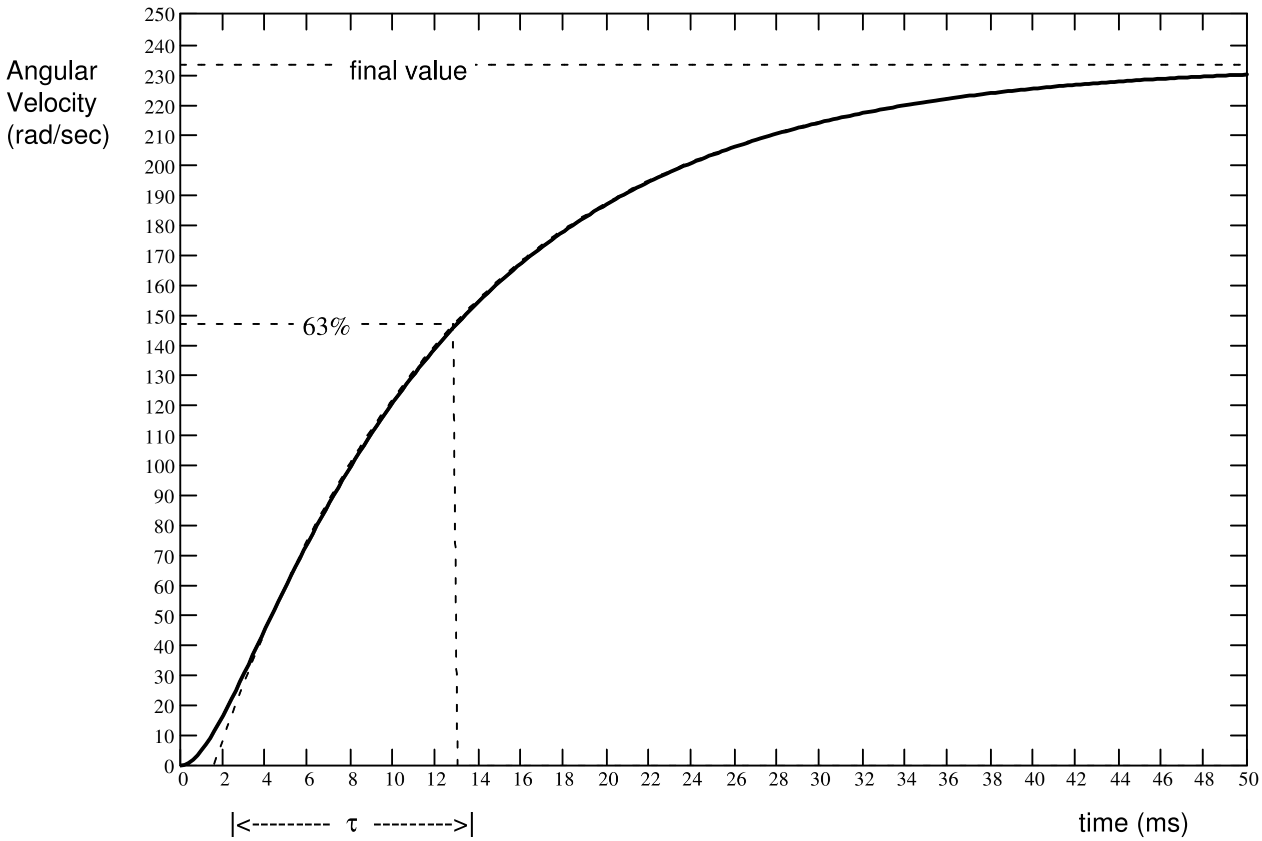
To get the Motor's moment of inertia (with coupler) we'll go back to the very simple motor transfer function used in the first lab:

$$\frac{\omega(s)}{V_a(s)} = \frac{K_T}{J \cdot R_a \cdot s + \left(B_m \cdot R_a + K_T \cdot K_V\right)} = \frac{\dfrac{K_T}{J \cdot R_a}}{s + \dfrac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot R_a}} = \frac{k}{s+a} \qquad a = \frac{1}{\tau}$$

$$\tau = \frac{J \cdot R_a}{B_m \cdot R_a + K_T \cdot K_V}$$

Also set up the equipment as you did in the first lab.

Use a 25V step input and take data to get curve similar to what you did in the first lab. Accurately measure and record the step voltage (the 25V), don't just depend on what the slider says. Move this data into Matlab and make a plot like the one below. You'll need to get the time constant of the big curve from this plot. If you can do that from the computer screen then you don't need to print the plot. Don't include the first part of the curve in the time constant, see below.



Measurement:

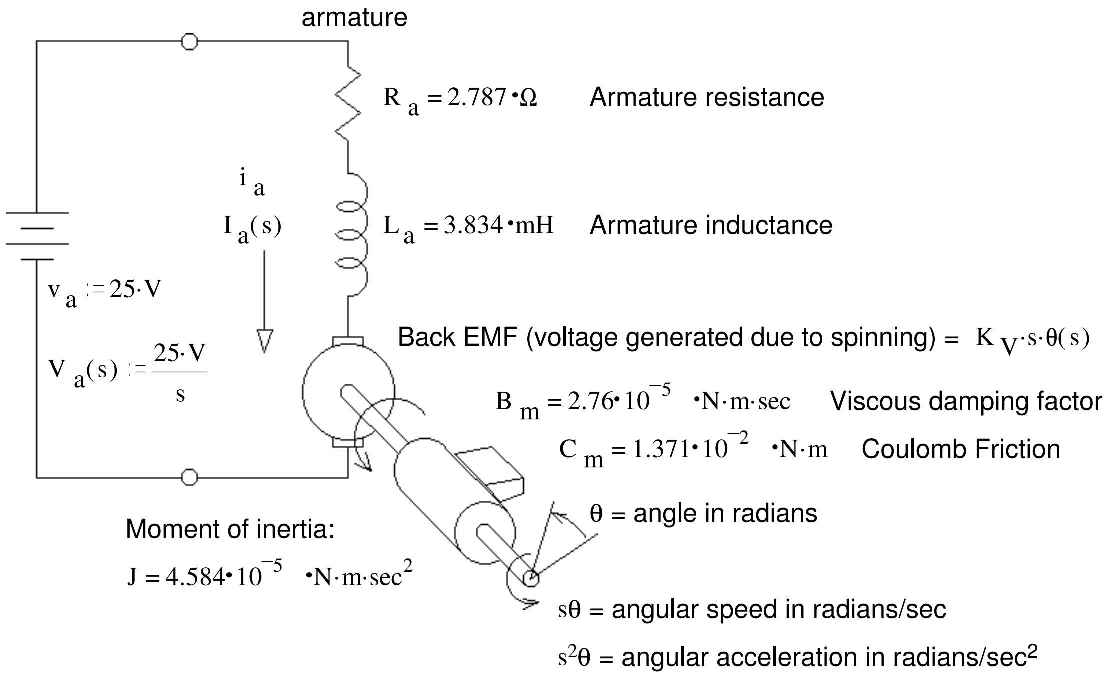
$\tau := 0.0115 \cdot \text{sec}$ $\qquad a := \dfrac{1}{\tau}$ $\qquad a = 86.957 \cdot \text{sec}^{-1}$ $\qquad J := \dfrac{\tau \cdot \left(B_m \cdot R_a + K_T \cdot K_V\right)}{R_a}$ $\qquad J = 4.584 \cdot 10^{-5} \cdot \text{kg} \cdot \text{m}^2$

Compare to the specs for this motor: $Js := 8.8 \cdot 10^{-3} \cdot \text{ozf} \cdot \text{in} \cdot \text{sec}^2$ $=$ $Js = 6.214 \cdot 10^{-5} \cdot \text{kg} \cdot \text{m}^2$

Not too close on this one, I wonder about this measurement and about the spec. You'll get a chance to tweak this value later.

Full model of DC permanent-magnet motor

It' s time to develop the full model of DC permanent-magnet motor, including all the parameters that we' ve just found. You don' t have to enter this in your notebook, just try to follow along. I added in the C_m as a constant in one direction because in the step response the motor is always running in only one direction. In general C_m is much more complex to include.



Torque: $T(s) = J\cdot\left(s^2\cdot\theta(s)\right) + B_m\cdot(s\cdot\theta(s)) + \frac{C_m}{s} = K_T\cdot I_a(s) = K_T\cdot\frac{V_a(s) - K_V\cdot s\cdot\theta(s)}{R_a + L_a\cdot s}$

$$J\cdot\left(s^2\cdot\theta(s)\right) + B_m\cdot(s\cdot\theta(s)) + \frac{C_m}{s} = K_T\cdot\frac{V_a(s) - K_V\cdot s\cdot\theta(s)}{R_a + L_a\cdot s}$$

$$\left[J\cdot\left(s^2\cdot\theta(s)\right) + B_m\cdot(s\cdot\theta(s)) + \frac{C_m}{s}\right]\cdot\left(R_a + L_a\cdot s\right) = K_T\cdot\left(V_a(s) - K_V\cdot s\cdot\theta(s)\right)$$

$$J\cdot s^2\cdot\theta(s)\cdot R_a + J\cdot s^3\cdot\theta(s)\cdot L_a + B_m\cdot s\cdot\theta(s)\cdot R_a + B_m\cdot s^2\cdot\theta(s)\cdot L_a + C_m\cdot L_a + \frac{C_m}{s}\cdot R_a = K_T\cdot V_a(s) - K_T\cdot K_V\cdot s\cdot\theta(s)$$

$$J\cdot s^2\cdot\theta(s)\cdot R_a + J\cdot s^3\cdot\theta(s)\cdot L_a + B_m\cdot s\cdot\theta(s)\cdot R_a + B_m\cdot s^2\cdot\theta(s)\cdot L_a + C_m\cdot L_a + \frac{C_m}{s}\cdot R_a + K_T\cdot K_V\cdot s\cdot\theta(s) = K_T\cdot V_a(s)$$

$$\left(J\cdot s^2\cdot R_a + J\cdot s^3\cdot L_a + B_m\cdot s\cdot R_a + B_m\cdot s^2\cdot L_a + K_T\cdot K_V\cdot s\right)\cdot\theta(s) = V_a(s)\cdot K_T - \left(C_m\cdot L_a + \frac{C_m}{s}\cdot R_a\right)$$

$$\theta(s) = V_a(s)\cdot\frac{K_T}{J\cdot L_a\cdot s^3 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s^2 + \left(B_m\cdot R_a + K_T\cdot K_V\right)\cdot s} - \frac{C_m\cdot L_a + \frac{C_m}{s}\cdot R_a}{J\cdot L_a\cdot s^3 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s^2 + \left(B_m\cdot R_a + K_T\cdot K_V\right)\cdot s}$$

$$\omega(s) = V_a(s)\cdot\frac{K_T}{J\cdot L_a\cdot s^2 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s + \left(B_m\cdot R_a + K_T\cdot K_V\right)} - \frac{C_m\cdot L_a + \frac{C_m}{s}\cdot R_a}{J\cdot L_a\cdot s^2 + \left(J\cdot R_a + B_m\cdot L_a\right)\cdot s + \left(B_m\cdot R_a + K_T\cdot K_V\right)}$$

$$\omega(s) = V_a(s)\cdot\frac{\dfrac{K_T}{J\cdot L_a}}{s^2+\left(\dfrac{R_a}{L_a}+\dfrac{B_m}{J}\right)\cdot s+\left(\dfrac{B_m\cdot R_a+K_T\cdot K_V}{J\cdot L_a}\right)} - \frac{\dfrac{C_m}{J}+\dfrac{C_m}{s}\cdot\dfrac{R_a}{J\cdot L_a}}{s^2+\left(\dfrac{R_a}{L_a}+\dfrac{B_m}{J}\right)\cdot s+\left(\dfrac{B_m\cdot R_a+K_T\cdot K_V}{J\cdot L_a}\right)}$$

Find the poles: $s^2+\left(\dfrac{R_a}{L_a}+\dfrac{B_m}{J}\right)\cdot s+\left(\dfrac{B_m\cdot R_a+K_T\cdot K_V}{J\cdot L_a}\right) = 0$

$$a_1 := \frac{1}{2}\cdot\left(\frac{R_a}{L_a}+\frac{B_m}{J}\right)+\sqrt{\frac{1}{4}\cdot\left(\frac{R_a}{L_a}+\frac{B_m}{J}\right)^2-\left(\frac{B_m\cdot R_a+K_T\cdot K_V}{J\cdot L_a}\right)} \qquad a_1 = 626.622\cdot sec^{-1}$$

$$a_2 := \frac{1}{2}\cdot\left(\frac{R_a}{L_a}+\frac{B_m}{J}\right)-\sqrt{\frac{1}{4}\cdot\left(\frac{R_a}{L_a}+\frac{B_m}{J}\right)^2-\left(\frac{B_m\cdot R_a+K_T\cdot K_V}{J\cdot L_a}\right)} \qquad a_2 = 100.871\cdot sec^{-1}$$

$$\frac{\dfrac{K_T}{J\cdot L_a}}{(s+a_1)\cdot(s+a_2)}\cdot\frac{v_a}{s} \qquad k_1 := \frac{K_T\cdot v_a}{J\cdot L_a}$$

$$\frac{k_1}{s\cdot(s+a_1)\cdot(s+a_2)} = \frac{A}{s} + \frac{B}{s+a_1} + \frac{C}{s+a_2}$$

$$= \frac{k_1}{(a_1\cdot a_2)}\cdot\frac{1}{s}+\frac{k_1}{a_1\cdot(a_1-a_2)}\cdot\frac{1}{(s+a_1)}+\frac{k_1}{a_2\cdot(a_2-a_1)}\cdot\frac{1}{(s+a_2)}$$

Deal with the part due to Coulomb friction:

$$\frac{-\left(\dfrac{C_m}{J}+\dfrac{C_m\cdot R_a}{J\cdot L_a}\cdot\dfrac{1}{s}\right)}{(s+a_1)\cdot(s+a_2)} = \frac{-\left(\dfrac{C_m}{J}\right)}{(s+a_1)\cdot(s+a_2)} + \frac{-\left(\dfrac{C_m\cdot R_a}{J\cdot L_a}\right)}{s\cdot(s+a_1)\cdot(s+a_2)} \qquad k_2 := -\left(\frac{C_m}{J}\right)$$

$$= \frac{k_2}{(s+a_1)\cdot(s+a_2)} + \frac{k_3}{s\cdot(s+a_1)\cdot(s+a_2)} \qquad k_3 := -\left(\frac{C_m\cdot R_a}{J\cdot L_a}\right)$$

$$\frac{k_2}{(s+a_1)\cdot(s+a_2)} = \frac{-k_2}{(a_1-a_2)}\cdot\frac{1}{(s+a_1)}+\frac{-k_2}{(a_2-a_1)}\cdot\frac{1}{(s+a_2)}$$

$$\frac{k_3}{s\cdot(s+a_1)\cdot(s+a_2)} = \frac{k_3}{(a_1\cdot a_2)}\cdot\frac{1}{s}+\frac{k_3}{a_1\cdot(a_1-a_2)}\cdot\frac{1}{(s+a_1)}+\frac{k_3}{a_2\cdot(a_2-a_1)}\cdot\frac{1}{(s+a_2)}$$

$$\omega(t) := \frac{k_1+k_3}{a_1\cdot a_2}+\frac{1}{(a_1-a_2)}\cdot\left(\frac{k_1}{a_1}-k_2+\frac{k_3}{a_1}\right)\cdot e^{-a_1\cdot t}+\frac{1}{(a_2-a_1)}\cdot\left(\frac{k_1}{a_2}-k_2+\frac{k_3}{a_2}\right)\cdot e^{-a_2\cdot t}$$

Plot and Compare

Now, you have to do the following to plot your theoretical curve on the same plot as your measured data.

Enter your parameters:

$R_a = 2.787 \cdot \Omega$ $\quad L_a = 3.834 \cdot 10^{-3} \cdot \text{henry}$ $\quad K_V = 0.105 \cdot \text{V} \cdot \text{sec}$ $\quad K_T := K_V$

$B_m = 2.76 \cdot 10^{-5} \cdot \text{N} \cdot \text{m} \cdot \text{sec}$ $\quad C_m = 0.014 \cdot \text{N} \cdot \text{m}$ $\quad J = 4.584 \cdot 10^{-5} \cdot \text{kg} \cdot \text{m}^2$

Calculate the following (and make it automatic, so you can play with the numbers above and see the effects on the plot):

$$a_1 := \frac{1}{2}\cdot\left(\frac{R_a}{L_a} + \frac{B_m}{J}\right) + \sqrt{\frac{1}{4}\cdot\left(\frac{R_a}{L_a} + \frac{B_m}{J}\right)^2 - \left(\frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot L_a}\right)} \qquad a_1 = 626.622 \cdot \text{sec}^{-1}$$

$$a_2 := \frac{1}{2}\cdot\left(\frac{R_a}{L_a} + \frac{B_m}{J}\right) - \sqrt{\frac{1}{4}\cdot\left(\frac{R_a}{L_a} + \frac{B_m}{J}\right)^2 - \left(\frac{B_m \cdot R_a + K_T \cdot K_V}{J \cdot L_a}\right)} \qquad a_2 = 100.871 \cdot \text{sec}^{-1}$$

$$k_1 := \frac{K_T \cdot v_a}{J \cdot L_a} \qquad k_2 := -\left(\frac{C_m}{J}\right) \qquad k_3 := -\left(\frac{C_m \cdot R_a}{J \cdot L_a}\right)$$

$$\omega(t) := \frac{k_1 + k_3}{a_1 \cdot a_2} + \frac{1}{(a_1 - a_2)}\cdot\left(\frac{k_1}{a_1} - k_2 + \frac{k_3}{a_1}\right)\cdot e^{-a_1 \cdot t} + \frac{1}{(a_2 - a_1)}\cdot\left(\frac{k_1}{a_2} - k_2 + \frac{k_3}{a_2}\right)\cdot e^{-a_2 \cdot t}$$

You don' t have to add this junk to your plot:

$$t := 0, 0.2 \cdot \text{ms} .. 50 \cdot \text{ms}$$

$$\text{fin} := \frac{k_1 + k_3}{a_1 \cdot a_2} \qquad \text{fin} = 233 \cdot \frac{\text{rad}}{\text{sec}} \qquad \text{tc} := 0.632 \cdot \text{fin} \qquad \text{dot}(t) := \text{if}(\omega(t) < \text{tc}, \text{tc}, 0)$$

$$f := \frac{a_1}{a_2} \qquad f = 6.212 \qquad \text{delay}(t) := \text{fin} \cdot \left(1 - e^{-a_2 \cdot t + \frac{1}{f}}\right)$$ = simple delayed exponential