DC permanent-magnet motor

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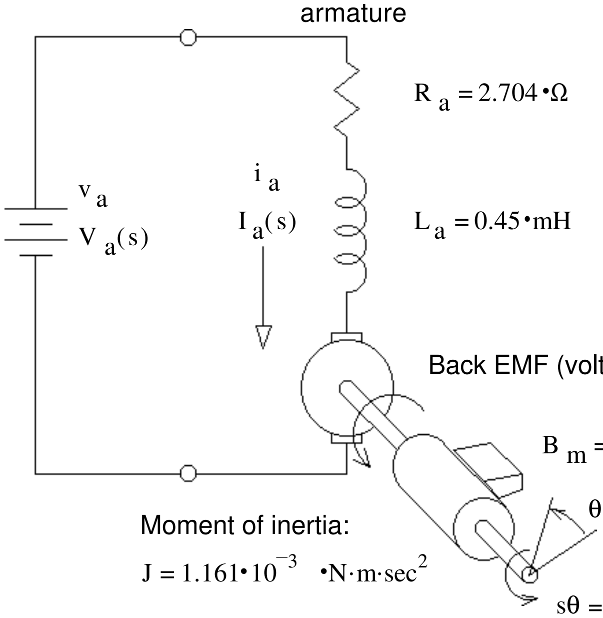
Torque constant $K_T = 0.12 \cdot \frac{N \cdot m}{A}$ $K_V := K_T$ Voltage constant $K_V = 0.12 \cdot \frac{V}{\left(\frac{rad}{sec}\right)}$



$R_a = 2.704 \cdot \Omega$ Armature resistance

$L_a = 0.45 \cdot mH$ Armature inductance

Back EMF (voltage generated due to spinning) = $K_V \cdot s \cdot \theta(s)$

$B_m = 0.012 \cdot N \cdot m \cdot sec$ Viscous damping factor

θ = angle in radians

Moment of inertia:
$J = 1.161 \cdot 10^{-3} \cdot N \cdot m \cdot sec^2$

$s\theta$ = angular speed in radians/sec

$s^2\theta$ = angular acceleration in radians/sec^2

Torque: $T(s) = J \cdot \left(s^2 \cdot \theta(s)\right) + B_m \cdot (s \cdot \theta(s)) = K_T \cdot I_a(s) = K_T \cdot \frac{V_a(s) - K_V \cdot s \cdot \theta(s)}{R_a + L_a \cdot s}$

$$J \cdot \left(s^2 \cdot \theta(s)\right) + B_m \cdot (s \cdot \theta(s)) = K_T \cdot \frac{V_a(s) - K_V \cdot s \cdot \theta(s)}{R_a + L_a \cdot s}$$

$$\left[J \cdot \left(s^2 \cdot \theta(s)\right) + B_m \cdot (s \cdot \theta(s))\right] \cdot \left(R_a + L_a \cdot s\right) = K_T \cdot \left(V_a(s) - K_V \cdot s \cdot \theta(s)\right)$$

$$J \cdot s^2 \cdot \theta(s) \cdot R_a + J \cdot s^3 \cdot \theta(s) \cdot L_a + B_m \cdot s \cdot \theta(s) \cdot R_a + B_m \cdot s^2 \cdot \theta(s) \cdot L_a = K_T \cdot V_a(s) - K_T \cdot K_V \cdot s \cdot \theta(s)$$

$$J \cdot s^2 \cdot \theta(s) \cdot R_a + J \cdot s^3 \cdot \theta(s) \cdot L_a + B_m \cdot s \cdot \theta(s) \cdot R_a + B_m \cdot s^2 \cdot \theta(s) \cdot L_a + K_T \cdot K_V \cdot s \cdot \theta(s) = K_T \cdot V_a(s)$$

$$\left(J \cdot s^2 \cdot R_a + J \cdot s^3 \cdot L_a + B_m \cdot s \cdot R_a + B_m \cdot s^2 \cdot L_a + K_T \cdot K_V \cdot s\right) \cdot \theta(s) = K_T \cdot V_a(s)$$

$$\left[J \cdot L_a \cdot s^3 + \left(J \cdot R_a + B_m \cdot L_a\right) \cdot s^2 + \left(B_m \cdot R_a + K_T \cdot K_V\right) \cdot s\right] \cdot \theta(s) = K_T \cdot V_a(s)$$

Motor transfer function
(With inductance, L_a):

$$\frac{\theta(s)}{V_a(s)} = \frac{K_T}{J \cdot L_a \cdot s^3 + \left(J \cdot R_a + B_m \cdot L_a\right) \cdot s^2 + \left(B_m \cdot R_a + K_T \cdot K_V\right) \cdot s}$$

$$= \frac{K_T}{s \cdot \left[J \cdot L_a \cdot s^2 + \left(J \cdot R_a + B_m \cdot L_a\right) \cdot s + \left(B_m \cdot R_a + K_T \cdot K_V\right)\right]}$$

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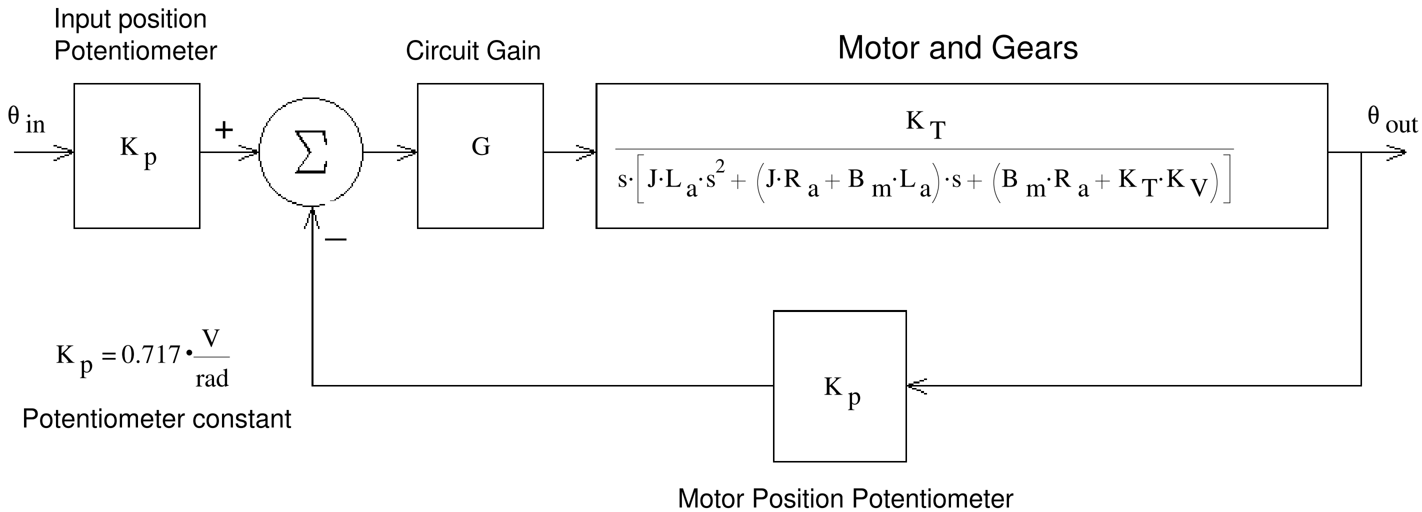
The system block diagram



Overall transfer function:

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = K_p \cdot \frac{G \cdot \dfrac{K_T}{s \cdot \left[J \cdot L_a \cdot s^2 + (J \cdot R_a + B_m \cdot L_a) \cdot s + (B_m \cdot R_a + K_T \cdot K_V) \right]}}{1 + K_p \cdot G \cdot \left[\dfrac{K_T}{s \cdot \left[J \cdot L_a \cdot s^2 + (J \cdot R_a + B_m \cdot L_a) \cdot s + (B_m \cdot R_a + K_T \cdot K_V) \right]} \right]}$$

$$\times \; \frac{s \cdot \left[J \cdot L_a \cdot s^2 + (J \cdot R_a + B_m \cdot L_a) \cdot s + (B_m \cdot R_a + K_T \cdot K_V) \right]}{s \cdot \left[J \cdot L_a \cdot s^2 + (J \cdot R_a + B_m \cdot L_a) \cdot s + (B_m \cdot R_a + K_T \cdot K_V) \right]}$$

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{G \cdot K_T \cdot K_p}{s \cdot \left[J \cdot L_a \cdot s^2 + (J \cdot R_a + B_m \cdot L_a) \cdot s + (B_m \cdot R_a + K_T \cdot K_V) \right] + K_p \cdot G \cdot K_T}$$

This transfer function is a bit messy, so we'll neglect the motor inductance and proceed.

Simple motor (no inductance)

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{G \cdot K_T \cdot K_p}{s \cdot \left[J \cdot \underset{0}{L_a} \cdot s^2 + (J \cdot R_a + B_m \cdot \underset{0}{L_a}) \cdot s + (B_m \cdot R_a + K_T \cdot K_V) \right] + K_p \cdot G \cdot K_T}$$

Motor transfer function (simple, no inductance):

$$\frac{\theta(s)}{V_a(s)} = \frac{K_T}{R_a \cdot J \cdot s^2 + (R_a \cdot B_m + K_T \cdot K_V) \cdot s}$$

Overall transfer function:

(simple, no inductance)

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = K_p \cdot \frac{G \cdot \dfrac{K_T}{R_a \cdot J \cdot s^2 + (R_a \cdot B_m + K_T \cdot K_V) \cdot s}}{1 + K_p \cdot G \cdot \dfrac{K_T}{R_a \cdot J \cdot s^2 + (R_a \cdot B_m + K_T \cdot K_V) \cdot s}} \quad \times \quad \frac{R_a \cdot J \cdot s^2 + (R_a \cdot B_m + K_T \cdot K_V) \cdot s}{R_a \cdot J \cdot s^2 + (R_a \cdot B_m + K_T \cdot K_V) \cdot s}$$

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{G \cdot K_T \cdot K_p}{R_a \cdot J \cdot s^2 + (R_a \cdot B_m + K_T \cdot K_V) \cdot s + K_p \cdot G \cdot K_T}$$

Overall transfer function:
(simple, no inductance)

$$\frac{\theta_{out}(s)}{\theta_{in}(s)} = \frac{G \cdot K_T \cdot K_p}{R_a \cdot J \cdot s^2 + \left(R_a \cdot B_m + K_T \cdot K_V\right) \cdot s + K_p \cdot G \cdot K_T}$$

The characteristic equation $0 = R_a \cdot J \cdot s^2 + \left(R_a \cdot B_m + K_T \cdot K_V\right) \cdot s + K_p \cdot G \cdot K_T$

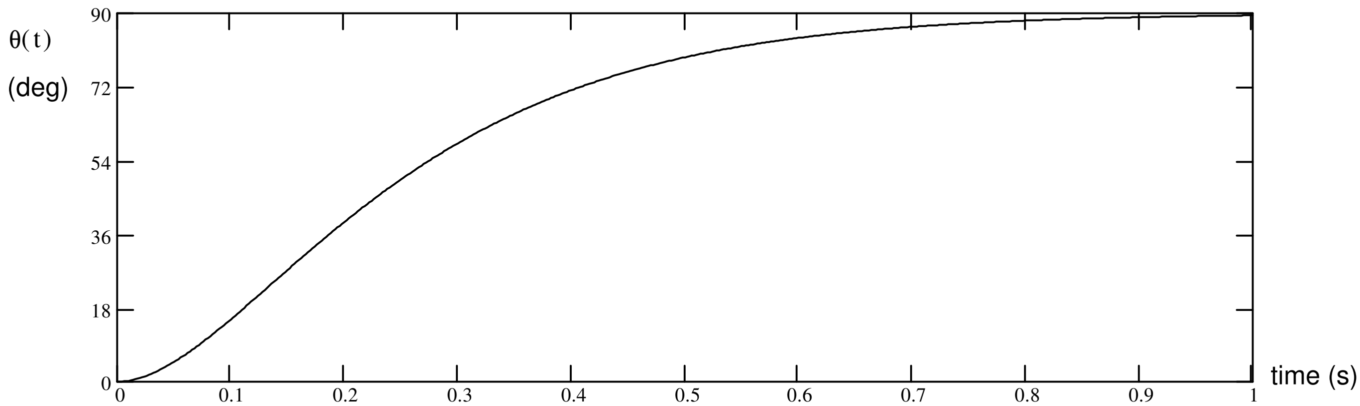
for Gain = $G := 2$ $\quad a := R_a \cdot J$ $\quad b := \left(R_a \cdot B_m + K_T \cdot K_V\right)$ $\quad k := K_p \cdot G \cdot K_T$

$$s_1 := \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot k}}{2 \cdot a} \qquad s_1 = -6.561 \cdot sec^{-1} \qquad s_2 := \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot k}}{2 \cdot a} \qquad s_2 = -8.334 \cdot sec^{-1}$$

For example:

$\theta_{init} := 0 \cdot deg$ $\quad \theta_{fin} := 90 \cdot deg$ $\quad \frac{d}{dt}\theta(0) = 0$ no initial angular speed

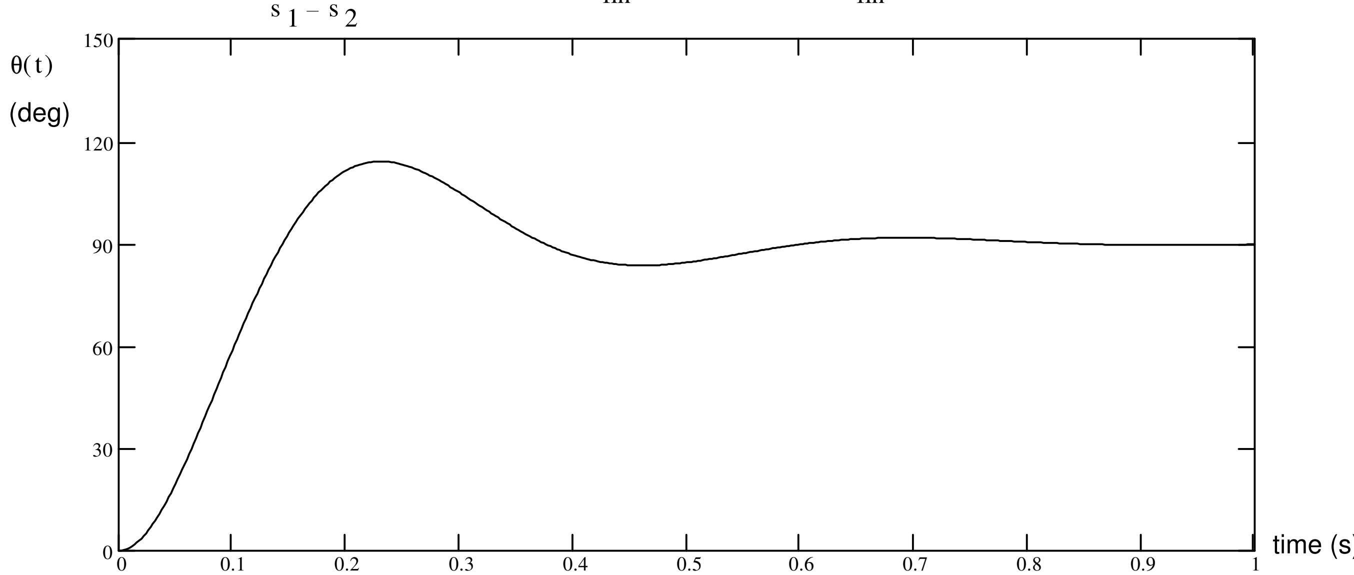
$$B := \frac{\theta_{fin} \cdot s_2}{s_1 - s_2} \qquad D := -B - \theta_{fin} \qquad \theta(t) := \theta_{fin} + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$$



for Gain = $G := 8$ $\quad a := R_a \cdot J$ $\quad B_m := .008 \cdot N \cdot m \cdot sec$ $\quad b := \left(R_a \cdot B_m + K_T \cdot K_V\right)$ $\quad k := K_p \cdot G \cdot K_T$

$$s_1 := \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot k}}{2 \cdot a} \qquad s_1 = -5.725 + 13.636j \cdot sec^{-1} \qquad s_2 := \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot k}}{2 \cdot a}$$

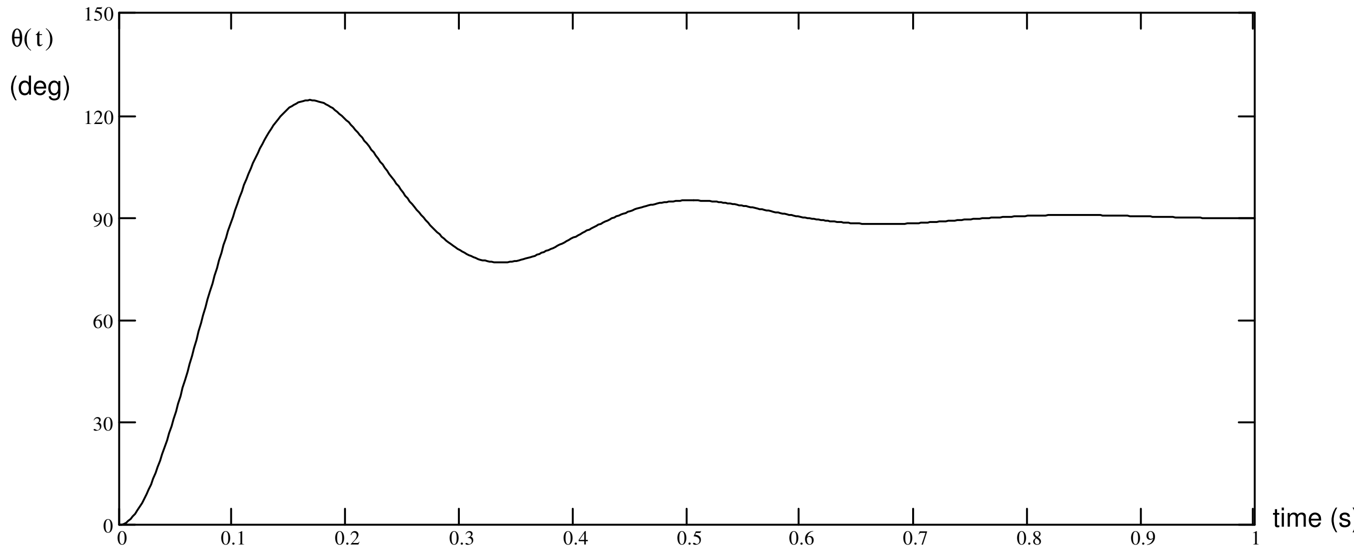
For example: $B := \frac{\theta_{fin} \cdot s_2}{s_1 - s_2}$ $\quad D := -B - \theta_{fin}$ $\quad \theta(t) := \theta_{fin} + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$



for Gain = $G := 14$ $a := R_a \cdot J$ $b := (R_a \cdot B_m + K_T \cdot K_V)$ $k := K_p \cdot G \cdot K_T$

$s_1 := \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot k}}{2 \cdot a}$ $s_1 = -5.725 + 18.708j \cdot sec^{-1}$ $s_2 := \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot k}}{2 \cdot a}$

For example: $B := \frac{\theta_{fin} \cdot s_2}{s_1 - s_2}$ $D := -B - \theta_{fin}$ $\theta(t) := \theta_{fin} + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$

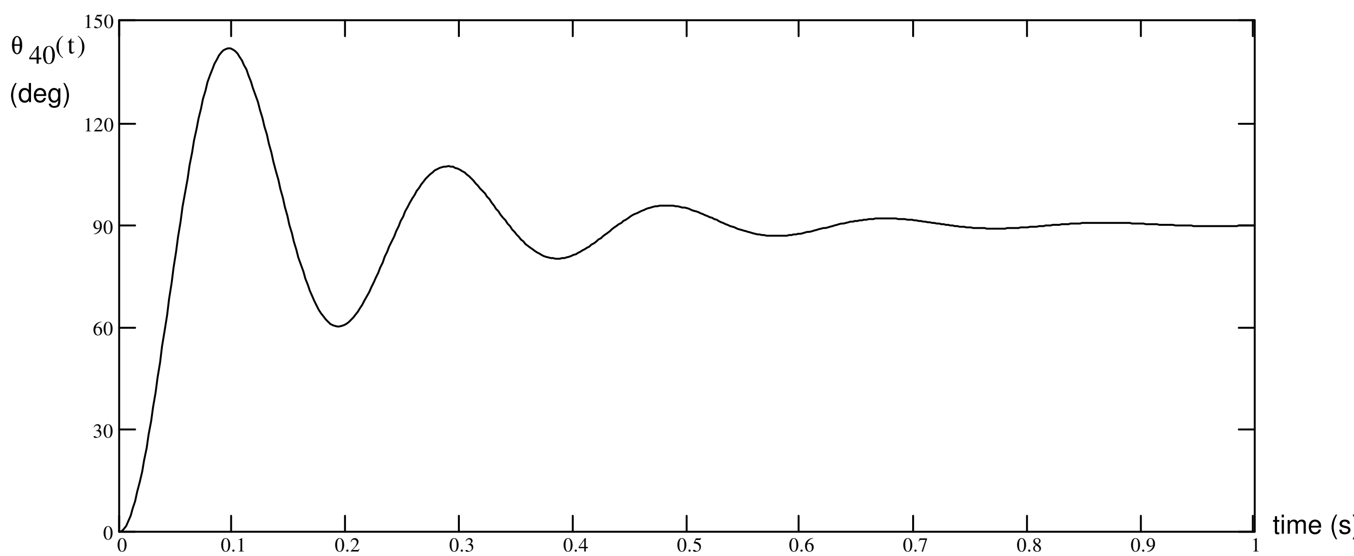


for Gain = $G := 40$ $a := R_a \cdot J$ $b := (R_a \cdot B_m + K_T \cdot K_V)$ $k := K_p \cdot G \cdot K_T$

$s_1 := \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot k}}{2 \cdot a}$ $s_1 = -5.725 + 32.57j \cdot sec^{-1}$ $s_2 := \frac{-b - \sqrt{b^2 - 4 \cdot a \cdot k}}{2 \cdot a}$

For example: $B := \frac{\theta_{fin} \cdot s_2}{s_1 - s_2}$ $B = -0.785 + 0.138j$ $D := -B - \theta_{fin}$ $D = -0.785 - 0.138j$

$\theta_{40}(t) := \theta_{fin} + B \cdot e^{s_1 \cdot t} + D \cdot e^{s_2 \cdot t}$



Our simple motor example doesn't account for some nonlinearities which force the poles toward the right-half plane, so it doesn't oscillate.