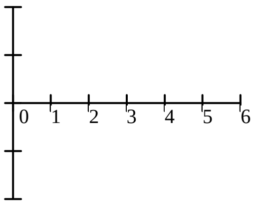
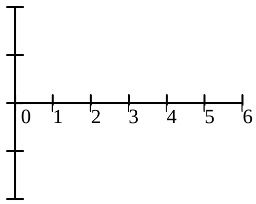
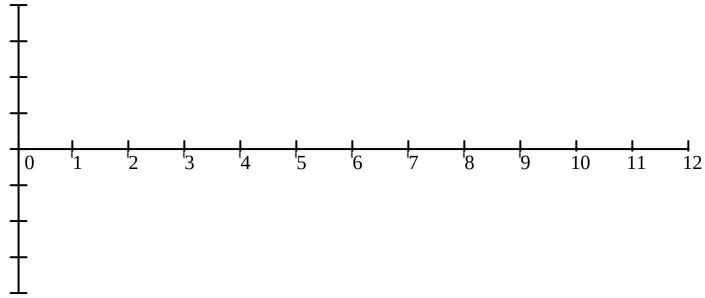
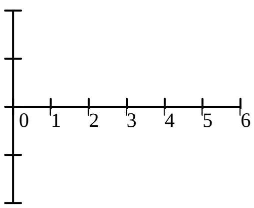
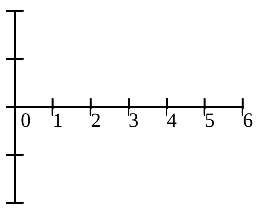
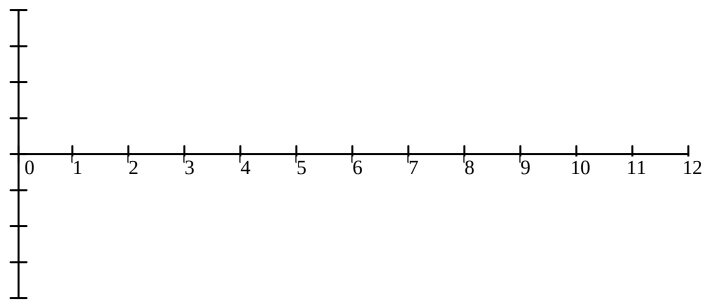
Name______________________ ECE 3510 Homework Z1 A.Stolp d

1. Like problem 6.4 in the text. Sketch the time function $x(k)$ that you would associate with the following poles. Each signal transform has two poles. Only a sketch is required, but be as precise as possible. You may wish to use Matlab or a spreadsheet to plot these. Actual signal magnitudes and/or phase angles cannot be determined from the pole locations. For the real poles, plot the individual signals first, then add them together to get the final plot.

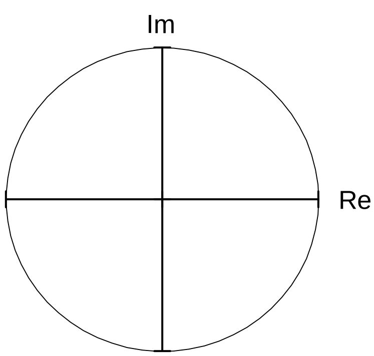
c) $p_1 = 0.3$ , $p_2 = 0.9$

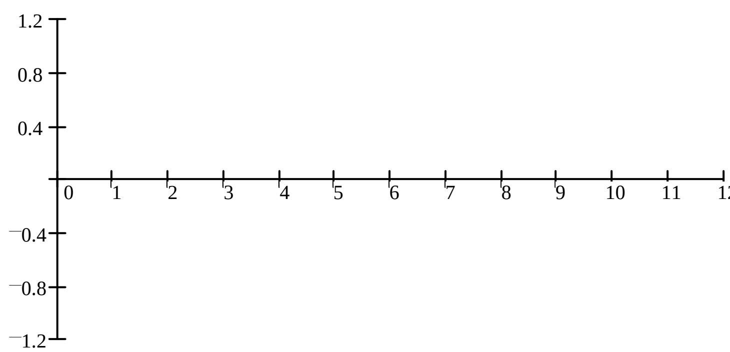




b) $p_1 = 1$ , $p_2 = -1$

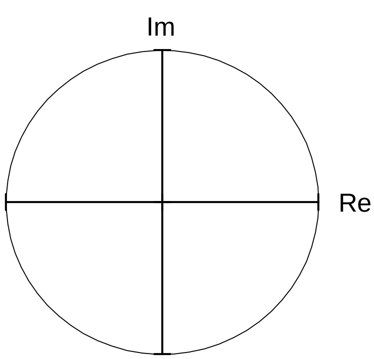




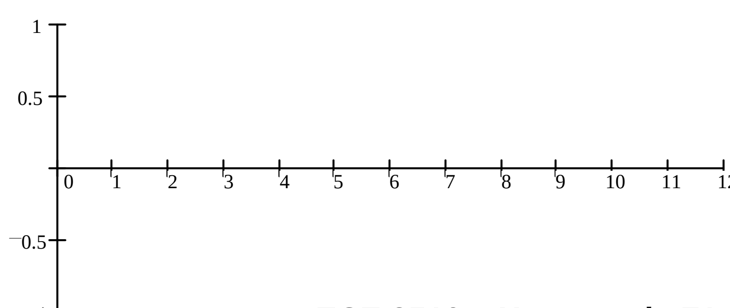
d) $p_1 := e^{j\cdot\frac{\pi}{6}}$ , $p_2 := e^{-j\cdot\frac{\pi}{6}}$ For the complex poles, first plot the poles on the complex plane & unit circle.





a) $p_1 := 0.9\cdot j$ , $p_2 := -0.9\cdot j$



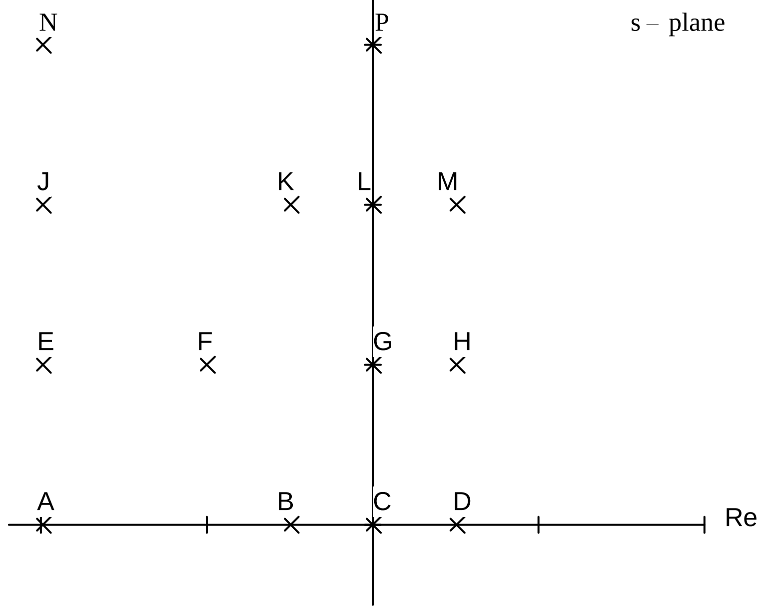


2. For each of the pole locations shown on the s-plane below, Draw and label a similar pole location on the z-plane.

Top line represents Maximum (Nyquist) frequency

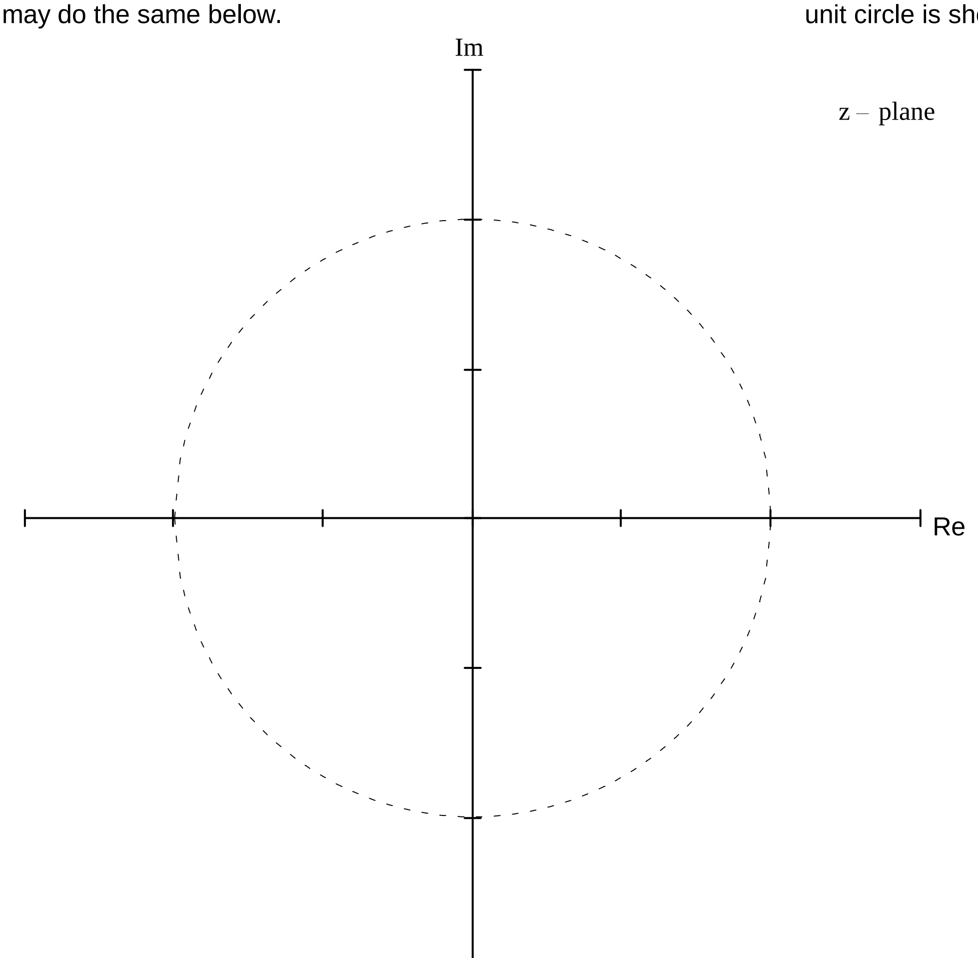
$$\frac{\omega_s}{2} = 2\cdot\pi\cdot\frac{f_s}{2} = \frac{2\cdot\pi}{2\cdot T_s}$$

where $f_s$ is the digital sampling frequency, so this line is 1/2 the sampling frequency




Note: The poles on both planes do come in complex-conjugate pairs, but I have only shown those above the real axis. You may do the same below.

unit circle is shown as dotted line



3. Problem 6.1 in the Bodson text. Find $x(0)$ if the z-transform of $x(k)$ is

   a) $\mathbf{X}(z) = \dfrac{a \cdot z - 1}{z - 1}$

   b) $\mathbf{X}(z) = \dfrac{z}{z^2 - a \cdot z + a^2}$

4. Problem 6.7 in the text.
   For the signals whose z-transforms are given below, indicate whether the time functions $x(k)$ are bounded, converge to some value, or vanish in finite time.

   <u>Poles or pole magnitudes</u> <u>Bounded</u> <u>Converges</u> <u>x(∞)</u>

   a) $\mathbf{X}(z) = \dfrac{z + 1}{(z + 0.5) \cdot (z - 0.7 + 0.7 \cdot j) \cdot (z - 0.7 - 0.7 \cdot j)}$

   b) $\mathbf{X}(z) = \left(1 - 2 \cdot z^{-1}\right) \cdot \left(1 + 3 \cdot z^{-1}\right)$

   c) $\mathbf{X}(z) = \dfrac{z - 1}{(z + 1) \cdot (z + 0.5)^2}$

   d) $\mathbf{X}(z) = \dfrac{z + 1}{(z - 1) \cdot (z + 0.5)^2}$

   e) $\mathbf{X}(z) = \dfrac{z + 1}{z \cdot (z - 1)}$

   f) $\mathbf{X}(z) = \dfrac{z^{10}}{(z + 5)}$

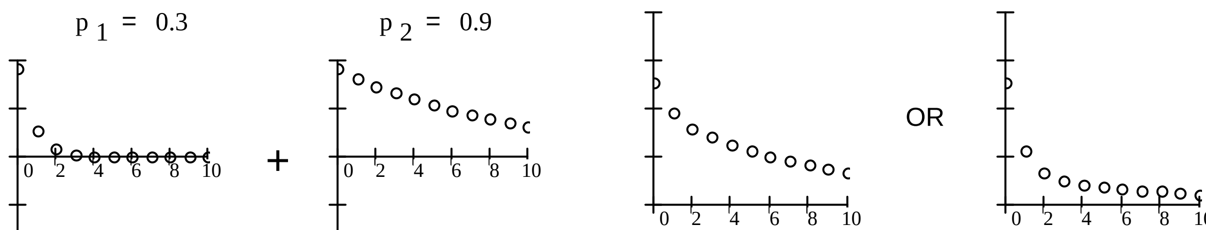
g) $\mathbf{X}(z) = \dfrac{(z+1)^2}{\left(z^2+1\right)\cdot(z-0.5)}$

h) $\mathbf{X}(z) = \dfrac{z-2}{z^3\cdot(z-1)}$

**<u>Answers</u>**

1. Actual signals may have different magnitudes and/or phase angles. You can't tell those things from the pole locations.

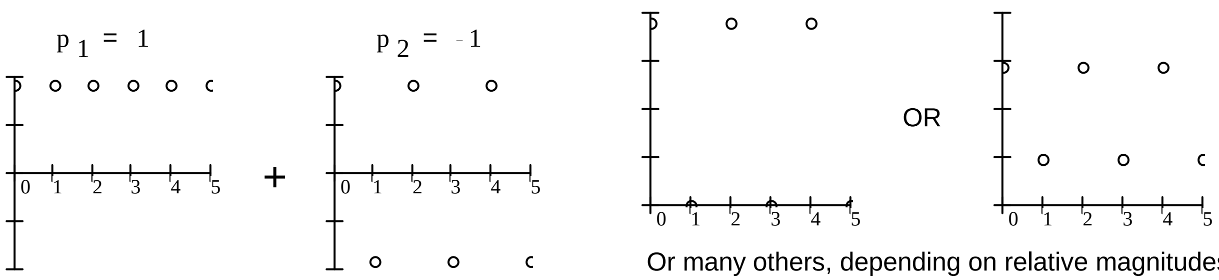
c) $p_1 = 0.3$ $p_2 = 0.9$




Fast decay Slow decay

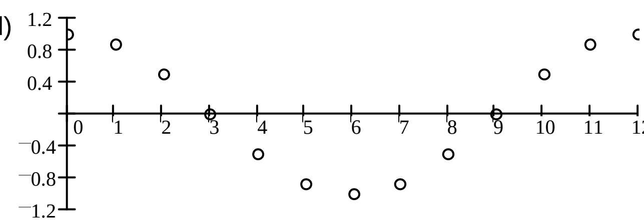
Or many others, depending on relative magnitudes

b) $p_1 = 1$ $p_2 = -1$

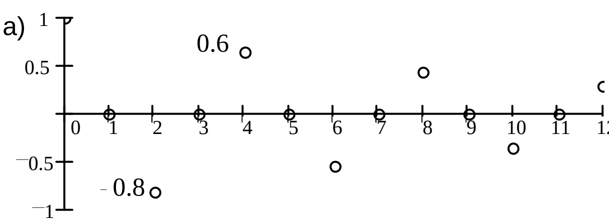



Or many others, depending on relative magnitudes

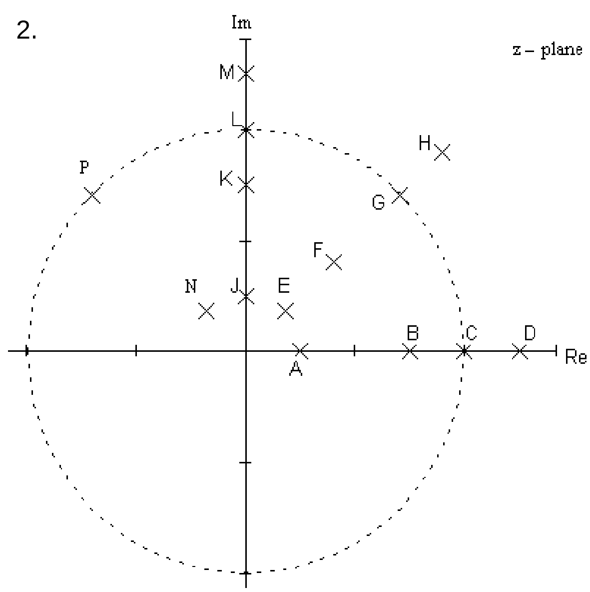
d)



a)




2.




3. a) $a$ b) $0$

4. (6.7)

| | Bounded | Converges | $x(\infty)$ |
|---|---|---|---|
| a) | yes | yes | 0 |
| b) | yes | yes | 0 vanishes in a finite time (all poles are at zero) |
| c) | yes | no | |
| d) | yes | yes | 8/9 |
| e) | yes | yes | 2 |
| f) | no | | |
| g) | yes | no | |
| h) | yes | yes | 1 |