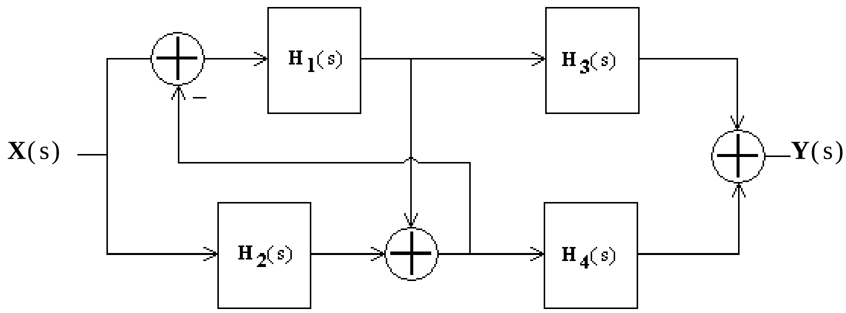
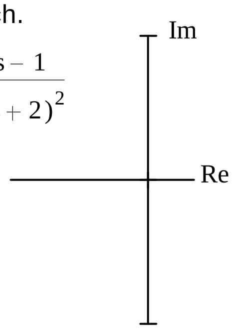
Name________________________ ECE 3510 Homework 5 Transfer Functions d3

1. Problem 3.2b, p.54 in Bodson text. Find the transfer function from **X**(s) to **Y**(s) for the system shown.

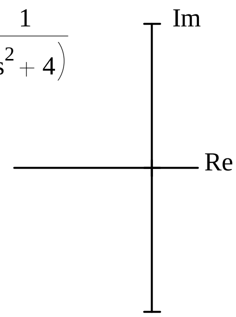


2. Problem 3.3 in Bodson text. Determine which transfer functions are stable. For the unstable systems, give an example of a bounded input that yields an unbounded output. As part of your work to reach a solution, draw the pole diagram for each.

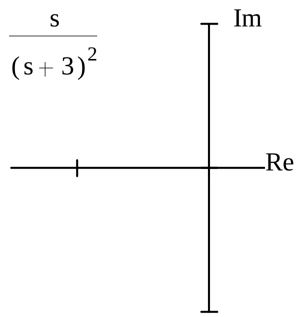
a) $\dfrac{s-1}{(s+2)^2}$



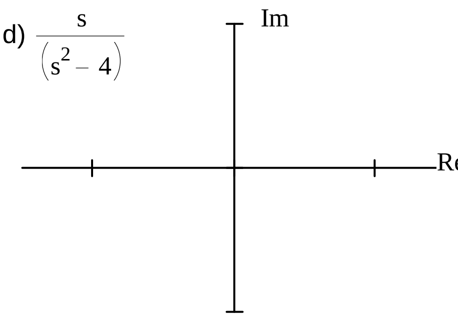
b) $\dfrac{1}{\left(s^2+4\right)}$



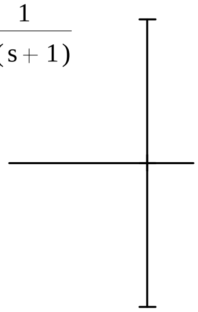
c) $\dfrac{s}{(s+3)^2}$



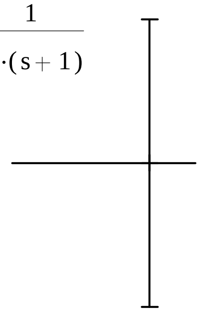
d) $\dfrac{s}{\left(s^2-4\right)}$



e) $\dfrac{1}{s\cdot(s+1)}$



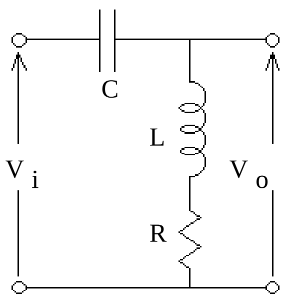
f) $\dfrac{1}{s^2\cdot(s+1)}$



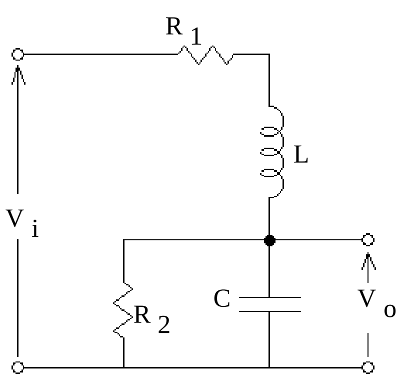
	Stable	Example of a Problem input
a)		
b)		
c)		
d)		
e)		
f)		

3. Find the transfer function $\mathbf{H}(s) = \dfrac{\mathbf{V_o}(s)}{\mathbf{V_i}(s)}$ for these circuits. Properly simplify all your expressions for $\mathbf{H}(s)$ like you did in HW 4.

a)



b)



4. Find the step response of: $\mathbf{H}(s) = \frac{k}{(s+a_1)\cdot(s+a_2)}$ Step input: $x(t) = x_m \cdot u(t)$ $\mathbf{X}(s) = \frac{x_m}{s}$

Show the steps necessary to arrive at the steady-state and transient responses shown as equation(s) 3.40 on p.35 of the text, repeated below.

$$y_{ss}(t) = \frac{k}{a_1 \cdot a_2} \cdot x_m \qquad y_{tr}(t) = \frac{k}{a_1 \cdot (a_1 - a_2)} \cdot x_m \cdot \left(e^{-a_1 \cdot t}\right) + \frac{k}{a_2 \cdot (a_2 - a_1)} \cdot x_m \cdot \left(e^{-a_2 \cdot t}\right) \qquad \text{eq(3.40)}$$

5. Find the step response of: $$\mathbf{H}(s) = \frac{k\cdot s}{(s+a)^2+b^2} = \frac{k\cdot s}{s^2+2\cdot s\cdot a+\left(a^2+b^2\right)}$$ where b is real

Show the steps necessary to arrive at the steady-state and transient responses.

6. For the transfer functions below, find the DC gain and the full step responses. You may use the results found in section 3.3.2 of the text as well as problem 4, above.

a) $\mathbf{H}(s) = \dfrac{2}{s^2 + 2\cdot s + 1}$

b) $\mathbf{H}(s) = \dfrac{-s - 2}{s^2 + 2\cdot s + 2}$

Hint: Notice how easily this will split into two parts that you already have answers for.

Answers

1. $\dfrac{\mathbf{H}_1\cdot\mathbf{H}_4 + \mathbf{H}_2\cdot\mathbf{H}_4 - \mathbf{H}_1\cdot\mathbf{H}_2\cdot\mathbf{H}_3 + \mathbf{H}_1\cdot\mathbf{H}_3}{1 + \mathbf{H}_1}$

3. a) $\dfrac{s^2 + \frac{R}{L}\cdot s}{s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C}}$ b) $\dfrac{\frac{1}{L\cdot C}}{s^2 + \left(\frac{1}{C\cdot R_2} + \frac{R_1}{L}\right)\cdot s + \left(1 + \frac{R_1}{R_2}\right)\cdot\frac{1}{L\cdot C}}$

2.	Stable	Example of a Problem input
a)	yes	
b)	no	$\cos(2\cdot t)$
c)	yes	
d)	no	any input, even noise
e)	no	1 DC
f)	no	any input, even noise

u(t) is assumed

4. $y(\infty) = \dfrac{x_m\cdot k}{a_1\cdot a_2}$ $y_{tr}(t) = x_m\cdot\left[\dfrac{k}{a_1\cdot(a_1 - a_2)}\cdot e^{-a_1\cdot t} + \dfrac{k}{a_2\cdot(a_2 - a_1)}\cdot e^{-a_2\cdot t}\right]$

OR: $y(t) = x_m\cdot\left[\dfrac{k}{a_1\cdot a_2} + \dfrac{k}{a_1\cdot(a_1 - a_2)}\cdot e^{-a_1\cdot t} + \dfrac{k}{a_2\cdot(a_2 - a_1)}\cdot e^{-a_2\cdot t}\right]$

5. $y(\infty) = 0$ $y_{tr}(t) = x_m\cdot\dfrac{k}{b}\cdot e^{-a\cdot t}\cdot\sin(b\cdot t)$ OR: $y(t) = 0 + x_m\cdot\dfrac{k}{b}\cdot e^{-a\cdot t}\cdot\sin(b\cdot t)$

6. a) $x_m\cdot\left(2 - 2\cdot e^{-t} - 2\cdot t\cdot e^{-t}\right)$ b) $x_m\cdot\left(-1 + e^{-t}\cdot\cos(t)\right)$