Root Locus Design

You may sketch root locus plots and make calculations using a computer program.
Questions and problems from Nise are the same for 3rd & 4th editions unless specified otherwise.

1. Nise Ch. 9 review questions: 3, 4, 5, 9, 10, 11, & 12.

    3. What kind of compensation improves the steady-state error?

    4. What kind of compensation improves the transient response?

    5. What kind of compensation improves both steady-state error and transient response?

    9. In order to speed up a system without changing the percent overshoot, where must the compensated system's poles on the s-plane be located in comparison to the compensated system's poles?

    10. Why is there more improvement in the steady-state error if a PI controller is used instead of a lag network?

    11. When compensating for steady-state error, what effect is sometimes noted in the transient response?

    12. A lag compensator with the zero 25 times as far from the imaginary axis as the compensator pole will yield approximately how much improvement in the steady-state error?

2. Nise Ch. 9 problem 1. For an explanation of the static error constants & calculation of steady-state error, see Nise, section 7.3 or Root Locus Design Crib Sheet. If you use Bodson eq 4.6, include the gain factor (multiply $\mathbf{P}(0)\mathbf{C}(0)$ by $\mathbf{K}$).

    Use $\mathbf{G}(s)$ and damping ratio (factor) from 3rd ed:
    $$\mathbf{G}_{\mathbf{uc}}(s) := \frac{1}{(s+3)\cdot(s+6)} \qquad \zeta := 0.707$$
    $$uc = \text{uncompensated}$$

    a) Design a PI controller and show that it works to drive the step-response error to zero.
    The system operates with a damping ratio of 0.707. Compare the specifications of the compensated and uncompensated systems.

3. Nise Ch. 9 problem 3   Use $\mathbf{G}(s)$ and 10% overshoot from 3rd ed: $\mathbf{G_{uc}}(s) := \dfrac{1}{(s+1)\cdot(s+3)\cdot(s+5)}$

   a) Find the static error constant (K_p on our Crib Sheet).

   b) Find the transfer function of lag network to improve the static error constant to $K_p = 4$ without significantly changing the positions of the dominant CL poles.

   c) Use the SISO tool to show the improvement.

ECE 3510   homework RL7   p2

4. Nise Ch. 9 problem 6   Use $\mathbf{G}(s)$ from 3rd ed: $\mathbf{G}_{\mathbf{uc}}(s) := \dfrac{1}{(s+1)\cdot(s+2)\cdot(s+3)\cdot(s+6)}$ use: $\zeta := 0.707$

The system operates with a damping ratio of 0.707.

Design a PD controller so that settling time is reduced by a factor of 2 without significantly changing the positions of the dominant CL poles. Compare the transcient and steady-state performances of the compensated and uncompensated systems. Describe any remaining problems that you can see.

5. Nise Ch. 9 problem 8   Use $\mathbf{G}(s)$ and 20% overshoot from 3rd ed: $\mathbf{G}_{\mathbf{uc}}(s) := \dfrac{1}{s \cdot (s+5) \cdot (s+15)}$

a) Design a PD compensator to shorten settling time to 1/4 of what it is without PD compensation. Keep the system overshoot = 20%. Compare the specifications of the compensated and uncompensated systems.

b) Change the design to a lead compensator. Move the zero you found in part a) to -3 and finding the required pole. Compare the specifications of the compensated and uncompensated systems.

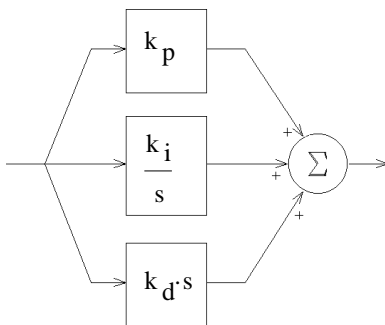6. You have designed a compensator with the following:

    A pole at the origin   A zero at -0.5   A zero at -10   Gain of 20

    Find the k_p, k_i, & k_d of a PID controller.

Answers

1. ANSWERS TO REVIEW QUESTIONS

1. Chapter 8: Design via gain adjustment. Chapter 9: Design via cascaded or feedback filters.
2. A. Permits design for transient responses not on original root locus and unattainable through simple gain adjustments.
   B. Transient response and steady-state error specifications can be met separately and independently without the need for tradeoffs
3. PI or lag compensation   4. PD or lead compensation   5. PID or lag-lead compensation
6. A pole is placed on or near the origin to increase or nearly increase the system type, and the zero is placed near the pole in order not to change the transient response.
7. The zero is placed closer to the imaginary axis than the pole. The total contribution of the pole and zero along with the previous poles and zeros must yield 180º at the design point. Placing the zero closer to the imaginary axis tends to speed up a slow response.
8. A PD controller yields a single zero, while a lead network yields a zero and a pole. The zero is closer to the imaginary axis.
9. Further out along the same radial line drawn from the origin to the uncompensated poles
10. The PI controller places a pole right at the origin, thus increasing the system type and driving the error to zero. A lag network places the pole only close to the origin yielding improvement but not zero error.
11. The transient response is approximately the same as the uncompensated system, except after the original settling time has passed. A slow movement toward the new final value is noticed.
12. 25 times; the improvement equals the ratio of the zero location to the pole location.
13. No; the feedback compensator's zero is not a zero of the closed-loop system.
14. A. Response of inner loops can be separately designed; B. Faster responses possible; C. Amplification may not be necessary since signal goes from high amplitude to low.

2. Uncompensated: CL pole $s_{uc} := -4.5 + 4.5 \cdot j$ $K_{uc} := 22.5$ 44.4% steady-state error

$$\%OS = 4.32\% \qquad T_S = 0.889 \text{ sec}$$

For: $C(s) = \dfrac{s + 0.1}{s}$ CL pole $s_c := -4.472 + 4.472 \cdot j$ $K_{uc} := 22.5$ no steady-state error

$$\%OS = 4.32\% \qquad T_S = 0.894 \text{ sec} \qquad \text{Using 2nd-order approximation}$$

3. Uncompensated: $s_{uc} := -1.4 + 1.91 \cdot j$ $K := 19.9$ steady-state error is about 43%

Compensated, want $K_p = 4$, steady-state error of 20%   Try: $C(s) = \dfrac{s + 0.3}{s + 0.1}$ That should yield a 3x improvement in K_p.

Matlab output shows a good reduction in steady-state error.

4. Uncompensated: $s_{uc} := -1.05 + 1.05 \cdot j$ $K := 16.65$

Want $s_c := -2.1 + 2.1 \cdot j$ Need zero at -0.604

Possible problems with   Pole at -0.771 is not close enough to the zero at -0.604 to cancel it.
the 2nd-order assumption: Pole at -7.03 is not 5 times farther from jω axis than -2.1.

b) 0.753 75% error! That zero close to the origin is NOT OK.

5. Uncompensated: $s_{uc} := -1.809 + 3.533 \cdot j$ $K := 258$

Want $s_c := -7.236 + 14.132 \cdot j$ Need zero at -5.422

Compare to example 9.7 (table 9.8), similar to compensated system except gain.
Gain is similar to uncompensated system.

b) $C(s) = \dfrac{s + 3}{s + 94.43}$

6. $210, 100, 20$