Name: ______________________ Homework # BP3 a

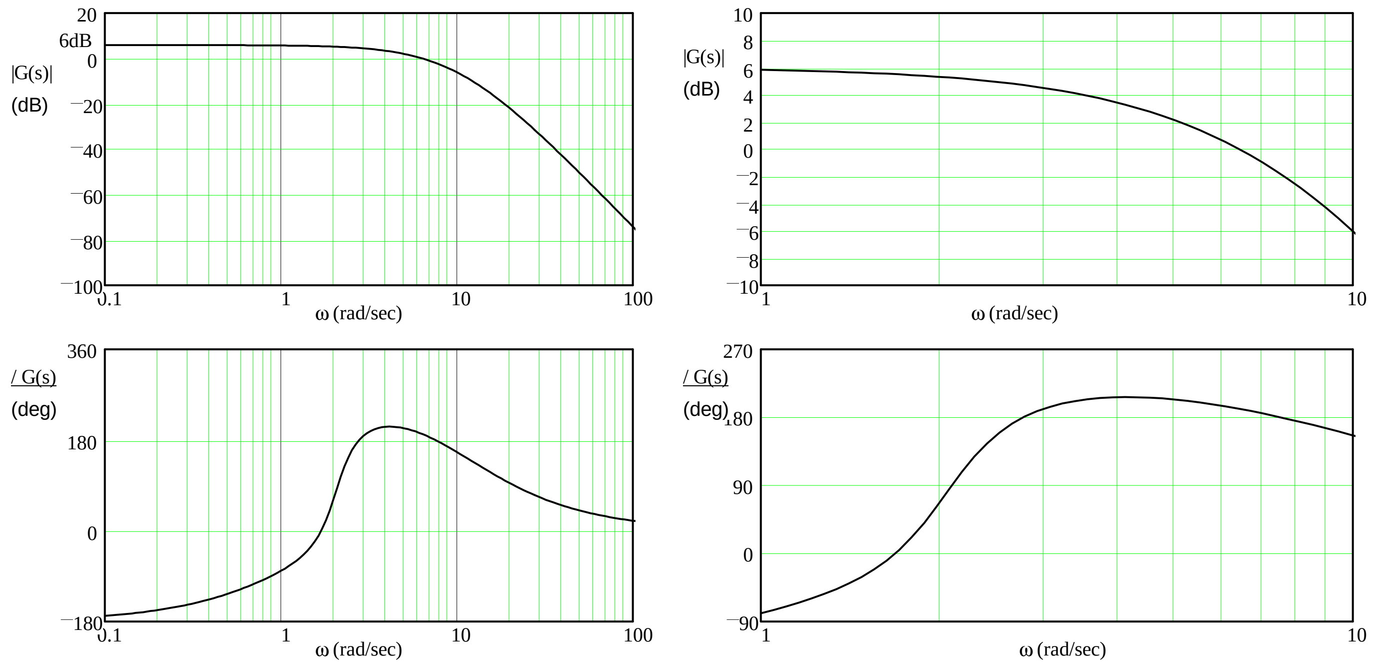
1. Problem 5.13 b & new c & d in the text.

b) Bode plots of the open-loop transfer function of a feedback system are shown below, with the detail from 1 to 10 rad/sec shown on the left. For this system:

• How much can the open-loop gain be changed (increased and/or decreased) before the closed-loop system becomes unstable ?

• What is a rough estimate of the phase margin of the feedback system? Show on the graph how the results are obtained. The numerical results do not have to be precise.

• How much time delay can there be in feedback system before the phase margin disappears.

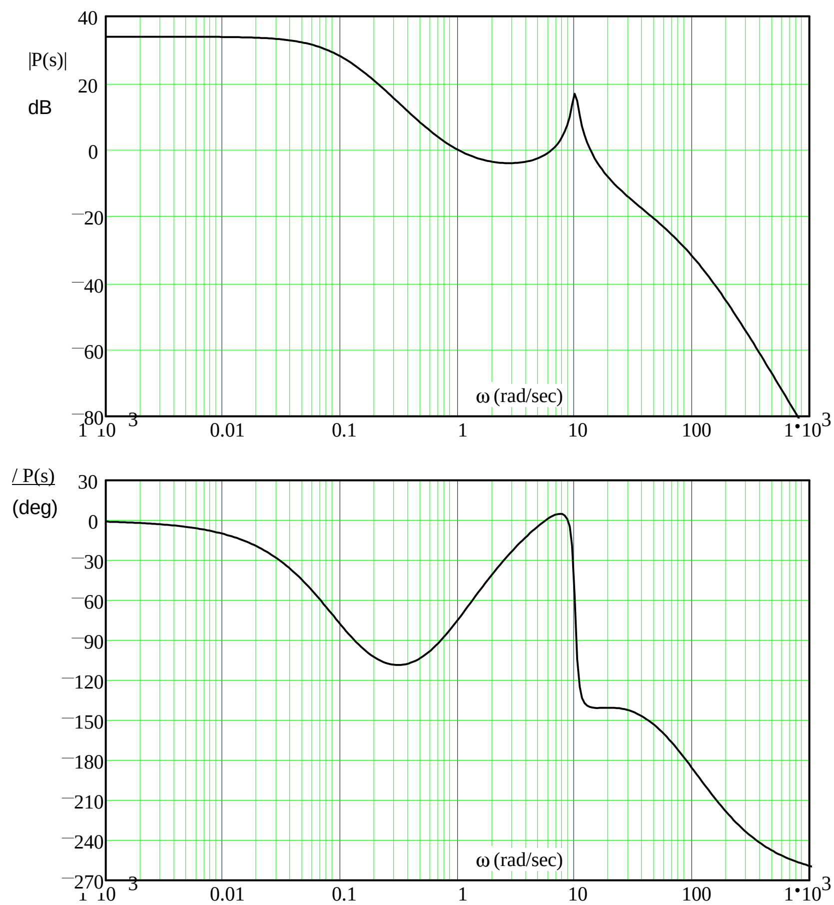


c) For the system of part (a), give the steady-state response of the open-loop system an input $x(t) = 4\cos(10t)$. express the answer in the time-domain.

d) Give the steady-state response of the closed-loop system for the same input. Hint: closed loop output is: $\text{input}\cdot\frac{G(10\cdot j)}{1+G(10\cdot j)}$

2. Like problem 5.9a in the Bodson text.

   a) Give the gain margin, the phase margin and the delay margin of the system whose Bode plots are shown at right (the plots are for the open-loop transfer function and the closed-loop transfer function is assumed to be stable).



Add another sheet of paper the following:

3. A system has a delay of $D := 0.01 \cdot sec$ How many degrees of phase does this represent at:

   a) $f := 1 \cdot Hz$ $\quad f := 10 \cdot Hz$ $\quad f := 100 \cdot Hz$ $\quad f := 1 \cdot kHz$

   b) $\omega := 1 \cdot \frac{rad}{sec}$ $\quad \omega := 10 \cdot \frac{rad}{sec}$ $\quad \omega := 100 \cdot \frac{rad}{sec}$ $\quad \omega := 1000 \cdot \frac{rad}{sec}$

4. a) If the phase response of a pure time delay were plotted on linear phase vs. linear frequency plot, what would be the shape of the curve?

   b) If the phase response of a pure time delay were plotted on linear phase vs. logarithmic frequency plot, what would be the shape of the curve?

5. Ch. 10, prob.30 in Nise, 3rd Ed., page 677. Similar to example 10.17 on page 658 (in section 10.12)
   Ch. 10, prob.30 in Nise, 4th Ed., page 682. Similar to example 10.17 on page 663 (in section 10.12)
   Ch. 10, prob.30 in Nise, 5th Ed., page ___. Similar to example 10.17 on page ___ (in section 10.12)
   Ch. 10, prob.30 in Nise, 6th Ed., page 614. Similar to example 10.17 on page 600 (in section 10.12)

   Given a unity feedback system with a forward-path transfer function $G(s) = \frac{K}{s\cdot(s+3)\cdot(s+12)}$

   and a delay of 0.5 second, make a second-order approximation and estimate the percent overshoot if $K := 40$

   Use Bode Plots and frequency response techniques or you may calculate where $|G(j\omega)| = 1$ to find ω and then the the phase margin. You may also use Matlab.

   You may use the estimate from equation in Bodson text: $\zeta \simeq \frac{PM}{100\cdot deg}$ (PM in degrees and may include delay effects)

   Also helpful: $\%OS = 100\%\cdot e^{\frac{-\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}$

6. In section 5.3.9 of Bodson's book, he discusses using a lead controller to stabilize a system (plant) represented by a double integrator. Give one or more examples of real systems that are essentially double integrators.

7. Problem 5.14 in the text.

   a) Consider the lead controller for the double integrator. For the design that makes the crossover frequency equal to ω_C, obtain the polynomial that specifies the closed-loop poles (as a function of a/b and ω_C). Show that one closed-loop pole is at s = - ω_C no matter what a/b is.

   Hints: Find: $G(s) = P(s)\cdot C(s)$

   Find the denominator of the closed-loop transfer function: $D_G + N_G$

   Substitute in a, b, and k_c like eq. 5.71 in book.

   Use polynomial division to show that $D_G + N_G$ can be divided by $(s + \omega_c)$ with no remainder.

   b) Compute the other closed-loop poles, as functions of ω_C, when a/b = 5.83, 9, and 13.9.

   Hint: The "other" roots are the roots of the quotient.

   c) Use Matlab SISO or other software of your choice to confirm the results of part c) and the % overshoot figures expected from the phase margins. ($20\cdot\%$ $14\cdot\%$ $9.5\cdot\%$)

<u>Answers</u>

1. b) Gain may be increased by ~ 2dB and reduced by ~ 4.4dB. $PM \simeq 13\cdot deg$ $DM \simeq 36\cdot ms$
   c) $2\cdot\cos(10\cdot t + 158\cdot deg)$ d) $3.5\cdot\cos(10\cdot t + 140\cdot deg)$

2. a) $GM \simeq 30\cdot dB$ $PM \simeq 40\cdot deg$ $DM \simeq 50\cdot ms$

3. a) $3.6\cdot deg$ $36\cdot deg$ $360\cdot deg$ $3600\cdot deg$ b) $0.573\cdot deg$ $5.73\cdot deg$ $57.3\cdot deg$ $573\cdot deg$

4. a) A straight line of negative slope, ωD, where D is the time delay.

   b) A negative sloping line with a slope of ωD. Since the frequency increases by a factor of 10 each decade, so would the downward slope of the line.

5. Calculated 0dB freq: $\omega := 1.045$ expect about 30% overshoot

6. Any system with mass where a force is the input and position is the "output". $F = m\cdot a = m\cdot\frac{d^2}{dt^2}x$

7. b) a/b = 5.83: $(-0.7071 - 0.7071\cdot j)\cdot\omega_c$ & $(-0.7071 + 0.7071\cdot j)\cdot\omega_c$

   a/b = 9: $-\omega_c$ & $-\omega_c$ (making 3 all at ω_c)

   a/b = 13.9: $-0.436\cdot\omega_c$ & $-2.292\cdot\omega_c$

   c) Poles are confirmed with $\omega_c := 10$ $k_p := 1$ gain = 1

   Overshoots are about double expected values $35\cdot\%$ $27\cdot\%$ $21\cdot\%$