ECE 3510 homework # 5 Due: Mon, 1/30/06

a

1. Find the step response of: $H(s) = \dfrac{k}{(s+a_1)\cdot(s+a_2)}$ Step input: $x(t) = x_m \cdot u(t)$

$$X(s) = \frac{x_m}{s}$$

Show the steps necessary to arrive at the steady-state and transient responses shown as equation(s) 3.35 on p.33 of the text.

2. Find the step response of: $H(s) = \dfrac{k\cdot s}{(s+a)^2 + b^2} = \dfrac{k\cdot s}{s^2 + 2\cdot s\cdot a + (a^2+b^2)}$

Show the steps necessary to arrive at the steady-state and transient responses.

3. For the transfer functions below, find the DC gain and the full step responses. You may use the results found in section 3.3.2 of the text as well as problem 2, above.

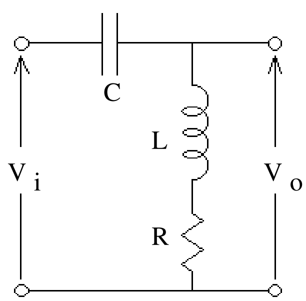
a) $H(s) = \dfrac{2}{s^2 + 2\cdot s + 1}$

b) $H(s) = \dfrac{-s-2}{s^2 + 2\cdot s + 2}$

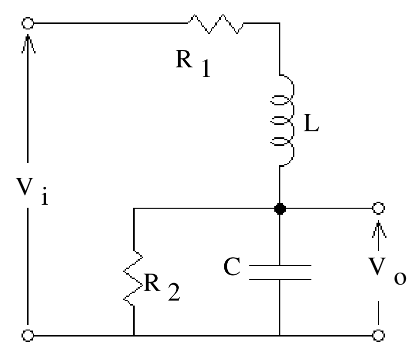
Hint: Notice how easily this will split into two parts that you already have answers for.

4. Find the transfer function $H(s) = \dfrac{V_o(s)}{V_i(s)}$ for these circuits. Write H(s) in the normal form, as a ratio of polynomials.

a)



b)



Answers

1. $y(\infty) = \dfrac{x_m\cdot k}{a_1\cdot a_2}$

$$y_{tr}(t) = x_m\cdot\left[\frac{k}{a_1\cdot(a_1-a_2)}\cdot e^{-a_1\cdot t} + \frac{k}{a_2\cdot(a_2-a_1)}\cdot e^{-a_2\cdot t}\right]$$

OR: $$y(t) = x_m\cdot\left[\frac{k}{a_1\cdot a_2} + \frac{k}{a_1\cdot(a_1-a_2)}\cdot e^{-a_1\cdot t} + \frac{k}{a_2\cdot(a_2-a_1)}\cdot e^{-a_2\cdot t}\right]$$

2. $y(\infty) = 0$ $\quad y_{tr}(t) = x_m\cdot\dfrac{k}{b}\cdot e^{-a\,t}\cdot\sin(b\cdot t)$ OR: $y(t) = 0 + x_m\cdot\dfrac{k}{b}\cdot e^{-a\,t}\cdot\sin(b\cdot t)$

3. a) $x_m\cdot\left(2 - 2\cdot e^{-t} - 2\cdot t\cdot e^{-t}\right)$ b) $x_m\cdot\left(-1 + e^{-t}\cdot\cos(t)\right)$

4. a) $$\frac{s^2 + \frac{R}{L}\cdot s}{s^2 + \frac{R}{L}\cdot s + \frac{1}{L\cdot C}}$$

b) $$\frac{\frac{1}{L\cdot C}}{s^2 + \left(\frac{1}{C\cdot R_2} + \frac{R_1}{L}\right)\cdot s + \left(1 + \frac{R_1}{R_2}\right)\cdot\frac{1}{L\cdot C}}$$