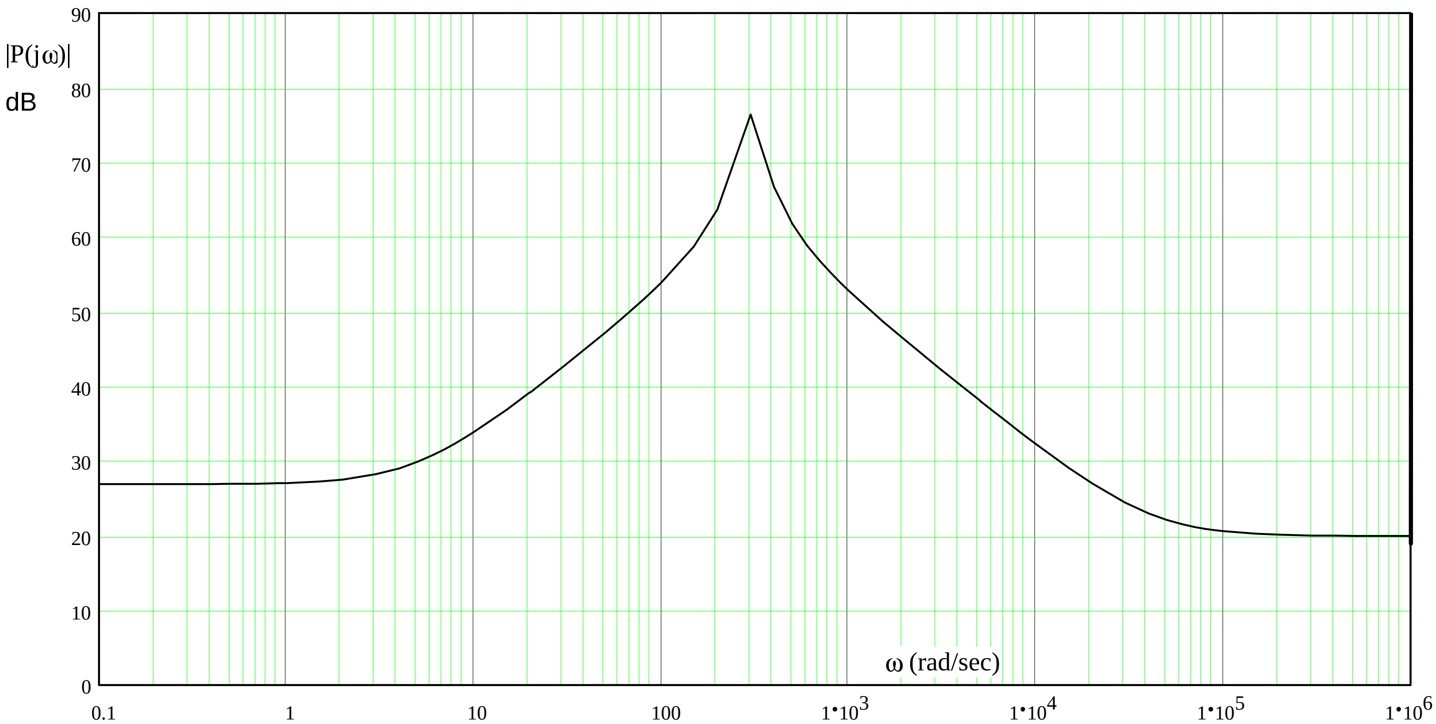
ECE 3510 Final given: Spring 21

DO NOT use eraseable ink

Show all work to receive credit. Circle answers, show units, and round off reasonably

1. (20 pts) Given the magnitude Bode plot of a system, estimate the transfer function of the system.
   Assume there are no negative signs in the transfer function (all poles and zeros are in the left-half plane).
   Use a straight edge and show your work (how you made your estimate).



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2. (40 pts) Consider the transfer function: $\mathbf{G}(s) := \dfrac{s+6}{(s+0.5)\cdot\left(s^2+6\cdot s+13\right)}$

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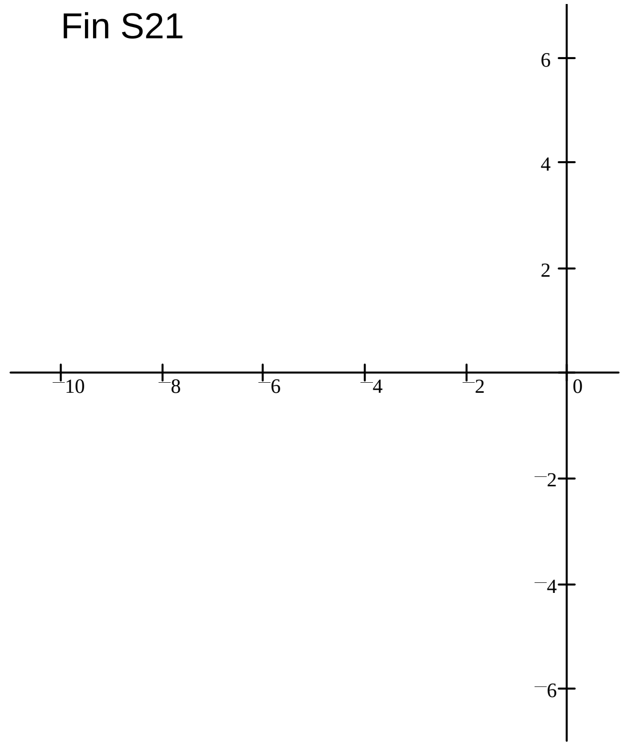
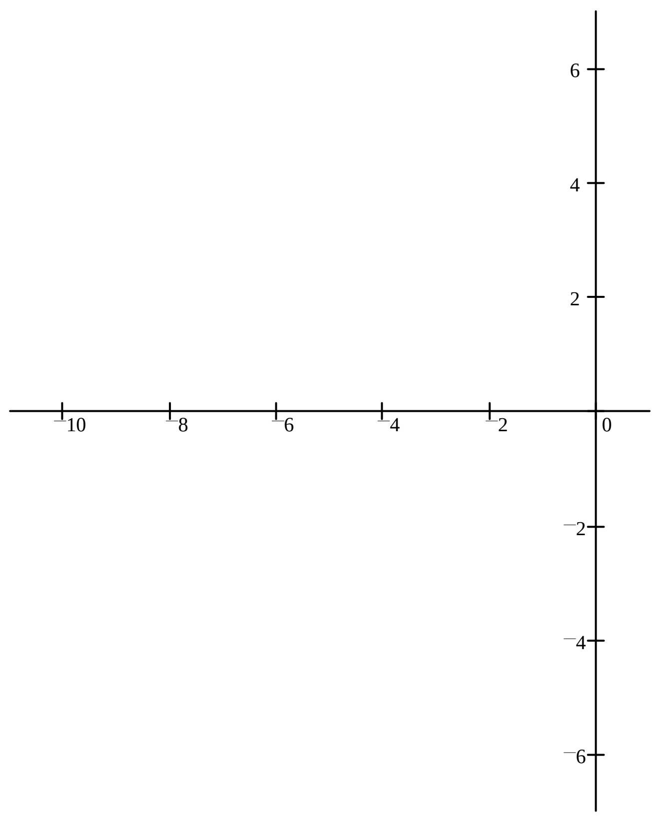
a) Sketch the root-locus plot



b) Does the root-locus pass through the point $s := -2.15 + 2\cdot j$
Show your work or state what did in your calculator.

c) Where should the closed-loop poles be located to decrease the settling time to 0.8 sec and set the ringing frequency to 0.6366 Hz ? (Use the second-order approximation.)



d) Add a compensator so that the closed-loop poles can be placed at the location found in part c). Use the angle summing method and show your work, esp. the angles that you find.

Note: If you can't calculate the zero location or doubt your calculation, assume it is at -3 for the rest of this problem.
For the remaining problem, the compensator in place and a closed-loop pole at the location desired in part d)

e) What is the gain?

f) What is the steady-state error to a unit-step input?

g) What is the simplest way to improve the steady-state error to a unit-step input (below 2%)? If it is another compensator, give specifics (numbers). Determine if the location from part c) is still close to the RL plot. Justify your answer.

3. (9 pts) A Nyquist curve is shown at right (only the portion for $\omega > 0$ is plotted).

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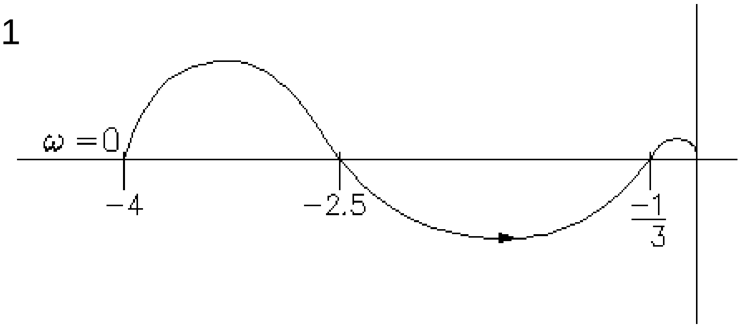
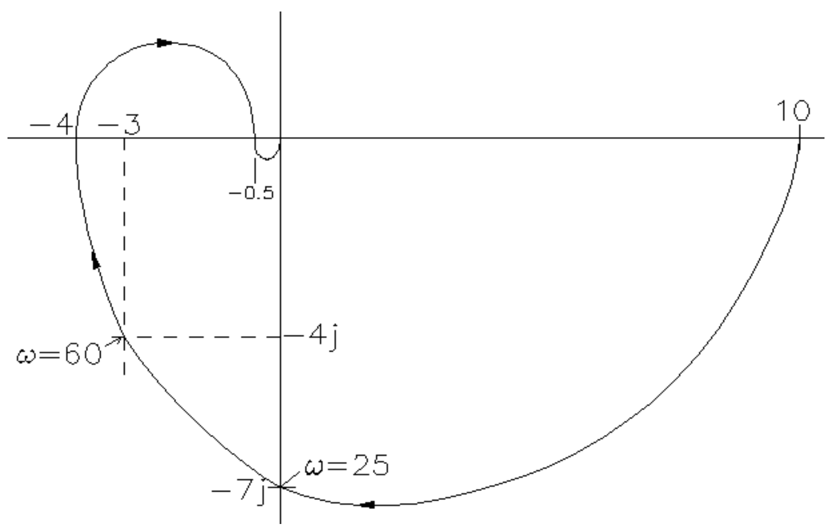


a) Knowing that the closed-loop system is stable, how many unstable poles could the open-loop system have?

b) What is (are) the gain margin(s)?

c) Does the open-loop system have more poles than zeros? If yes, how many? _____ / 9

4. (20 pts) All parts of this problem refer to the system whose Nyquist curve is shown at right (only the portion for $\omega > 0$ is plotted). Recall that the Nyquist curve represents the frequency response of the open-loop system, or $\mathbf{G}(j\omega)$. If $G(s)$ is the open-loop transfer function. The closed-loop transfer function is $\mathbf{G}(s)/(1 + \mathbf{G}(s))$. The open-loop system is stable.



a) Can the closed-loop system be stable? Show why or why not.

b) Could the closed-loop system be stable if
Show why or why not.ed by a factor of 4?

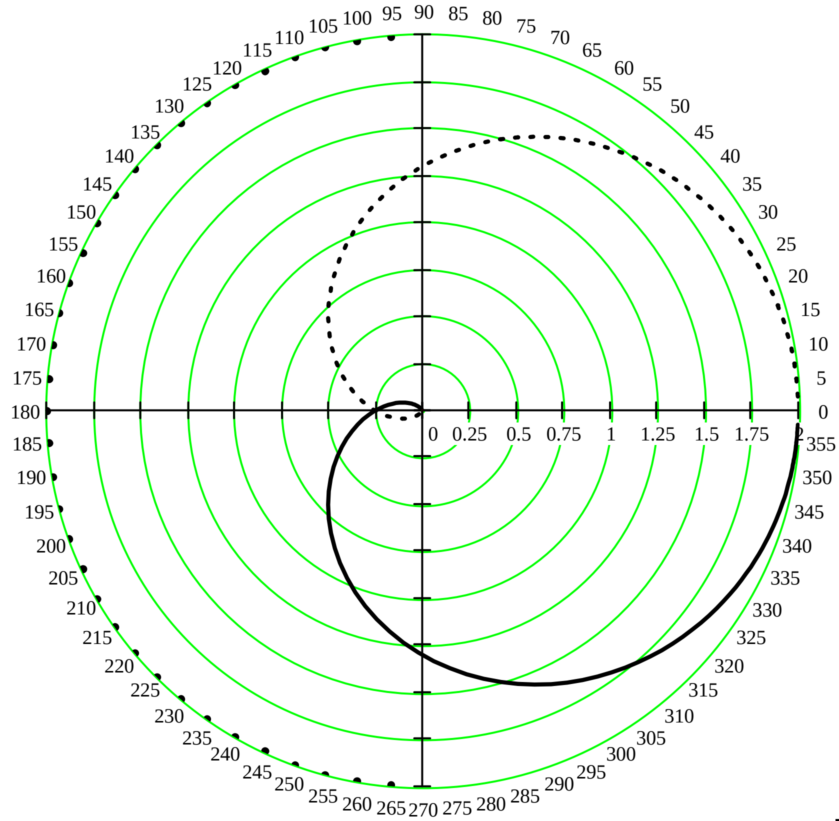
c) Give the steady-state response $y_{ss}(t)$ of the open-loop system to an input $x(t)=3\cos(25t)$.

The gain is reduced by a factor of 5 (multiplied by 0.2) for the remainder of this problem.

d) Is the closed-loop system stable now? Show why or why not.

e) Give the steady-state response $y_{ss}(t)$ of the closed-loop system to an input $x(t) = 9$

5. (6 pts) For the given Nyquist plot, find the gain margin and phase margin of the closed-loop system.
   Show your work on the drawing.



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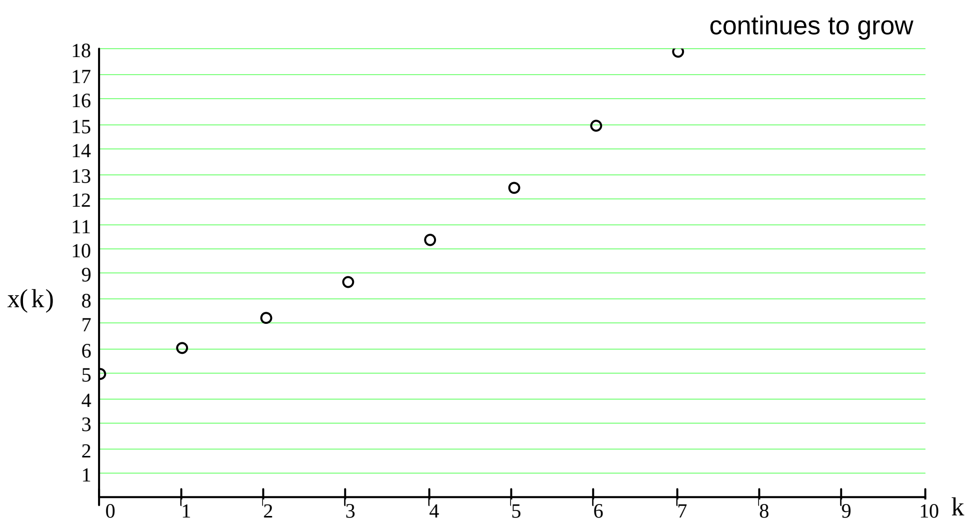
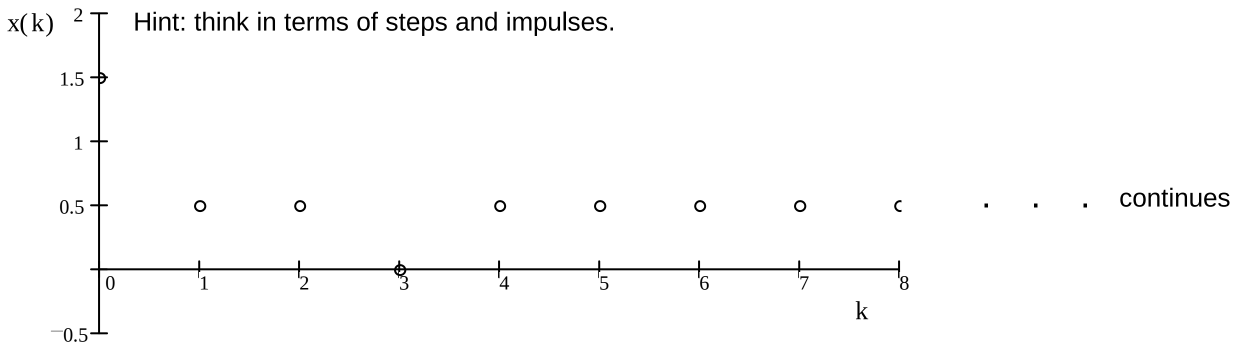
6. (20 pts) Discrete-time signals are shown below. Find x(k) and the z-transforms **X**(z) of these signals.

   a) Find the z-transform in closed form (not a series). Hint: What is the equivalent of an exponential in discrete time?



b) Find the z-transform in any form.



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7. (10 pts) Use partial fraction expansion to find $x(k)$ for the following z-transform: $\mathbf{X}(z) = \dfrac{12}{(z-4)\cdot(z+2)}$

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8. (25 pts) a) Use partial fraction expansion to find $x(k)$ for the following z-transform: $\mathbf{X}(z) = \dfrac{z\cdot(z+0.4)}{(z+0.6)\cdot\left(z^2-z+0.89\right)}$

b) Is the signal represented by part b) bounded? Does it converge? If yes, to what value? _____ / 25

9. (22 pts) a) Draw the block diagram of a simple direct implementation of the difference equation.

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$$y(k) = 2\cdot x(k) + 3\cdot x(k-1) - x(k-3) - \frac{1}{5}\cdot y(k-1) + \frac{1}{3}\cdot y(k-2)$$

b) Find the $\mathbf{H}(z)$ corresponding to the difference equation above. Show your work.
Express the z-transform of part b) in the normal form, as the ratio of polynomials or factors of polynomials.
(In other words combine the individual terms of $\mathbf{X}(z)$ with a common denominator and clean up.)

c) List the poles of $\mathbf{H}(z)$. Indicate multiple poles if there are any.

d) Is this system BIBO stable? Yes No How do you know?

e) Is this an FIR system? Yes No
If not, which terms in the difference equation are to blame?

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10. (8 pts) Draw a minimal implementation of a system with the following transfer function: $\mathbf{H}(z) = \dfrac{-z^3 + (z-2)\cdot(z+5)}{z\cdot\left(z^2 + \dfrac{z}{2} - 3\right)}$

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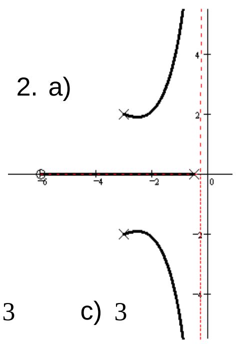
Answers

1. $\dfrac{10\cdot(s+5)\cdot(s+40000)}{\left(s^2 + 60\cdot s + 90000\right)}$

2. a) 

b) YES c) $s := -5 + 4\cdot j$ d) $\mathbf{C}(s) = s + 2.111$

e) 5.29 f) $8.84\cdot\%$ g) Just increase the gain

3. a) 1 b) $0.4 <$ gain < 3 c) 3

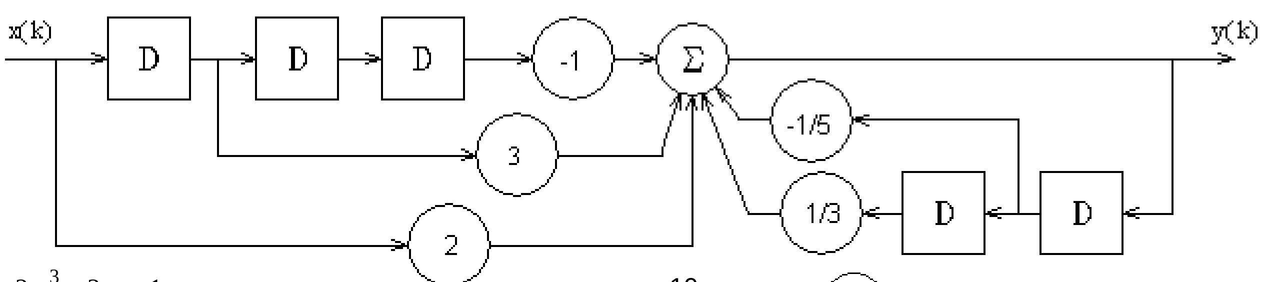
4. a) No b) Yes c) $21\cdot\cos(25\cdot t - 90\cdot deg)$ d) Yes e) 6

5. $GM := 4 \quad 67\cdot deg$

6. a) $5\cdot\dfrac{z}{z-1.2}$ b) $\dfrac{1}{2}\cdot\dfrac{z}{(z-1)} + 1 - \dfrac{1}{2\cdot z^3}$

7. $\left[-\dfrac{3}{2}\cdot\delta(k) + \dfrac{1}{2}\cdot(4)^k + 1\cdot(-2)^k\right]\cdot u(k)$

8. $\left[-0.108\cdot(-0.6)^k + (0.89)^{\frac{k}{2}}\cdot(0.108\cdot\cos(1.012\cdot k) + 1.101\cdot\sin(1.012\cdot k))\right]\cdot u(k)$

9. a) 

b) $\dfrac{2\cdot z^3 + 3\cdot z - 1}{z\cdot\left(z^2 + \dfrac{1}{5}\cdot z - \dfrac{1}{3}\right)}$ c) $0.486 \quad -0.686$

d) Yes, all poles are inside the unit circle

e) No $-\dfrac{1}{5}\cdot y(k-1)$ and $\dfrac{1}{3}\cdot y(k-2)$

10.