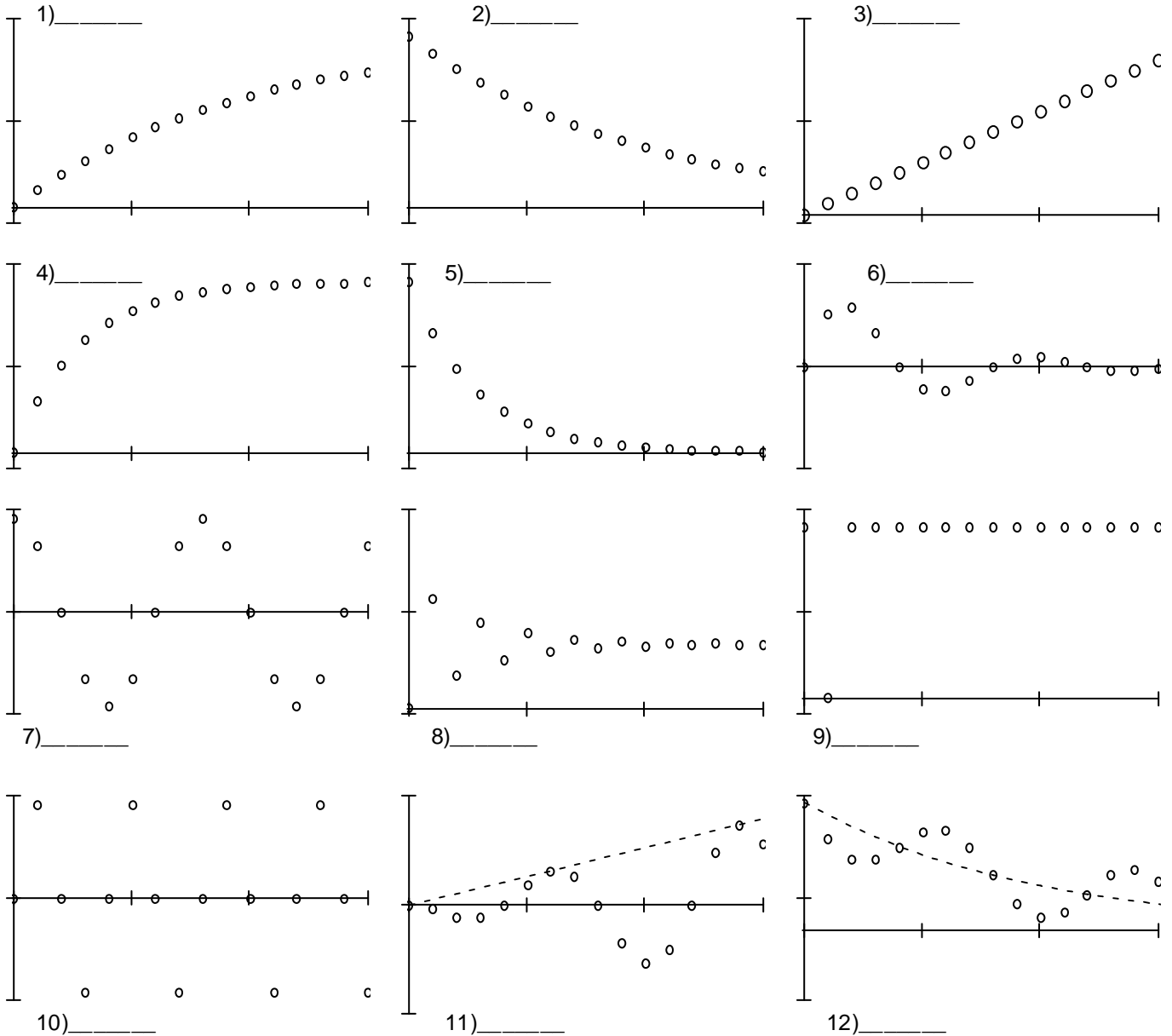


# ECE 3510 Final given: Spring 09

This part of the exam worth 46 points and is **Closed book, Closed notes, No Calculator**.

1. (36 pts) a) Match each of the following discrete-time **signals** to one of the answers on the separate answer sheet.

Find the single best match for each. Your answers should make sense relative to one another.



b) Match each of the following transfer functions to one of the answers on the separate answer sheet

1) $H(z) = \frac{10 \cdot z}{z - 0.9}$	_____	2) $H(z) = \frac{6 \cdot z}{(z - 1) \cdot (z + 0.7)}$	_____
3) $H(z) = \frac{z}{10 \cdot (z^2 + 1)}$	_____	4) $H(z) = \frac{2 \cdot z}{z^2 - 1.414 \cdot z + 1}$	_____

c) If the answers on the separate sheet are considered poles of **transfer functions**, list all that are BIBO stable.

2. (4 pts) a) What other compensator has a similar purpose to a PD compensator?

b) What other compensator has a similar purpose to a PI compensator?

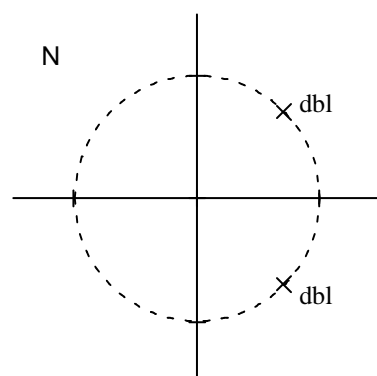
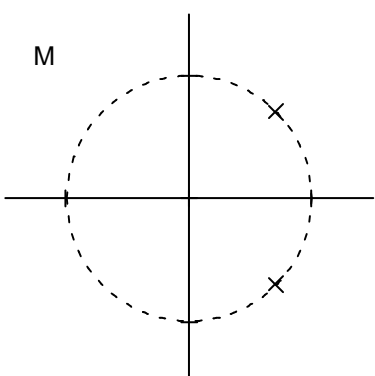
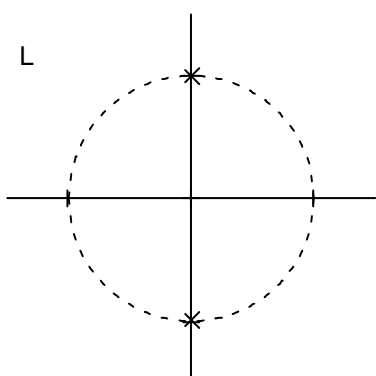
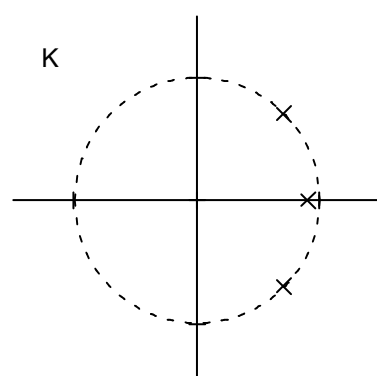
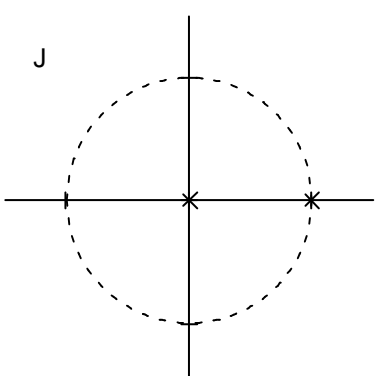
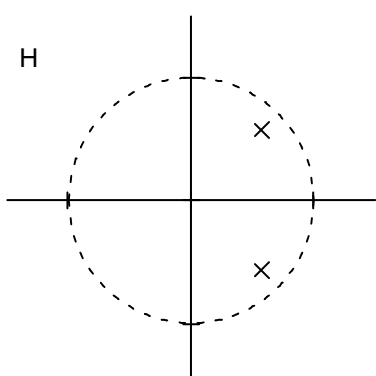
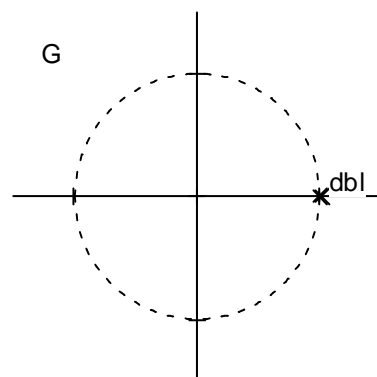
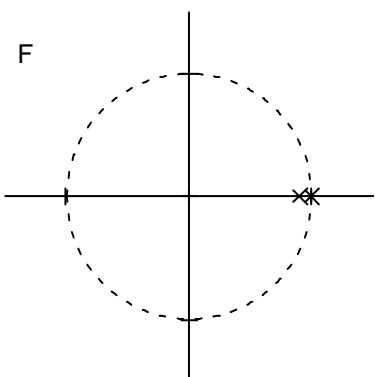
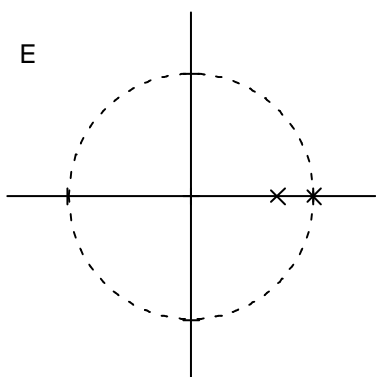
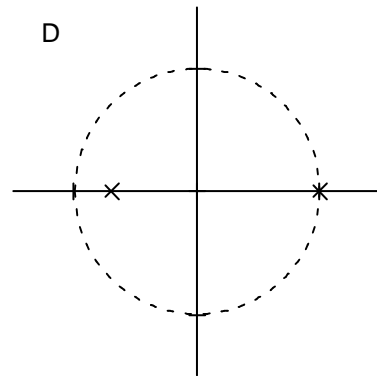
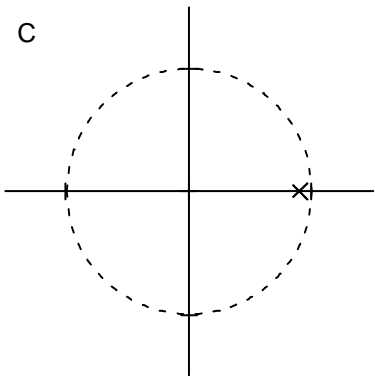
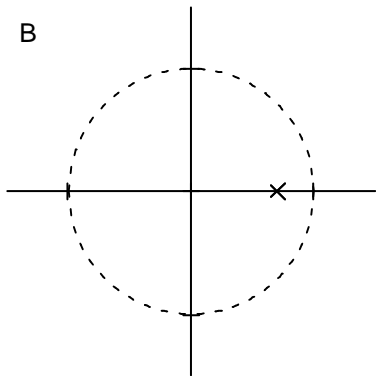
Each answer below is a z-plane showing the unit circle and usually some poles

Answers may be used more than once or not at all.

dbl = double pole at that location.

**ANSWERS**

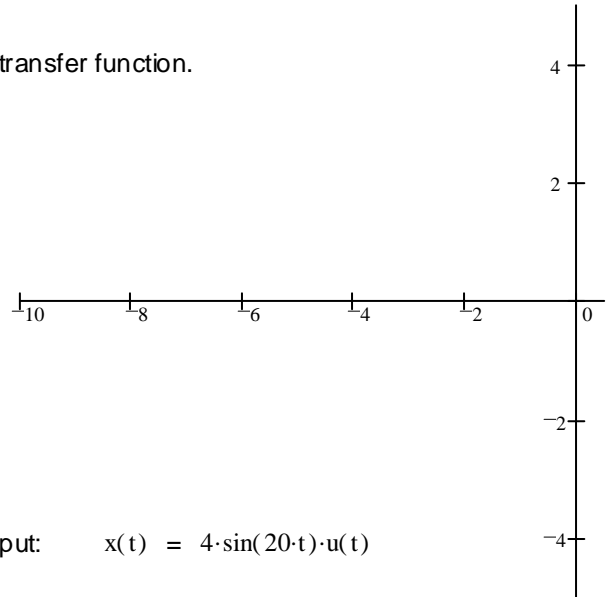
A None of these answers match.



**ECE 3510 Final given: Spring 09 p3**

3. (10 pts) Sketch the root-locus plot for the following open-loop transfer function.

a) 
$$G(s) = \frac{s + 2}{s \cdot (s + 5)^2 \cdot (s + 7)}$$



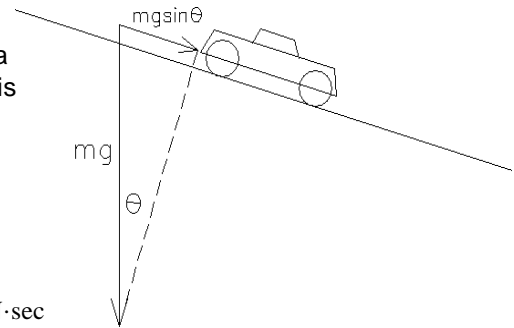
**Open-book Part**

1. (12 pts) This system:  $H(s) = \frac{6}{s + 12}$  Has this input:  $x(t) = 4 \cdot \sin(20 \cdot t) \cdot u(t)$

- a) Find an expression for  $Y(s)$   $Y(s) =$
- b) Separate  $Y(s)$  into partial fractions that you can find in the laplace transform table without using complex numbers. Show what they are, but don't find the coefficients.  $Y(s) =$
- c) Continue with the partial fraction expansion just far enough to find the **transient** coefficient as a number.
- d) Express the transient part as a function of time.  $y_{tr}(t) = ?$

2. (17 pts) Refer to the Electrical Analogies of Mechanical Systems handout.

A small car is rolling down a hill. It is using a DC motor to slow down. The DC motor is acting as a generator and is connected to a resistor. The "input" to this system is a force of  $mg \sin(\theta)$ , where  $m$  is the mass of the car and  $g$  is the acceleration of gravity.



Note: You may have to rethink some of the "transformer" "turns ratios" for parts that you usually use in the opposite way.

Car characteristics:	Mass:	0.5 · kg
	Viscous friction:	$0.002 \cdot \frac{\text{N} \cdot \text{sec}}{\text{m}}$
Wheel characteristics:	Moment of Inertia:	$3 \cdot 10^{-5} \cdot \text{kg} \cdot \text{m}^2 \cdot 4 \text{ wheels} = 12 \cdot 10^{-5} \cdot \text{kg} \cdot \text{m}^2$
	Radius:	0.02 · m

The DC motor is connected directly to the wheel shaft

The motor characteristics:	Armature resistance:	$2 \cdot \Omega$	Viscous friction:	$8 \cdot 10^{-5} \cdot \text{N} \cdot \text{m} \cdot \text{sec}$
	Armature Inductance:	50 · mH	Moment of Inertia:	$4 \cdot 10^{-5} \cdot \text{kg} \cdot \text{m}^2$
	Motor constant, K	$0.2 \cdot \text{V} \cdot \text{sec}$		

The resistor hooked to the motor is variable "R"

Anything else not listed here can be neglected.

- a) Draw an electrical equivalent of the system system, including "transformers".
- b) Use the values given above to assign electrical values to the parts. Use numbers and electrical units. Remember that base mechanical units relate to base electrical units with no conversion factors.
- c) Replace the rightmost transformer and the parts hooked to it with equivalent parts. You only need to relabel and reevaluate the parts that have changed. You do not need to go farther than just the one transformer.

**ECE 3510 Final given: Spring 09 p4**

3. (27 pts) Consider transfer function below.

$$G(s) := \frac{100}{(s + 25) \cdot (s + 40) \cdot (s + 70)}$$

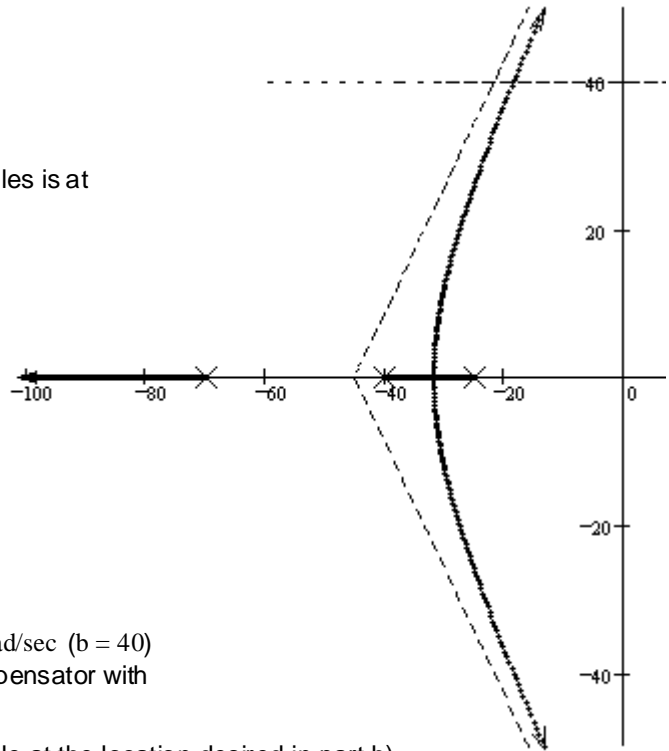
The gain is set at 452, so that one of the closed-loop poles is at

$$s := -24.48 + 27.2j$$

Further calculations yield:

Settling time: 0.163·sec

Steady-state error to a unit-step input: 60.8%



a) What is the % overshoot?

b) You wish to increase the frequency of ringing to 40 rad/sec ( $b = 40$ ) without changing the % overshoot at all. Add a compensator with a single zero so that you will be able to do this.

c) With the compensator in place and a closed-loop pole at the location desired in part b)

i) What is the gain?

ii) What is the 2% settling time? Use the second-order approximation.

iii) What is the steady-state error to a unit-step input?

iv) List those things that improved with this compensation.

d) Add another compensator:  $C_2(s) := \frac{s+2}{s}$  and maintain the gain of part c)

i) What is this type of compensator called and what is its purpose?

ii) Calculate what you need to show that this compensator achieved its purpose.

**NOTE: Problem 4 is on the next page**

5. (22 pts) Find the  $x(k)$  whose z-transform is given. Use partial fraction expansion.

Answers should not have complex numbers

a)  $X(z) = \frac{2.4}{(z - 0.3) \cdot (z + 0.2)}$

b)  $X(z) = \frac{3 \cdot z}{z^2 - 1.44 \cdot z + 0.81}$

6. (16 pts) a) Draw the block diagram of a simple direct implementation of the difference equation.

$$y(k) = 2 \cdot x(k) + \frac{1}{2} \cdot x(k-2) - \frac{1}{3} \cdot x(k-3) + \frac{1}{4} \cdot y(k-1) - \frac{1}{5} \cdot y(k-2)$$

b) Find the  $H(z)$  corresponding to the difference equation above. Show your work.

c) List the poles of  $H(z)$ . Indicate multiple poles if there are any.

**NOTE: Problem 7 is on the next page**

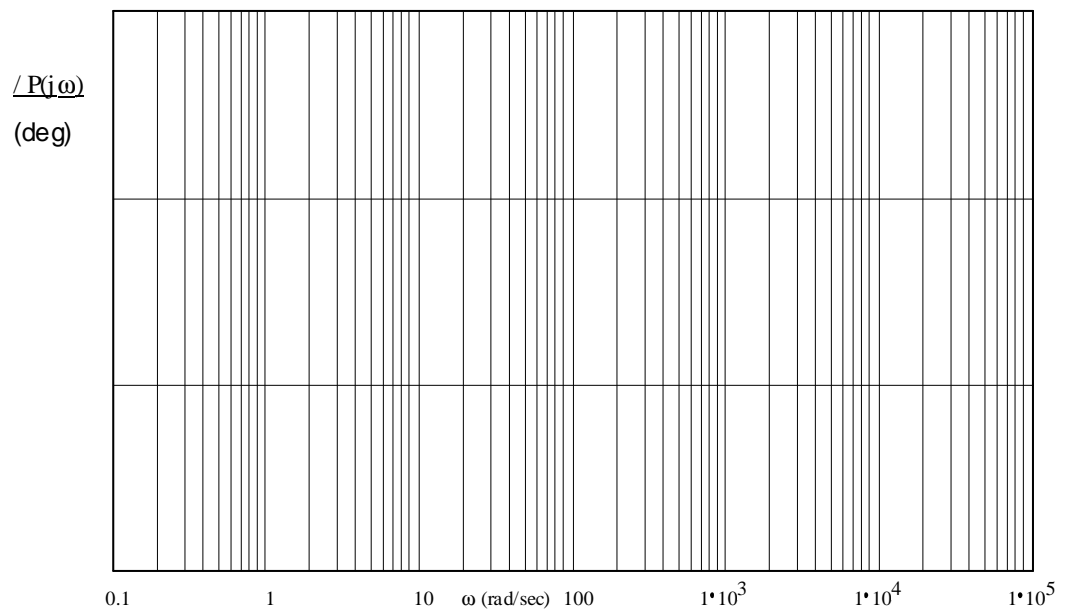
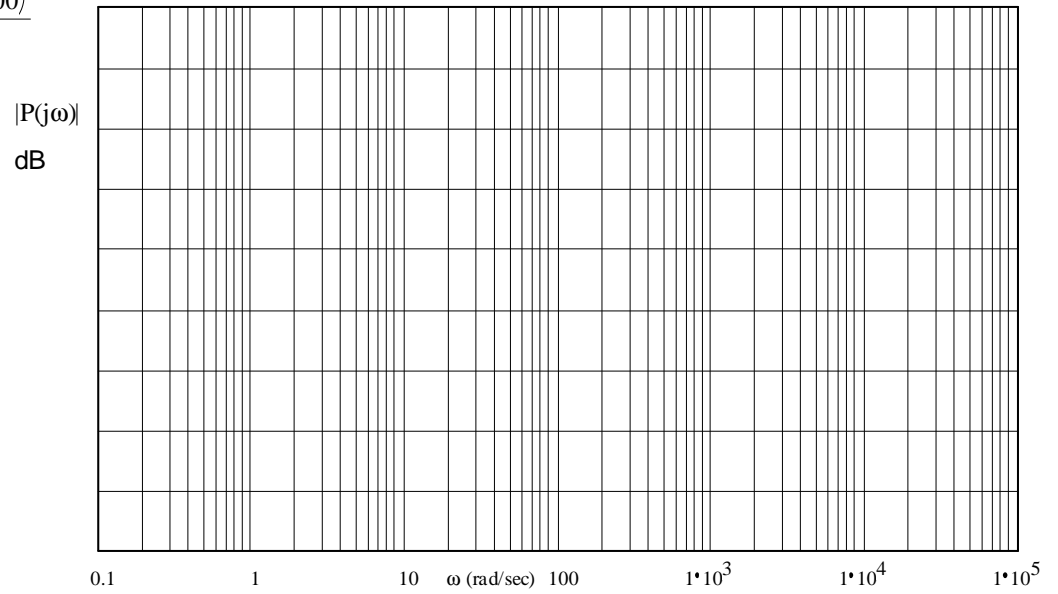
8. (8 pts) Draw a minimal implementation of a system with the following transfer function

$$H(z) = \frac{-z^3 + 4 \cdot z^2 - 3 \cdot z + 2}{z \cdot (z^2 - z + 5)}$$

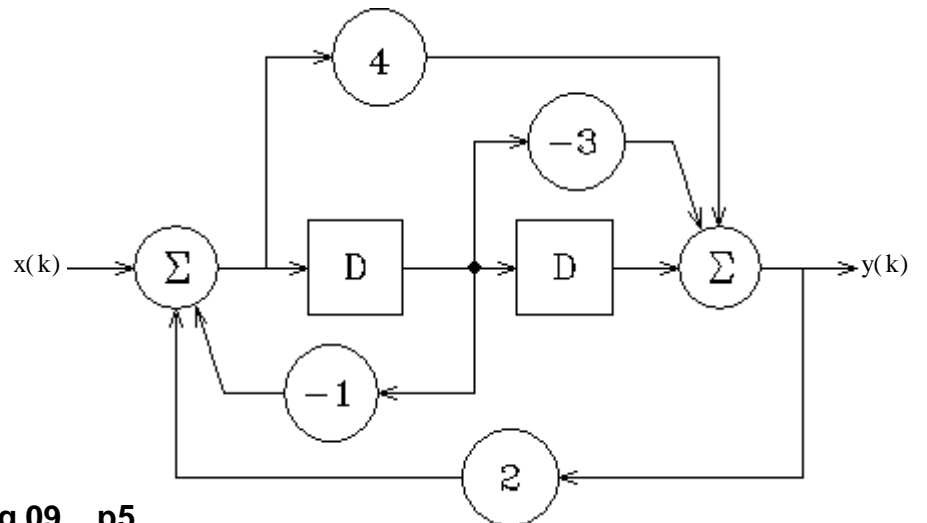
4. (14 pts) Sketch the Bode plots for the following transfer function.

$$P(s) = \frac{100 \cdot (s^2 + 3 \cdot s + 900)}{s \cdot (s + 2000)}$$

Make sure to label the graphs as necessary to show the magnitudes and slopes. Also draw the "smooth" lines.



7. (18 pts) a) Find the transfer function  $H(z) = Y(z)/X(z)$ .



b) Is this system BIBO stable?  
Give a reason for your answer.

**ECE 3510 Final given: Spring 09 p6**

9. Do you want your grade and scores posted on my door and on the Internet?  
 (Circle one) Yes No

If your answer is yes, then provide some sort of alias  
 or password: \_\_\_\_\_

The grades will be posted on my door in alphabetical order under the alias that you provide here. I will not post grades under your real name. The Internet version will be a pdf file which you can download. Both will show the homework, lab, and exam scores of everyone who answers yes here.

ECE 3510 Final

Name \_\_\_\_\_

Scores:

pages 1-2 \_\_\_\_\_ / 29 pts

pages 3-4 \_\_\_\_\_ / 27 pts

pages 5-6 \_\_\_\_\_ / 36 pts

pages 7-8 \_\_\_\_\_ / 34 pts

page 9 \_\_\_\_\_ / 8 pts

Closed Book \_\_\_\_\_ / 46

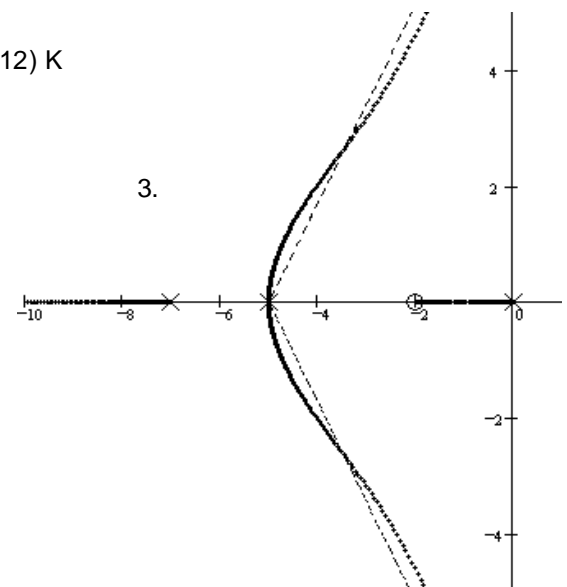
Total \_\_\_\_\_ / 180 pts

**Answers**

1. a) 1) F 2) C 3) G 4) E 5) B 6) H 7) M 8) D 9) J 10) L 11) N 12) K

b) 1) C 2) D 3) L 4) M c) B C H

2. a) Lead b) Lag



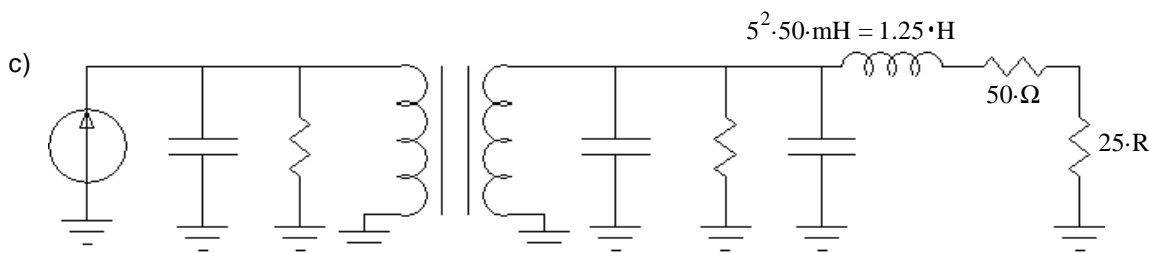
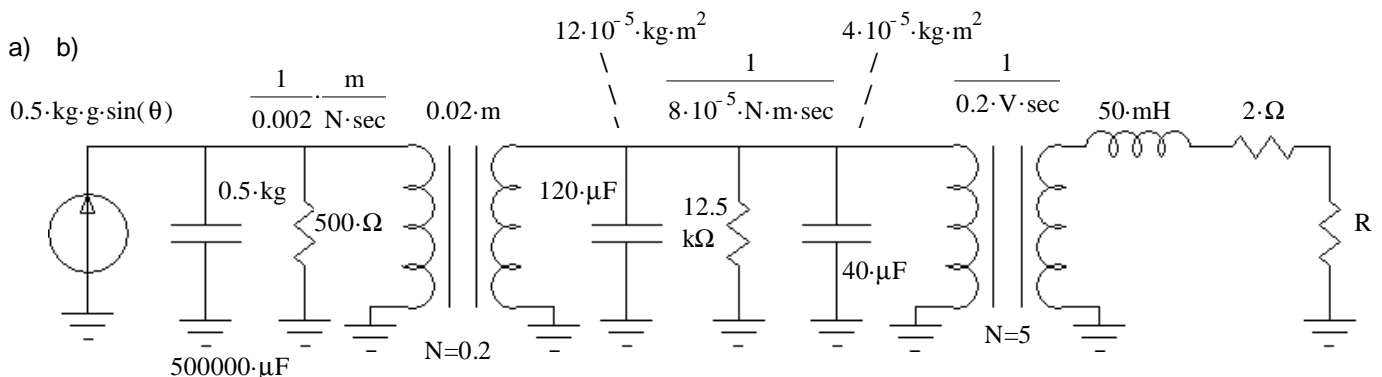
**Open-book Part**

1. a)  $Y(s) = \frac{6}{(s+12)} \cdot \frac{4 \cdot 20}{(s^2+400)}$

b)  $Y(s) = \frac{A}{s+12} + \frac{B \cdot s}{(s^2+400)} + \frac{C \cdot 20}{(s^2+400)}$

c) 0.882 d)  $y_{tr}(t) = 0.882 \cdot e^{-12t}$

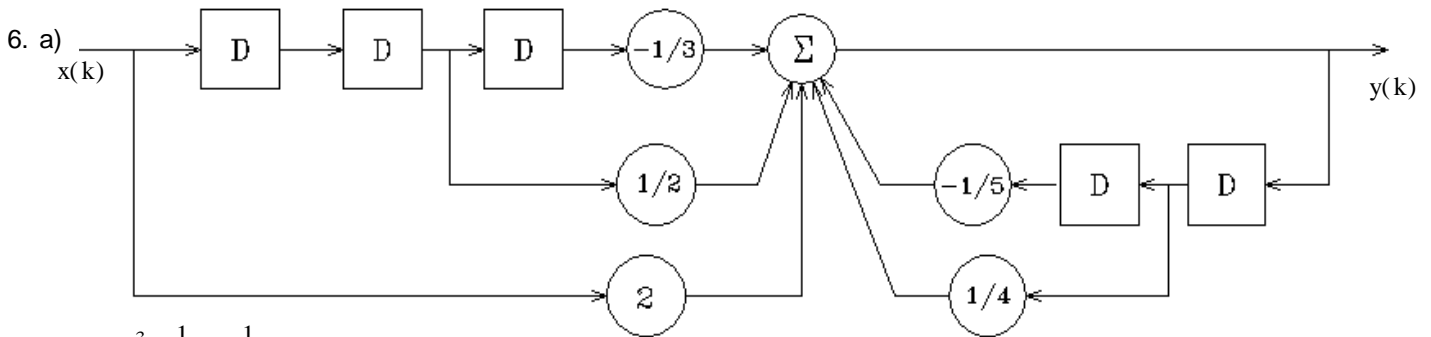
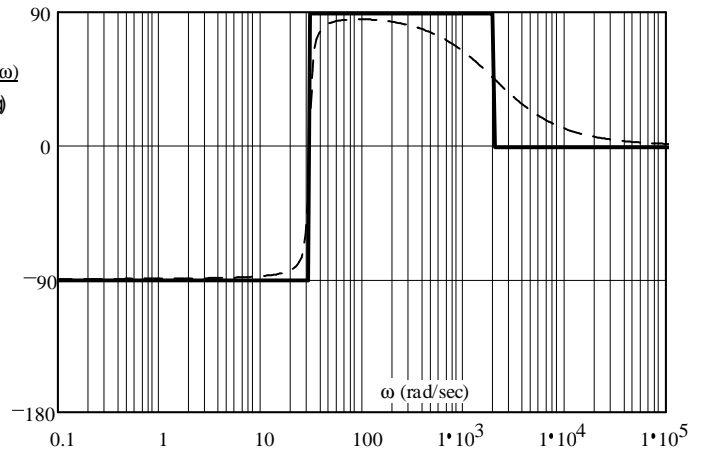
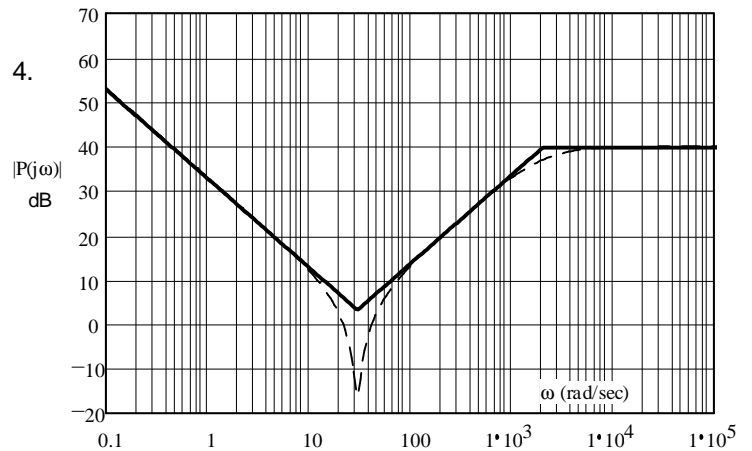
2. a) b)



**ECE 3510 Final given: Spring 09 p7**

3. a) 5.92%    b)  $(s + 59.75)$   
 c) i) 18.8    ii) 0.111-sec    iii) 38.4%  
 iv) Speed and steady-state error  
 d) i) PI, used to eliminate steady-state error  
 ii)  $\frac{1}{1 + k \cdot \infty} = 0\%$

5. a)  $-40 \cdot \delta(k) + 16 \cdot (0.3)^k + 24 \cdot (-0.2)^k$   
 b)  $2 \cdot 2.778 \cdot 0.9^k \cdot \cos\left(0.644 \cdot k - \frac{\pi}{2}\right)$   
 OR  
 $5.556 \cdot (0.9)^k \cdot \sin(0.644 \cdot k)$



b)  $\frac{2 \cdot z^3 + \frac{1}{2} \cdot z - \frac{1}{3}}{z \cdot \left(z^2 - \frac{1}{4} \cdot z + \frac{1}{5}\right)}$     c)  $0.125 + 0.429 \cdot j$   
 $0.125 - 0.429 \cdot j$  &  $0$

7. a)  $\frac{-4 \cdot z^2 + 3 \cdot z - 1}{7 \cdot z^2 - 7 \cdot z + 2} = \frac{-\frac{4}{7} \cdot z^2 + \frac{3}{7} \cdot z - \frac{1}{7}}{z^2 - z + \frac{2}{7}}$

b) YES, poles are inside the unit circle  
 poles  $\begin{pmatrix} 0.5 + 0.189 \cdot j \\ 0.5 - 0.189 \cdot j \end{pmatrix}$

