ECE 3510 Final given: Spring 08

1. (12 pts) This system: $H(s) = \dfrac{3}{s+8}$

   Has this input: Cosine input: $x(t) = 4\cdot\cos(10\cdot t)\cdot u(t)$ $\qquad X(s) = \dfrac{4\cdot s}{\left(s^2+10^2\right)}$

   Resulting in this output: $Y(s) = \dfrac{3}{(s+8)}\cdot\dfrac{4\cdot s}{\left(s^2+100\right)}$

   a) This separates into 3 partial fractions that you can find in the laplace transform table without using complex numbers. Show what they are, but don't find the coefficients.

   $$Y(s) = \frac{3}{s+8}\cdot\frac{4\cdot s}{s^2+100} =$$

   b) Continue with the partial fraction expansion just far enough to find the **transient** coefficient as a number.

   c) Express the transient part as a function of time. $y_{tr}(t) = ?$

   d) What is the time constant of this expression? $\tau = ?$

2. (18 pts) An electric motor is used to move a small car. It is hooked to the wheel through gears.

   V_{in} is the voltage supply hooked to the motor

The motor charateristics:	Armature resistance:	$2\cdot\Omega$
	Armature Inductance:	$50\cdot mH$
	Motor constant, K	$0.1\cdot V\cdot sec$
	Viscous friction:	$1\cdot 10^{-4}\cdot N\cdot m\cdot sec$
	Moment of Inertia:	$4\cdot 10^{-5}\cdot kg\cdot m^2$
Gear characteristics:	Motor gear has 10 teeth	
	Wheel-shaft gear has 50 teeth	
Wheel characteristics:	Moment of Inertia:	$2\cdot 10^{-5}\cdot kg\cdot m^2\cdot 4wheels = 8\cdot 10^{-5}\cdot kg\cdot m^2$
	Radius:	$0.02\cdot m$
Car characteristics:	Mass:	$0.2\cdot kg$
	Viscous friction:	$0.001\cdot\dfrac{N\cdot sec}{m}$

   Anything else not listed here can be neglected.

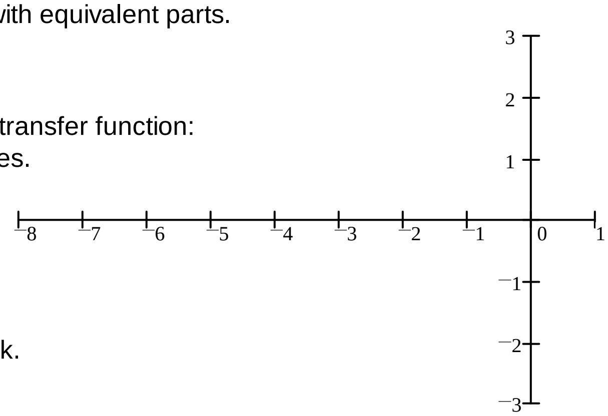
   a) Refer to page 6 of the Electrical Analogies of Mechanical Systems handout. Draw an electrical equivalent of this sytem, including "transformers".

   b) Use the values given above to assign electrical values to the parts. Use numbers and electrical units. Remember that base mechanical units relate to base electrical units with no conversion factors.

   c) Replace the right-most transformer and the parts hooked to it with equivalent parts.
   You only need to re-label the parts that have changed.
   You do not need to go farther than just the one transformer.

3. (12 pts) a) Sketch the root-locus plots for the following open-loop transfer function:
   Use only the main rules, that is, the first page of my root locus notes.
   Be sure to show your work for other calculations.

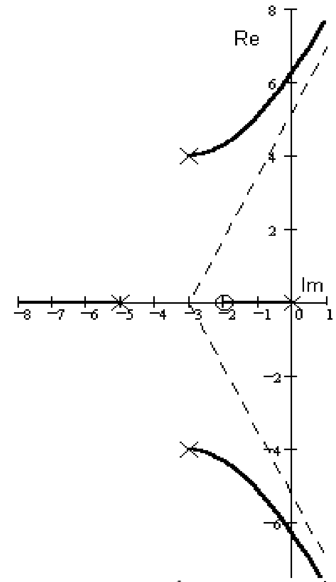
   $$G(s) = \frac{(s+4)}{s\cdot(s+3)}$$



   b) "Find all the "breakaway" and/or "arival" points. Show your work.

4. (11 pts) A root - locus is sketched at right.

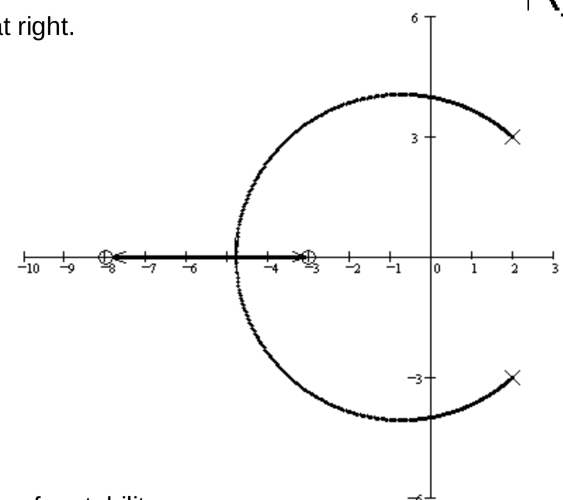
$$G(s) = \frac{3\cdot(s+2)}{s\cdot(s+5)\cdot\left(s^2+6\cdot s+25\right)}$$



a) Find the departure angle from the complex pole $-3 + 4j$.

5. (14 pts) The root locus for the transfer function below is shown at right.

$$G(s) = \frac{(s+3)\cdot(s+8)}{\left(s^2-4\cdot s+13\right)}$$



a) Does the root locus cross the $j\omega$ axis at 4?
Be sure to show the work and method you used to decide.

b) Regardless of what you found in part a, continue to assume that the root-locus crosses the $j\omega$ axis at 4. Give the range of gain k for which the system is closed-loop stable.

Remember, I asked for a **range** for stability

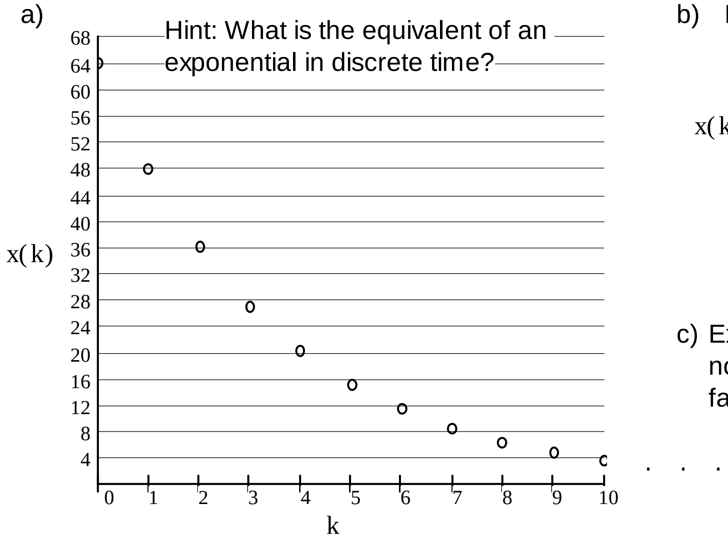
6. (22 pts) Find the $x(k)$ whose z-transform is given. Use partial fraction expansion. Answers should not have complex numbers

a) $X(z) = \dfrac{12}{(z-4)\cdot(z+2)}$

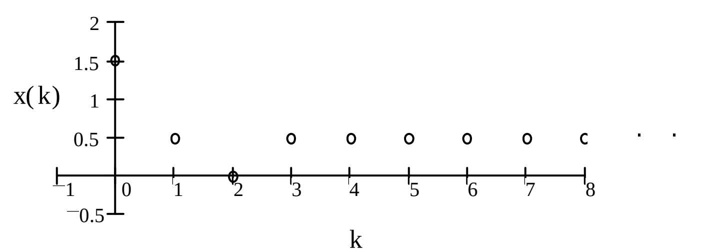
b) $X(z) = \dfrac{3\cdot z}{z^2-4\cdot z+13}$

7. (16 pts) Discrete-time signals are shown below.
Find the z-transforms of these signals in closed form (not a series).

a)



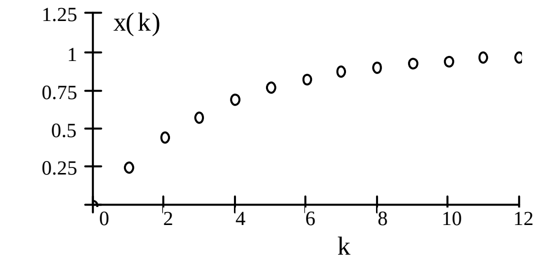
b) Hint: think in terms of steps and impulses.

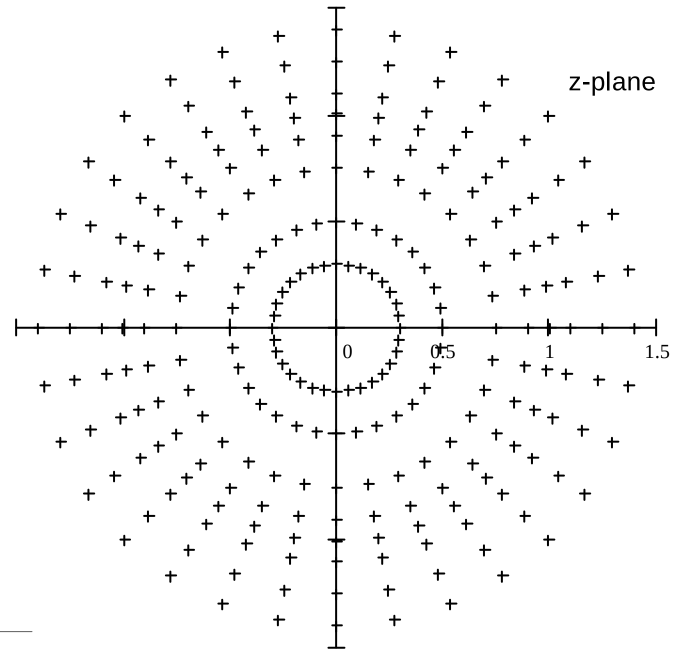


c) Express the z-transform of part b) in the normal form, as the ratio of polynomials or factors of polynomials.

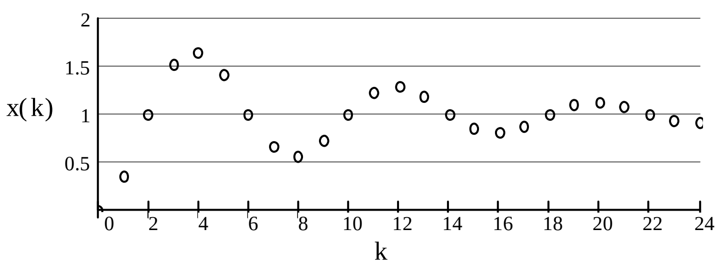
8. (16 pts) Discrete-time signals are shown below. Show the location of the pole or poles of the z-transform of each signal. Select the best position of each pole from among the marks shown and "X" or circle those marks.

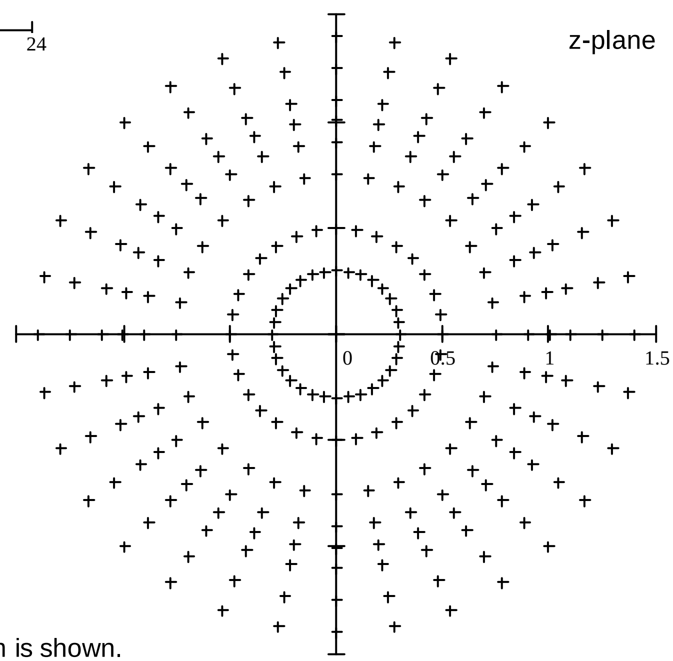
a) Show work



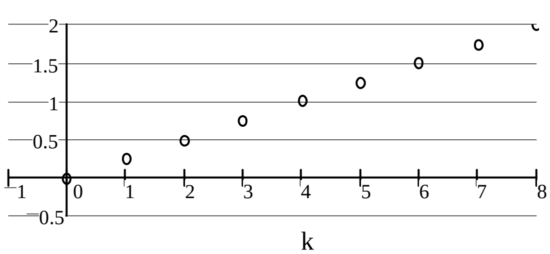


b) Show work





9. (7 pts) The impulse response of a discrete-time system is shown. Find the transfer function in closed form (not a series).



10. (16 pts) a) Find the H(z) corresponding to the difference equation below.

$$y(k) = 4\cdot x(k) - 2\cdot x(k-2) + 3\cdot x(k-3) + \frac{1}{4}\cdot y(k-2)$$

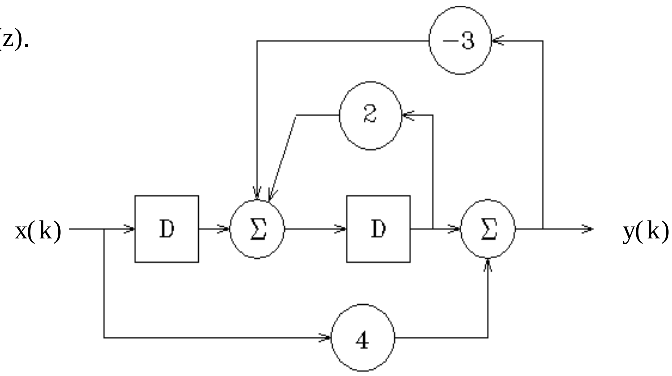
b) List the poles of H(z). Indicate multiple poles if there are any.

c) Draw the block diagram of a simple direct implementation of the difference equation.

11. (24 pts) Find the transfer function H(z) = Y (z)/X(z).

b) Is this system BIBO stable?
Give a reason for your answer.

c) Draw a minimal implementation of this system



12. (12 pts) a) Find the closed-loop transfer function H(s) = Y (s)/X(s).

Plant transfer function: $\dfrac{Y}{W} = \dfrac{k_1}{s\cdot(s+a)^2}$ Control law: $w = k_p\cdot\left(k_F\cdot x - y\right) + k_2\cdot\left[\int (x-y)\,dt\right]$

b) Find any poles or zeros that can be found using no more than the quadratic equation.

13. Do you want your grade and scores posted on my door and on the Internet?

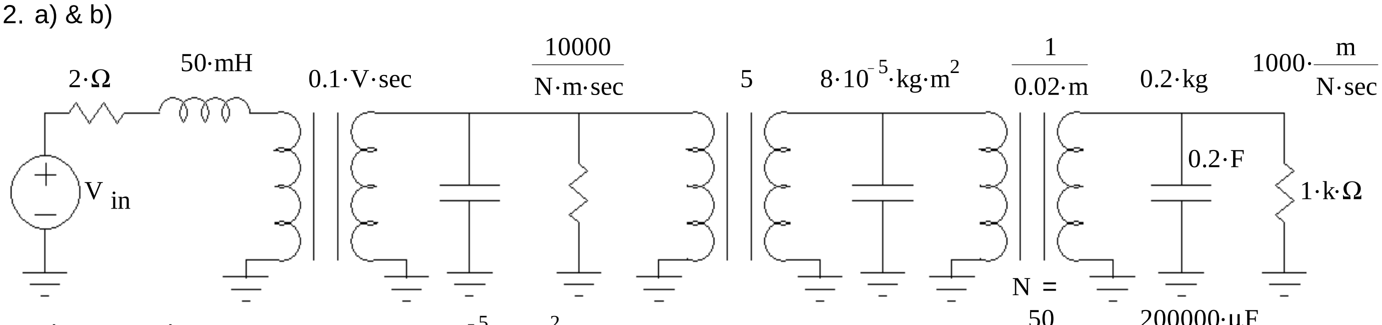
If your answer is yes, then provide some sort of alias or password: ______________________________
Otherwise, leave blank

The grades will be posted on my door in alphabetical order under the alias that you provide here. I will not post grades under your real name. The Internet version will be a pdf file which you can download. Both will show the homework, lab, and exam scores of everyone who answers yes here.

Answers

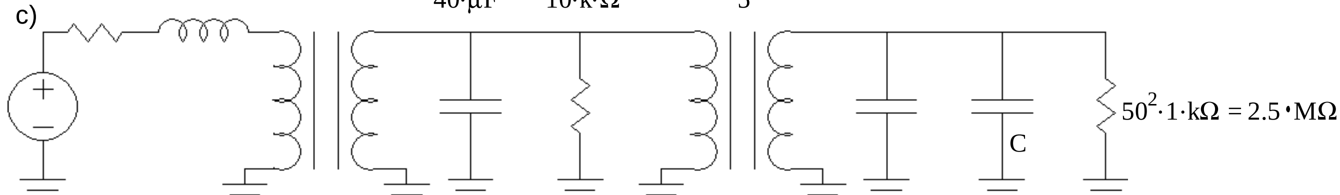
1. a) $\dfrac{A}{s+8} + \dfrac{B\cdot s}{\left(s^2+100\right)} + \dfrac{C\cdot 10}{\left(s^2+100\right)}$ b) -0.585 c) $-0.585\cdot e^{-8\cdot t}$ d) $125\cdot ms$

2. a) & b)



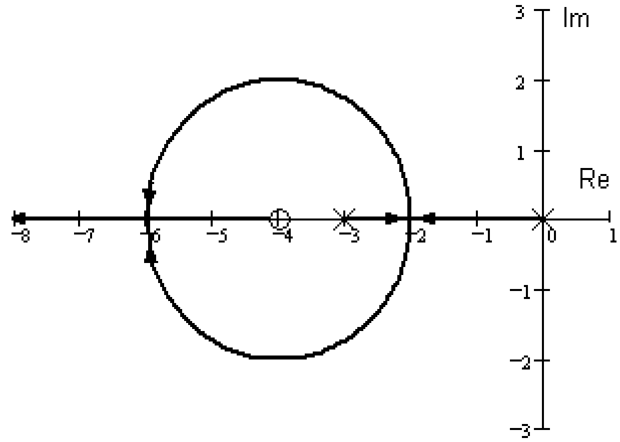
These numbers are for both drawings: N = 0.1 $4\cdot 10^{-5}\cdot kg\cdot m^2$ $40\cdot\mu F$ $10\cdot k\cdot\Omega$ N = 5 $80\cdot\mu F$

c)



$$C = \frac{200000\cdot\mu F}{50^2} = 80\cdot\mu F$$

3. a) at right



   b) $-2 \quad -6$

4. $3.73\cdot\text{deg}$

5. a) no, not quite $180.3\text{deg} \neq 180\cdot\text{deg}$

   b) $k > 0.364$

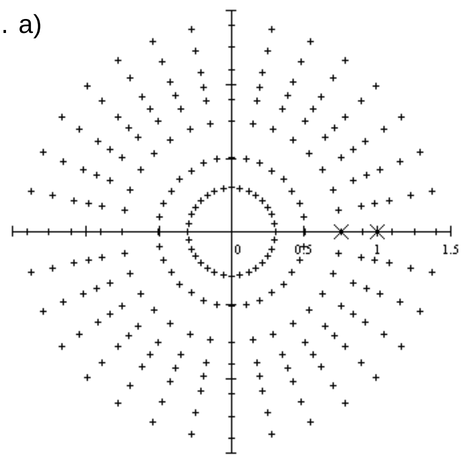
6. a) $x(k) = -\frac{3}{2}\cdot\delta(k) + \frac{1}{2}\cdot(4)^k + 1\cdot(-2)^k$ b) $\left(\sqrt{13}\right)^k\cdot\sin(0.983\cdot k)$

7. a) $64\cdot\frac{z}{z-0.75}$

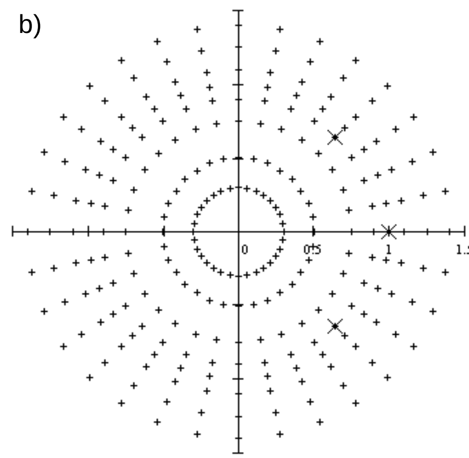
   b) $X(z) = \frac{1}{2}\cdot\frac{z}{(z-1)} + 1 - \frac{1}{2\cdot z^2}$

   c) $\dfrac{\frac{3}{2}\cdot z^3 - z^2 - \frac{1}{2}\cdot z + \frac{1}{2}}{z^3 - z^2}$

8. a)



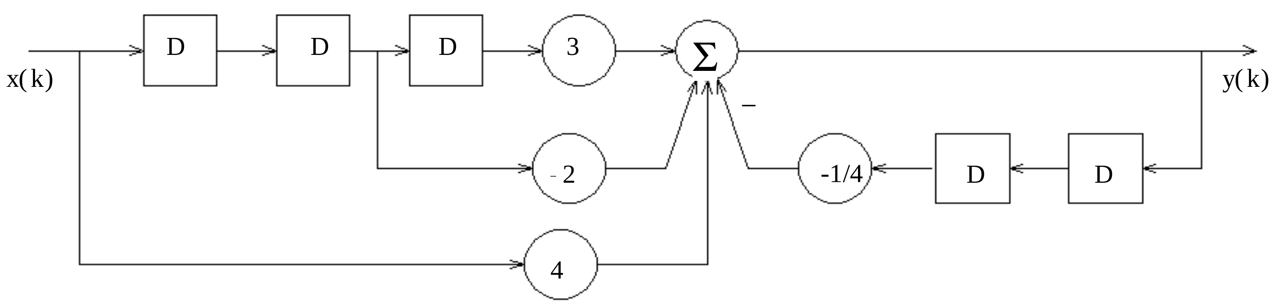
b)



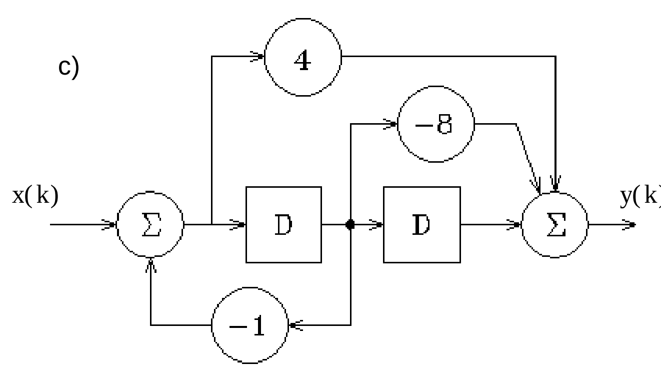
9. a) $H(z) = \frac{1}{4}\cdot\frac{z}{(z-1)^2}$

10. a) $\dfrac{4\cdot z^3 - 2\cdot z + 3}{z\cdot\left(z^2 - \frac{1}{4}\right)}$ b) $\pm\frac{1}{2}$ & 0

   c)



11. a) $\dfrac{4\cdot z^2 - 8\cdot z + 1}{z\cdot(z+1)}$ b) NO, pole is on the unit circle c)



12. a) $\dfrac{k_1\cdot k_p\cdot k_F\cdot s + k_1\cdot k_2}{s^4 + 2\cdot a\cdot s^3 + a^2\cdot s^2 + k_1\cdot k_p\cdot s + k_1\cdot k_2}$

   b) zero at $-\dfrac{k_2}{k_p\cdot k_F}$