ECE3510 Final given: Spring 06

1. (10 pts) Find the inverse Laplace transform of the following function:

   Use partial fraction expansion and the tables.

   Show all your work to get credit.

$$F(s) := \frac{1}{s} \cdot \frac{s+2}{s+3}$$

2. (12 pts) A system has this transfer function: $H(s) = \dfrac{6}{s+4}$

   a) What is the steady-state response of this system to the input: $x(t) = 8 \cdot u(t)$

   b) What is the steady-state response of this system to the input: $x(t) = 2 \cdot \sqrt{2} \cdot \sin(4 \cdot t) \cdot u(t)$

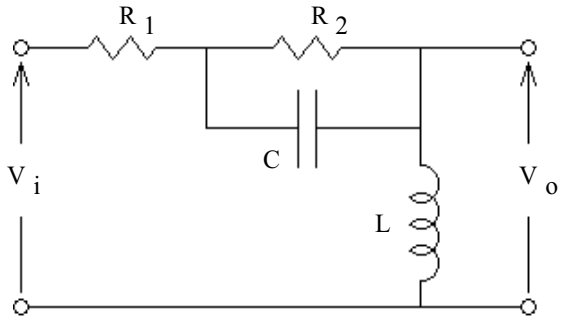
3. (14 pts) a) Find the Laplace transform of the following function:

   Combine any pieces you may find so that they have one common denominator.

$$f(t) = (3 - t) \cdot \sin(4 \cdot t) \cdot u(t)$$

   b) List the poles of the Laplace transform. Indicate multiple poles if there are any.

4. (15 pts) Find the transfer function $H(s) = \dfrac{V_o(s)}{V_i(s)}$ for this circuit.



   Write $H(s)$ in the normal form, as a ratio of polynomials.
   Don't divide the polynomials. Initial conditions are 0.

   You MUST show work to get credit.

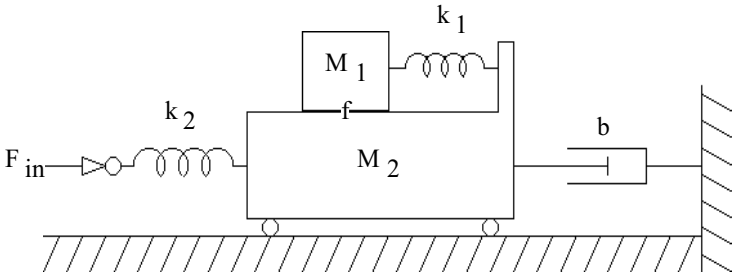
$$H(s) = ?$$

   b) Find the characteristic equation of the circuit shown.

   c) The solutions to the characteristic equation are called the _____ of the transfer function.

   d) Does the transfer function have one or more zeros? If yes, express it (them) in terms of R_1, R_2, C, & L.

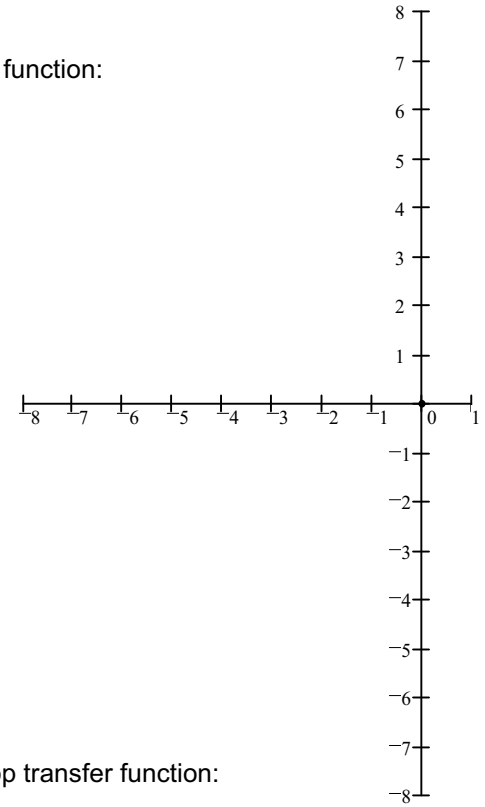
5. (8 pts) Find the equivalent electric circuit for the mechanical system shown. F_{in} is an input force.



6. (14 pts) Sketch the root-locus plots for the following open-loop transfer function:
   Use only the main rules, that is, don't sweat the details like
   breakaway points and departure angles.

   Be sure to show your work for other calculations.

$$G(s) = \frac{3\cdot(s+2)}{s\cdot(s+5)\cdot\left(s^2+6\cdot s+25\right)}$$

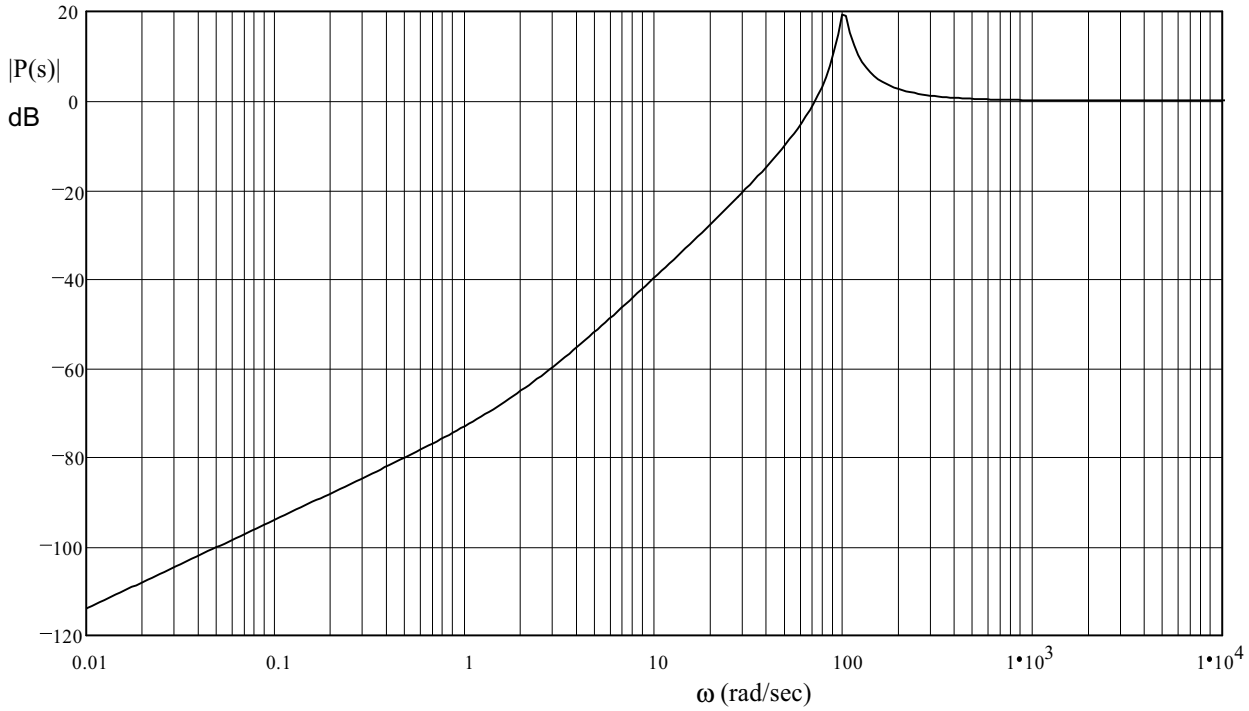


7. (9 pts) Is the point, $s = 3j$ on the root locus plot of the following open-loop transfer function:

$$G(s) = \frac{1}{s\cdot(s+2)\cdot(s+4)}$$

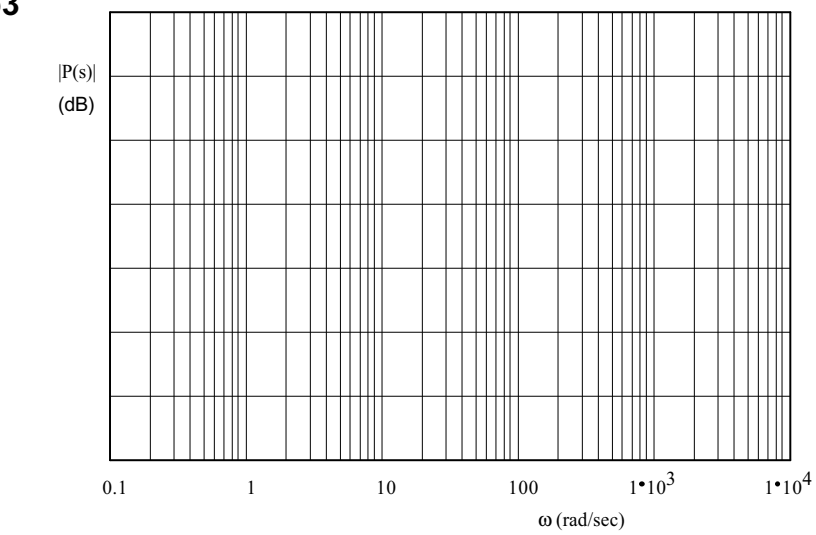
   Be sure to show the work and method you used to decide.

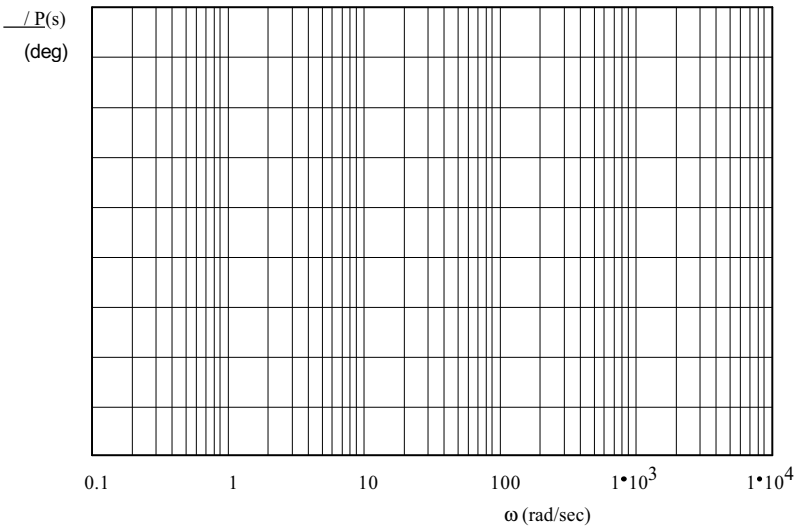
8. (10 pts) Given the magnitude Bode plot of a system, estimate of the transfer function of the system.
   Assume there are no negative signs in the transfer function (all poles and zeros are in the left-half plane).
   Show your work (how you made your estimate).



9. (14pts) Sketch the Bode plot for the following transfer function. Make sure to label the graphs, and to give the slopes of the lines in the magnitude plot. Also draw the "smooth" lines.

$$P(s) = \frac{s \cdot 10000}{(s+2)\cdot\left(s^2 + 40\cdot s + 10000\right)}$$





10. (4 pts) Find $x(0)$ if the z-transform of $x(k)$ is

$$X(z) = \frac{4\cdot\left(z^2+1\right)}{(z+1)\cdot(2\cdot z+3)}$$

11. (19 pts) Find the $x(k)$ whose z-transform is

a) Use partial fraction expansion

$$X(z) = \frac{1}{(z-1)\cdot\left(z+\frac{1}{2}\right)}$$

b) $$X(z) = \frac{3\cdot j\cdot z}{(z-(2+j))} + \frac{-3\cdot j\cdot z}{(z-(2-j))}$$

12. (16 pts) $$y(k) = 3\cdot x(k) - 2\cdot x(k-1) + x(k-4) + \frac{1}{2}\cdot y(k-2)$$

a) Find the $H(z)$ corresponding to the difference equation.

b) List the poles of $H(z)$. Indicate multiple poles if there are any.

c) Draw the block diagram of a simple direct implementation of the difference equation.

13. (20 pts) A signal $x(t)$ with transform $X(s) = \dfrac{1}{s\cdot(s+1)^2} = \dfrac{1}{s} - \dfrac{1}{(s+1)} - \dfrac{1}{(s+1)^2}$

(partial fraction expansion)

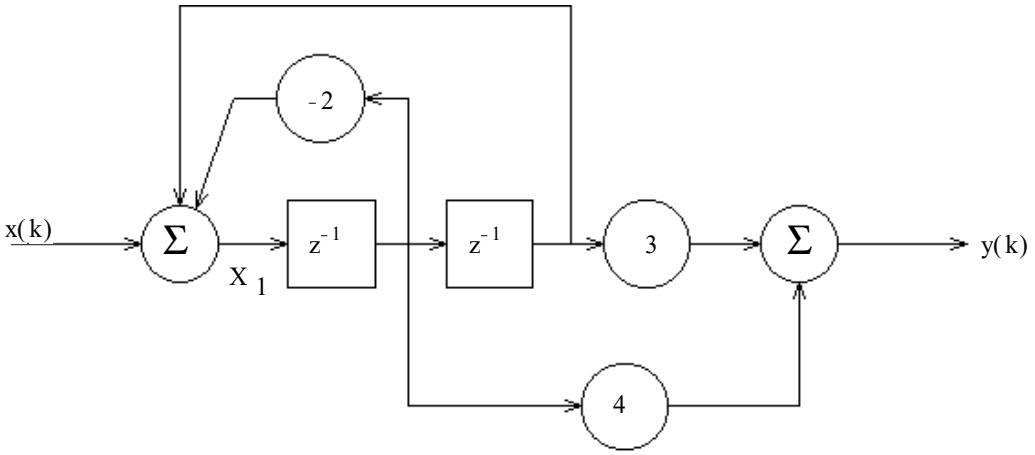
is sampled at time instants $t = kT$ to obtain $x_d(k)$. Find the transform $X_d(z)$ of the resultant signal.

a) Express $X_d(z)$ as one numerator over one denominator. You do not have to simplify beyond that.

Hint: it may help to remember that: $p^k = p\cdot p^{k-1}$

b) List the poles of $H(z)$. Indicate multiple poles if there are any.

14. (15 pts) Find the transfer function $H(z) = Y(z)/X(z)$.



15. Do you want your grade and scores posted on my door and on the internet? Yes No (Circle one)

If your answer is yes, then provide some sort of alias or password: _____

The grades will be posted on my door in alphabetical order under the alias that you provide here. I will not post grades under your real name. The internet version will be a pdf file which you can download. Both will show the homework, lab, and exam scores of everyone who answers yes here.

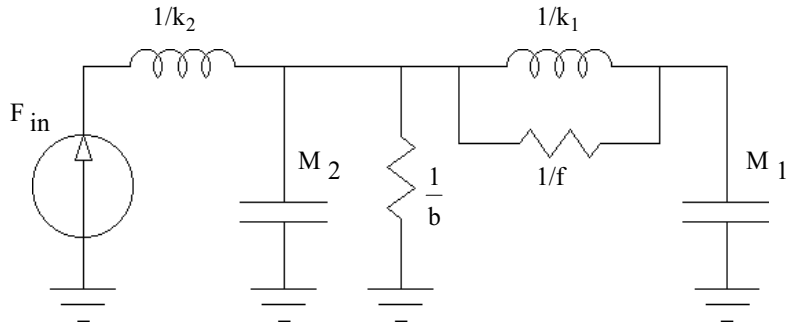
Answers

1. $\left(\frac{2}{3} + \frac{1}{3}\cdot e^{-3\cdot t}\right)\cdot u(t)$ 2. a) 12 b) $3\cdot\sin\left(4\cdot t - \frac{\pi}{4}\right)$ 3. a) $\dfrac{12\cdot s^2 - 8\cdot s + 12\cdot 16}{\left(s^2+16\right)^2}$ b) $4\cdot j$ $4\cdot j$ $-4\cdot j$ $-4\cdot j$

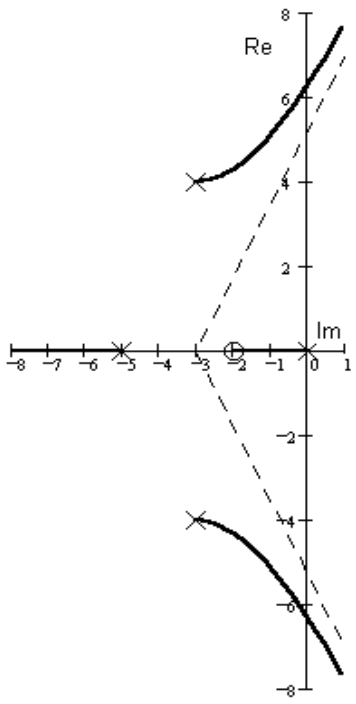
4. a) $\dfrac{s^2 + \frac{1}{C\cdot R_2}\cdot s}{s^2 + \left(\frac{R_1}{L} + \frac{1}{R_2\cdot C}\right)\cdot s + \left(1 + \frac{R_1}{R_2}\right)\cdot\frac{1}{L\cdot C}}$ b) $0 = s^2 + \left(\frac{R_1}{L} + \frac{1}{R_2\cdot C}\right)\cdot s + \left(1 + \frac{R_1}{R_2}\right)\cdot\frac{1}{L\cdot C}$

c) poles d) $0,\ -\dfrac{1}{C\cdot R_2}$

5.



6.



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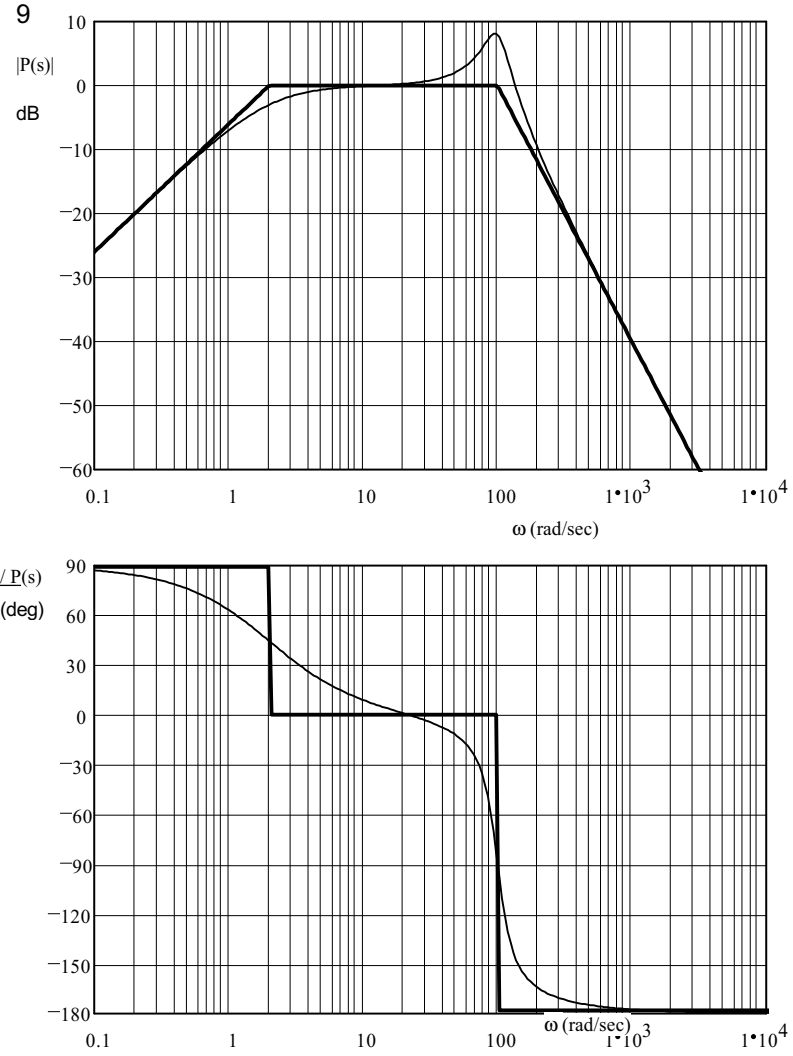


7. $\arg(G(3\cdot j)) = -183.2\cdot\text{deg}$

which is NOT equal to $\pm 180^{o}$

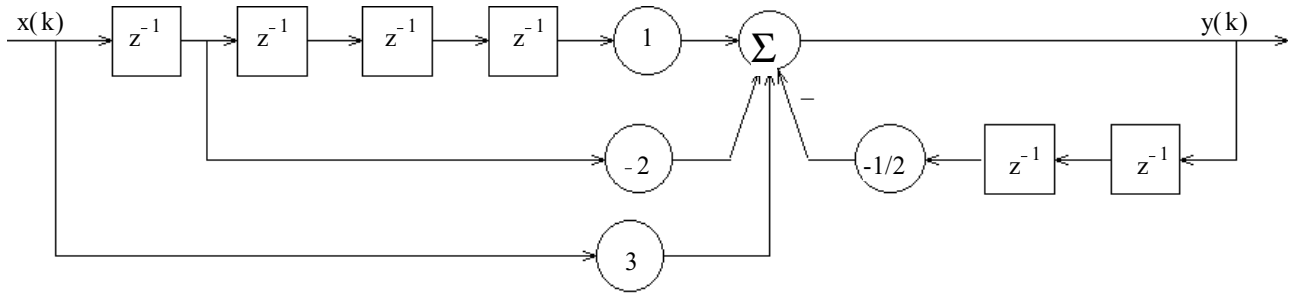
NO, the point, $s = 3j$ is NOT on the root locus plot.

8. $\dfrac{s\cdot(s+2)}{\left(s^2+10\cdot s+10000\right)}$

10. 2 11. a) $-2\cdot\delta(k)+\dfrac{2}{3}+\dfrac{4}{3}\cdot\left(-\dfrac{1}{2}\right)^{k}$ b) $6\cdot(5)^{\frac{k}{2}}\cdot\cos\left(0.464\cdot k+\dfrac{\pi}{2}\right)$

12. a) $\dfrac{3\cdot z^4-2\cdot z^3+1}{z^4-\dfrac{1}{2}\cdot z^2}$ b) $\pm\dfrac{1}{\sqrt{2}}$ double poles at 0

c)



13. a) $\dfrac{z\cdot\left(z-e^{-T}\right)^2-z\cdot(z-1)\cdot\left(z-e^{-T}\right)-T\cdot e^{-T}\cdot z\cdot(z-1)}{(z-1)\cdot\left(z-e^{-T}\right)^2}$ b) 1 e^{-T} 14. $\dfrac{4\cdot z+3}{z^2+2\cdot z-1}$