## ECE 3510 Final Exam Information Bode Plots

See also Exam 3 Information sheet
b) At complex poles and zeroes.
if complex pole is expressed:

$$
\begin{aligned}
& \left(\mathrm{s}^{2}+2 \cdot \zeta \cdot \omega_{\mathrm{n}} \cdot \mathrm{~s}+\omega_{\mathrm{n}}^{2}\right) \\
& \text { natural frequency } \omega_{\mathrm{n}}=\sqrt{\omega_{\mathrm{n}}^{2}} \\
& \text { damping factor: } \quad \zeta=\frac{2 \cdot \zeta \cdot \omega_{\mathrm{n}}}{2 \cdot \omega_{\mathrm{n}}}
\end{aligned}
$$

if complex pole is expressed:
$\left[(s+a)^{2}+b^{2}\right]=s^{2}+2 \cdot a \cdot s+\left(a^{2}+b^{2}\right)$
natural
frequency $\omega_{n}=\sqrt{a^{2}+b^{2}}$
$\begin{aligned} & \text { damping } \\ & \text { factor }\end{aligned} \zeta=\frac{\mathrm{a}}{\omega_{\mathrm{n}}}$
Near $\omega_{n}$, the difference between peak \& straight straight is: $\quad \frac{1}{2 \cdot \zeta} \quad$ in $\left.\mathrm{dB}: \quad 20 \cdot \log \left(\frac{1}{2 \cdot \zeta}\right)\right) ~(1) ~$
For angles, see drawing at right.

## GM, PM, \& DM Gain Margin (GM):

Find where angle plot crosses $180^{\circ}$ GM is -dB of mag plot at same freq.

This sheet and the Information sheets from Exams 1-3 are the only reference materials allowed at exam. Bring this page. You may add whatever you want to this sheet (both sides).



Phase Margin (PM): Find where mag plot crosses 0 dB . PM is $180^{\circ}+$ phase angle at same freq.
Delay Margin (DM): $\quad \mathrm{T}=\frac{2 \cdot \pi}{\omega_{\mathrm{PM}}} \quad$ OR $\frac{1}{\mathrm{f}_{\mathrm{PM}}} \quad \mathrm{DM}=\left(\frac{\mathrm{PM}}{360 \cdot \mathrm{deg}}\right) \cdot \mathrm{T}$

## Bode Plot to Transfer Function

Draw best-fit straight lines at slopes of $0 \mathrm{~dB} / \mathrm{dec}, \pm 20 \mathrm{~dB} / \mathrm{dec}, \pm 40 \mathrm{~dB} / \mathrm{dec}, \pm 60 \mathrm{~dB} / \mathrm{dec}$, etc..
Essentially reverse the plot procedure.
Find the magnitude at some part of the plot (usually a flat part) \& find any multiplying constant needed.

Finite-length signals have all poles at zero

Inverse z-transforms (partial fractions \& long division)

$$
\text { Divide by } z \text { first: } \frac{F(z)}{z}
$$



Poles on real Poles on real
axis (not at zero):
$f(k) \quad F(z)=\sum_{k=0}^{\infty} f(k) \cdot z^{-k}$

$$
(|\mathrm{p}|)^{\mathrm{k}} \cdot \cos \left(\theta_{\mathrm{p}} \cdot \mathrm{k}\right)
$$

$\underline{f(k)}$
$\delta(\mathrm{k})$
$\mathrm{u}(\mathrm{k})$
$p^{k}$
$(|\mathrm{p}|)^{\mathrm{k}} \cdot \cos \left(\theta_{\mathrm{p}} \cdot \mathrm{k}\right) \quad \frac{\mathrm{z} \cdot\left(\mathrm{z}-|\mathrm{p}| \cdot \cos \left(\theta_{\mathrm{p}}\right)\right)}{\mathrm{z}^{2}-2 \cdot|\mathrm{p}| \cdot \cos \left(\theta_{\mathrm{p}}\right) \cdot \mathrm{z}+(|\mathrm{p}|)^{2}}$

$$
(|\mathrm{p}|)^{\mathrm{k}} \cdot \sin \left(\theta_{\mathrm{p}} \cdot \mathrm{k}\right)
$$

$\underline{\mathbf{F}(\mathbf{z}) \quad \underline{f(k)}}$
$\mathrm{A} \cdot \delta(\mathrm{k})$
$B \cdot p^{k}$
$k \cdot p^{k}$
A

Properties of the z-transform
linear $\quad$ Right-shift = delay = multiply
Left-shift = advance = multiply by z
Initial value $=\mathrm{f}(0)=\mathrm{F}(\infty) \quad$ Final value $(\mathrm{DC})=\mathrm{f}(\infty)=(\mathrm{z}-1) \cdot \mathrm{F}(\mathrm{z})$
| $\mathrm{z}:=1$
Signals are bounded if all poles in inside unit circle, no double poles on unit circle
Converge to 0 if all poles inside unit circle. Converge to a non-zero value if a single pole is at 1
Difference equations, be able to get $\mathrm{H}(\mathrm{z})$
Discrete-time systems, FIR (all poles at zero), IIR (some poles not at zero)
BIBO Stability, all poles inside unit circle.
Step \& Sinusoidal responses, effects of poles \& zeros, etc.

$$
\text { DC gain }=\mathrm{H}(1) \quad \text { sinusoidal: } \quad \mathrm{H}\left(\mathrm{e}^{\mathrm{j} \cdot \Omega} \mathrm{o}\right) \quad=|\mathrm{H}| \underline{\theta}_{\mathrm{H}} \quad \text { multiply magnitudes and add angles }
$$

Same Feedback system as in continuous-time and Root locus works the same but is interpreted very differently.

