ECE 3510 Final Exam Information

This sheet and the Information sheets from Exams 1 - 3 are the only reference materials allowed at exam. Bring this page. You **may add** whatever you want to this sheet (both sides).

Bode Plots

See also Exam 3 Information sheet

b) At complex poles and zeroes.

if complex pole is expressed:

$$\left(s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^{\,2}\right)$$

natural frequency $\omega_n = \sqrt{\omega_n^{\,2}}$

damping factor: $\zeta = \dfrac{2\cdot\zeta\cdot\omega_n}{2\cdot\omega_n}$

if complex pole is expressed:

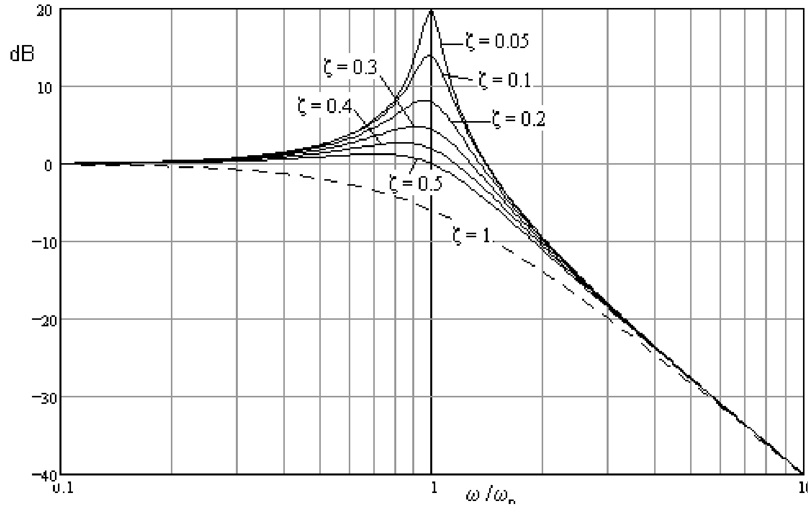
$$\left[(s+a)^2 + b^2\right] = s^2 + 2\cdot a\cdot s + \left(a^2 + b^2\right)$$

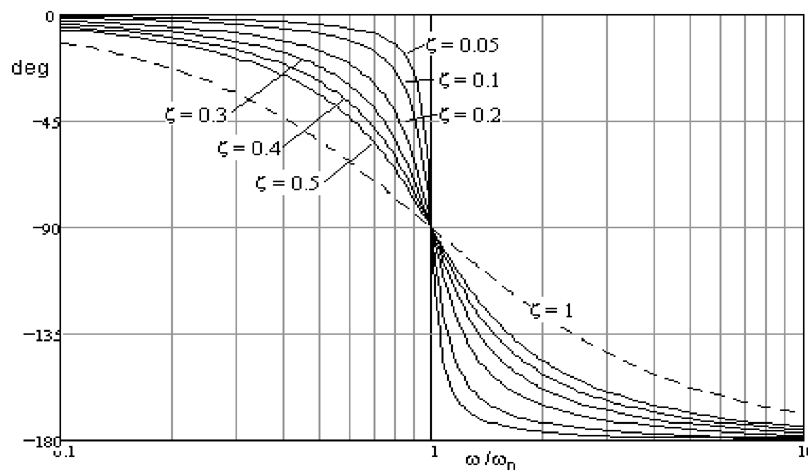
natural frequency $\omega_n = \sqrt{a^2 + b^2}$

damping factor $\zeta = \dfrac{a}{\omega_n}$

Near ω_n, the difference between peak & straight lines is: $\dfrac{1}{2\cdot\zeta}$ in dB: $20\cdot\log\left(\dfrac{1}{2\cdot\zeta}\right)$

For angles, see drawing at right.





GM, PM, & DM Gain Margin (GM):
Find where angle plot crosses 180^o.
GM is -dB of mag plot at same freq.

Phase Margin (PM): Find where mag plot crosses 0dB. PM is 180^o + phase angle at same freq.

Delay Margin (DM): $T = \dfrac{2\cdot\pi}{\omega_{PM}}$ OR $\dfrac{1}{f_{PM}}$ $\quad DM = \left(\dfrac{PM}{360\cdot deg}\right)\cdot T$

Bode Plot to Transfer Function

Draw best-fit straight lines at slopes of 0dB/dec, $\pm$20dB/dec, $\pm$40dB/dec, $\pm$60dB/dec, etc..

Essentially reverse the plot procedure.

Find the magnitude at some part of the plot (usually a flat part) & find any multiplying constant needed.

Nyquist plots
Polar Frequency Response plots

Start at G(0), the DC gain, a point on the real axis, + or -, may be at ∞ if pole(s) at origin.

If pole(s) at origin, arc at ∞ for -90^0 per pole.

Plot ends at $G(\infty)$, usually 0, approaches at final angle, given by: $(n - m)\cdot(-90\cdot deg)$

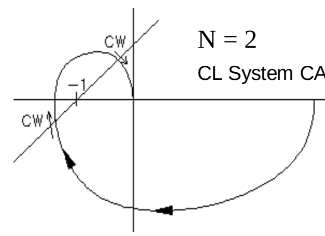
Plot the rest of the frequency response of $G(s)$. It may help to start with Bode plots.

Gain, k, makes entire plot grow in all directions (or shrink if $k<1$).

$Z = N + P$ $\quad P$ = OL poles in RHP (0 if open-loop stable)

N = CW encirclements of -1, CCW encirclements are counted as negative and may make up for P. Example:

Z = CL poles in RHP (must be zero (or $\leq$ 0) if closed-loop stable)



N = 2
CL System CANNOT be stable

To find the Phase Margin (PM):

1. Find where the Nyquist plot crosses the unit circle. These crossings separate the unit circle into regions.
2. Decide which of these regions have unacceptable CW encirclements.
3. Determine what phase change would cause the -1 point to be an unacceptable region, usually $180^o - \underline{\angle \text{crossing}}$

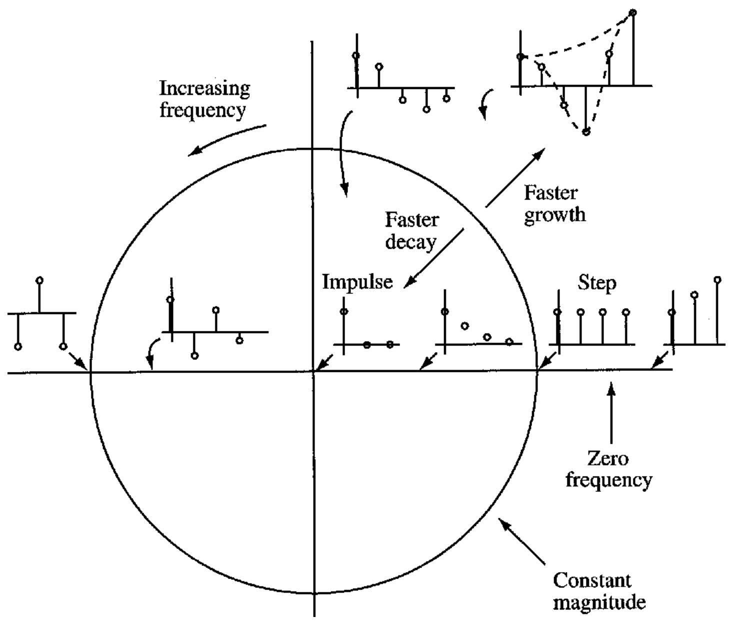
To find the Gain Margin (GM):

1. Find where the Nyquist plot crosses the negative real axis. These crossings separate the negative real axis into regions.
2. Decide which of these regions have unacceptable CW encirclements.
3. Determine what gain would cause the -1 point to be an unacceptable region, usually $\frac{1}{-\text{crossing}}$ into the unacceptable region.
4. Usually there is just one upper limit of gain-- in that case report that as the Gain Margin.
5. If there is a lower limit of gain, report the Gain Margin as: $GM = \left[\text{Lower limit , upper limit} \right]$

Discrete Signals, Systems and z-transforms

$f(k) \qquad F(z) = \sum_{k=0}^{\infty} f(k)\cdot z^{-k}$

Finite-length signals have all poles at zero



f(k)	**F(z)**
$\delta(k)$	1
$u(k)$	$\frac{z}{z-1}$
p^k	$\frac{z}{z-p}$
$(\lvert p\rvert)^k\cdot\cos(\theta_p\cdot k)$	$\frac{z\cdot(z-\lvert p\rvert\cdot\cos(\theta_p))}{z^2-2\cdot\lvert p\rvert\cdot\cos(\theta_p)\cdot z+(\lvert p\rvert)^2}$
$(\lvert p\rvert)^k\cdot\sin(\theta_p\cdot k)$	$\frac{z\cdot(\lvert p\rvert\cdot\sin(\theta_p))}{z^2-2\cdot\lvert p\rvert\cdot\cos(\theta_p)\cdot z+(\lvert p\rvert)^2}$

Inverse z-transforms (partial fractions & long division)

Divide by z first: $\frac{F(z)}{z}$

	F(z)	**f(k)**
	A	$A\cdot\delta(k)$
Poles on real axis (not at zero):	$\frac{B\cdot z}{(z-p)}$	$B\cdot p^k$
	$\frac{B\cdot p\cdot z}{(z-p)^2}$	$k\cdot p^k$
Complex poles:	$\frac{B\cdot z}{(z-p)}+\frac{\overline{B}\cdot z}{(z-\overline{p})}$	$2\cdot\lvert B\rvert\cdot(\lvert p\rvert)^k\cdot\cos(\theta_p\cdot k+\theta_B)$

Properties of the z-transform
- linear
- Right-shift = delay = multiply by $z^{-1} = \frac{1}{z}$
- Left-shift = advance = multiply by z
- Initial value = $f(0) = F(\infty)$ Final value (DC) = $f(\infty) = (z-1)\cdot F(z) \Big|_{z:=1}$

Signals are bounded if all poles in inside unit circle, no double poles on unit circle
Converge to 0 if all poles inside unit circle. Converge to a non-zero value if a single pole is at 1

Difference equations, be able to get $H(z)$

Discrete-time systems, FIR (all poles at zero), IIR (some poles not at zero)

BIBO Stability, all poles inside unit circle.

Step & Sinusoidal responses, effects of poles & zeros, etc.

DC gain = $H(1)$ sinusoidal: $H(e^{j\cdot\Omega_o}) = \lvert H\rvert \underline{\angle\theta_H}$ multiply magnitudes and add angles

Same Feedback system as in continuous-time and Root locus works the same but is interpreted very differently.