ECE 3510 Exam 3 Information

This sheet and the Information sheets from Exams 1 & 2 are the only reference materials allowed at exam. Bring this page.

You **may add** whatever you want to this sheet (both sides).

Root-Locus Plots Additional Rules

The **breakaway/in points** are also solutions to: $\displaystyle\sum_{\text{all}} \frac{1}{(s + -p_i)} = \sum_{\text{all}} \frac{1}{(s + -z_i)}$

Phase angle of $G(s)$ at any point on the root locus: $\arg(G(s)) = \arg(N(s)) - \arg(D(s)) = \pm 180^o, \pm 540^o, \ldots$

Or: $\arg\left(\dfrac{1}{G(s)}\right) = \arg(D(s)) - \arg(N(s)) = \pm 180^o, \pm 540^o, \ldots$

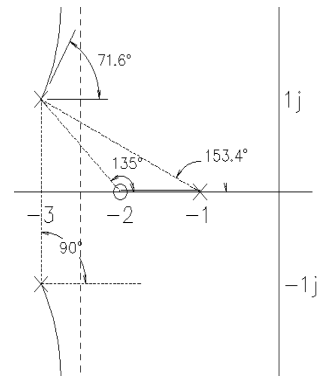
Note: arg(x) is $\underline{\angle(x)}$

Departure angle (θ_D) from a complex pole (p_c). Recall rule 9 (one of the most important rules):

for any point s on the root locus: $\arg(G(s)) = \arg(N(s)) - \arg(D(s)) = \pm 180^o \quad \pm 360^o \ldots$

Imagine a point ε-distance away from the complex pole. That point would have an angle of θ_D with respect to the complex pole, but it's angle relative to all the other poles and zeros would be essentially the same as the complex pole.

$$\sum_{\text{all zeroes}} (\text{angle of point } s \text{ relative to zero}) - \sum_{\text{all poles but } p_c} (\text{angle of point } s \text{ relative to pole}) - \theta_D = \pm 180^o \quad \pm 540^o \ldots$$



Example: $G(s) := \dfrac{s+2}{(s+1)\cdot\left[(s+3)^2 + 1^2\right]}$ Find the departure angle from the pole at: $p_c := -3 + 1\cdot j$

$135 - 153.4 - 90 - \theta_D = \pm 180^o \quad \pm 540^o \ldots$

rearrange: $\theta_D = 180 - 90 - 153.4 + 135 = 71.6 \text{ deg}$

Arrival angle (θ_A) to complex zero (z_c). Exactly the same idea.

To create an unconventional root locus plot:

1. Determine the gain factor if it can be adjusted, and make it part of the open-loop transfer function, $G(s)$. Hold it constant at some number.
2. Determine the denominator of the closed-loop transfer function, $H(s)$. Let's call it $D_H(s)$.
3. Rearrange $D_H(s)$ into this form: $D'(s) + x\cdot N'(s)$ where x is the variable for which you want to draw the root locus. Notice that x occupies exactly the same position the gain would normally occupy. Normal: $D_G(s) + k\cdot N_G(s)$
   Note: If you cannot rearrange $D_H(s)$ into this form, then you cannot use this method to create an root locus plot for the variable x. Now: $D'(s) + x\cdot N'(s)$
4. Now simply draw a root locus as though $D'(s)$ was the open-loop denominator and $N'(s)$ was the open-loop numerator.

Root Locus Design

PI To eliminate steady-state error (for constant inputs) & perfect rejection of constant disturbances
Add pole at 0 and zero at close by

LAG An alternative is a Lag Compensator, with a pole near the origin and a zero a little further away.

PD or PID To Improve the dynamic response, add a zero to affect angles.

LEAD An alternative to the differentiator is a Lead Compensator with a zero and a pole much further left.

Root Locus Design Crib Sheet

Using 2nd-order approximation: $\dfrac{N(s)}{(s+a)^2+b^2} = \dfrac{N(s)}{s^2+2\cdot a\cdot s+a^2+b^2} = \dfrac{N(s)}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^2}$

$\omega_n^2 = a^2+b^2$ ω_n = natural frequency $\zeta\cdot\omega_n = a$

$\zeta = \dfrac{a}{\omega_n} = \dfrac{a}{\sqrt{a^2+b^2}}$ = damping factor $\zeta = \sin\left(\operatorname{atan}\left(\dfrac{a}{b}\right)\right)$

Overshoot: $OS = e^{-\pi\cdot\frac{a}{b}}$ $\%OS = 100\%\cdot e^{-\pi\cdot\frac{a}{b}}$ $\dfrac{a}{b} = \dfrac{\ln(OS)}{-\pi}$

2% settling time: $T_s = \dfrac{4}{a} = \dfrac{4}{\zeta\cdot\omega_n}$ $\%OS = 100\%\cdot e^{\frac{-\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}$ $\zeta = \dfrac{-\ln(OS)}{\sqrt{\pi^2+(\ln(OS))^2}}$

Weird forms from Nise book

Static error constant (position): $K_p = \lim_{s\to 0} K\cdot C(s)\cdot G(s)$ $e_{step}(\infty) = e_{step} = \dfrac{1}{1+K_p}$

Lag compensation improves K_p, K_v and K_a by $\dfrac{z_c}{p_c}$ IE: $K_{pc} \simeq K_{puc}\cdot\dfrac{z_c}{p_c}$

angle of constant damping line: $90\cdot deg + \operatorname{atan}\left(\dfrac{a}{b}\right)$

Bode Plots

Sample transfer function: $P(s) = K\cdot\dfrac{(s+z_1)\cdot(s+z_2)}{s\cdot(s+p_1)\cdot(s+p_2)^2}$

Replace all s's with blanks: $P(\_) = K\cdot\dfrac{(\_+z_1)\cdot(\_+z_2)}{\_\cdot(\_+p_1)\cdot(\_+p_2)\cdot(\_+p_2)}$

Initial magnitude $= K\cdot\dfrac{z_1\cdot z_2}{(j\cdot\omega_{start})\cdot p_1\cdot(p_2)\cdot(p_2)}$

ω's in num --> + 20dB/dec each , j => 90º, - => +180º

ω's in den --> - 20dB/dec each , j => -90º, - => -180º

Cross out each pole or zero in turn & replace with jω $K\cdot\dfrac{(\_+z_1)\cdot(\_+z_2)}{(j\omega)\cdot(j\omega+\cancel{X}_1)\cdot(\_+p_2)\cdot(\_+p_2)}$

zeros turn up the slope --> + 20dB/decade
poles turn down the slope --> - 20dB/decade

zeros increase the phase angle --> + 90deg
poles decrease the phase angle --> - 90deg

Draw a smooth line through the bode plots to estimate the actual magnitude and phase.

Actual magnitude: -3dB at single poles, +3dB at single zeros; -6dB at double poles, +6dB at double zeros; etc.. Magnitude effects extend about 1 decade fore and aft.

Angle effects extend about 1.5 decade fore and aft. It is helpful to draw a line from 1 decade before to 1 decade after.