ECE 3510 Exam 2 given: Spring 17 (Some of the space between problems has been removed.)

This part of the exam is Closed book, Closed notes, No Calculator.

1. (12 pts) Several transfer functions are shown below. Without doing anything more than looking at the transfer function, try to determine if it is BIBO stable. Answer Y, N, or C for each.

   Y) definitely BIBO table
   N) definitely NOT BIBO table
   C) can't tell just by looking

   a) $\dfrac{s\cdot(s+6)}{s^6+4\cdot s^5+2\cdot s^3+6\cdot s^2+2\cdot s+1}$ ____

   b) $\dfrac{s+1}{s^5+3\cdot s^4+18\cdot s^3+3\cdot s^2+s}$ ____

   c) $\dfrac{s-4}{s^4+2\cdot s^3+1\cdot s^2+s+0.25}$ ____

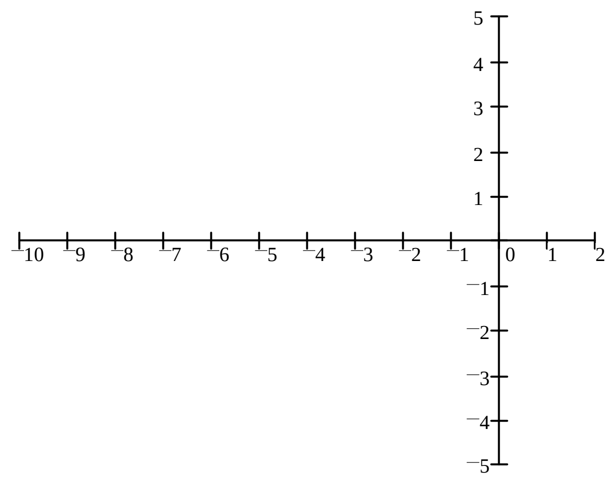
   d) $\dfrac{s^4+6\cdot s^2+2s}{\left(s^2+2\cdot s+5\right)\cdot\left(s^2+2\cdot s+2\right)}$ ____

   e) $\dfrac{10\cdot s+1}{(s+2)\cdot\left(s^2+2\cdot s+4\right)}$ ____

   f) $\dfrac{s+2}{5\cdot s^5+13\cdot s^4+8\cdot s^3+s^2+s+2}$ ____

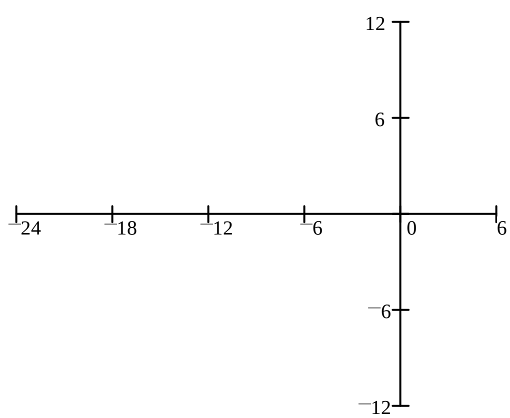
2. (12 pts) a) Sketch the root-locus plot for the open-loop transfer function below
   Sketch means: You may estimate details like breakaway points and departure angles from complex poles.
   Show your work where needed (like calculation of the centroid).

   $$\mathbf{G}(s) = \frac{s^2+2\cdot s+5}{(s-1)\cdot(s+6)\cdot\left(s^2+10\cdot s+29\right)}$$



   b) Find the range of gain (k) for which the system is closed-loop stable. Assume $k > 0$. The answer may be left as a fraction.

3. (17 pts) a) Sketch the root locus for: $\mathbf{G}(s) = \dfrac{s+6}{s^2-8\cdot s+52}$



   b) Does the root locus cross the $j\omega$ axis at $10j$?
   Be sure to show the work and method you used to decide.

   c) Regardless of what you found in part a, continue to assume that the root-locus crosses the $j\omega$ axis at 10.
   Give the range of gain k for which the system is closed-loop stable.

4. (25 pts) This system: $\mathbf{H}(s) = \dfrac{s+10}{s+4}$ Has this input: $x(t) = 3\cdot\sin(8\cdot t)\cdot u(t)$

a) Express the output, and separate into 3 partial fractions that you can find in the laplace transform table without using complex numbers. Show what they are, but don't find the coefficients.

$\mathbf{Y}(s) =$

b) Continue with the partial fraction expansion just far enough to find the **transient** coefficient as a number.

c) Use steady-state AC analysis to find the phasor representation of the steady-state output in polar form.

$\mathbf{Y}_{ss}(j\omega) = ?$

d) Express the complete (both transient and steady-state) output as a function of time. $y(t) = ?$
The steady-state part may be shown as a cosine with a phase angle.

e) What is the time constant of the transient part this expression? $\tau = ?$

5. (12 pts) When an electrical circuit is used as a representation if a mechanical system of translational motion, what do the following electrical quantities or parts represent in the mechanical system?

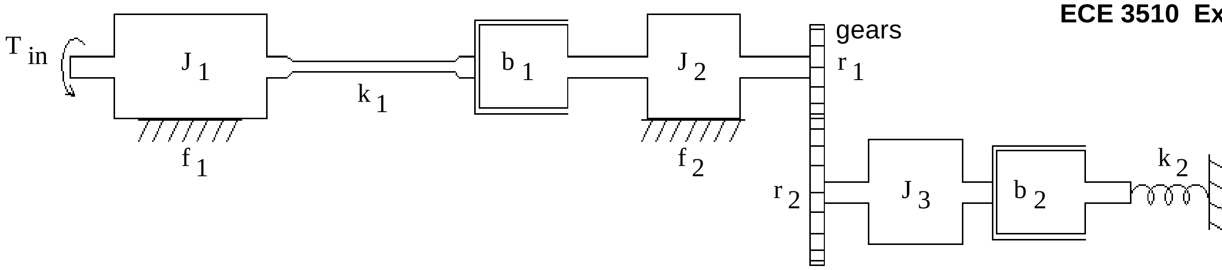
a) Current source = b) Branch current = c) Nodal voltage =

d) Ground = e) Resistor = f) Inductor =

g) Capacitor = h) Is the capacitor always hooked up in a certain way? If yes, say what.

i) Name two things represented by transformers. You may include items that rotate.

6. (22 pts) Find the equivalent electric circuit for the mechanical system shown. T_{in} is the input.



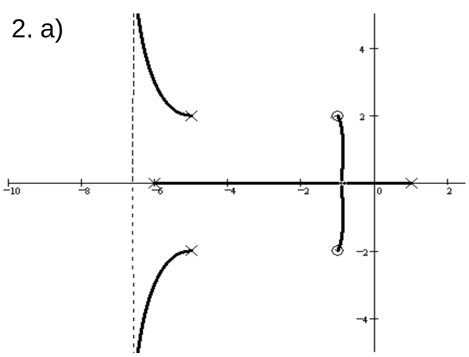
a) show the circuit with a transformer. Show the parts in terms of J's, k's, b's, etc., above.

b) Show the circuit without a transformer, just like you did in the homework. Show the parts in terms of J's, k's, b's, etc., above. Only last few parts need to be shown.

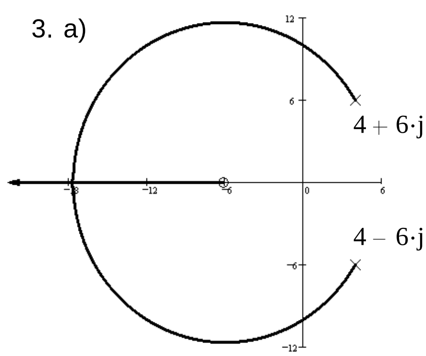
Answers

1. a) N b) N c) C d) Y e) Y f) C

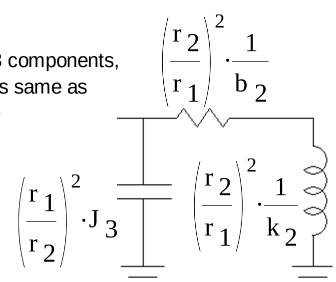
2. a)



3. a)



6. b) Last 3 components, Rest is same as above



$$\left(\frac{r_2}{r_1}\right)^2 \cdot \frac{1}{b_2} \qquad \left(\frac{r_1}{r_2}\right)^2 \cdot J_3 \qquad \left(\frac{r_2}{r_1}\right)^2 \cdot \frac{1}{k_2}$$

2. b) $k > 34.8$ 3. b) yes, by angles c) $k > 8$

4. a) $\dfrac{(s+10)}{(s+4)} \cdot \dfrac{3 \cdot 8}{(s^2+64)} = \dfrac{A}{s+4} + \dfrac{B \cdot s}{(s^2+64)} + \dfrac{C \cdot 8}{(s^2+64)}$ b) 1.8 c) $4.296 \cdot \cos(8 \cdot t - 114.8 \cdot deg)$

d) $\left(1.8 \cdot e^{-4 \cdot t} + 4.296 \cdot \cos(8 \cdot t - 114.8 \cdot deg)\right) \cdot u(t)$ e) 0.25

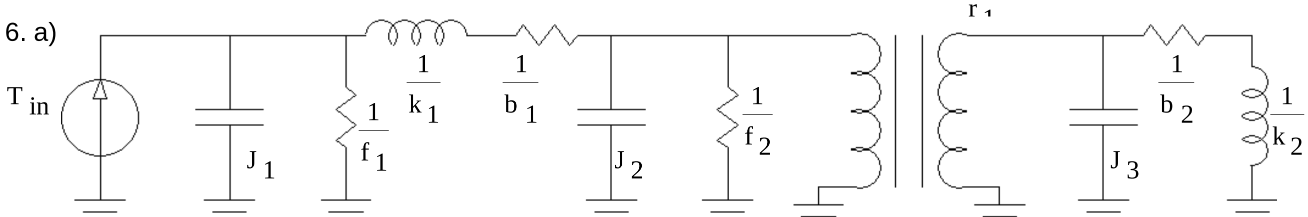
5. a) Force input b) Force c) Velocity d) Stationary reference of zero velocity e) Friction or damping

f) Spring g) Mass h) Yes, one side is always hooked to ground

i) Levers Wheels Belts Gears Electric motors
2 of these

$N = \dfrac{r_2}{r_1}$ 6. b) is shown above

6. a)



$$T_{in} \quad J_1 \quad \frac{1}{f_1} \quad \frac{1}{k_1} \quad \frac{1}{b_1} \quad J_2 \quad \frac{1}{f_2} \quad J_3 \quad \frac{1}{b_2} \quad \frac{1}{k_2}$$