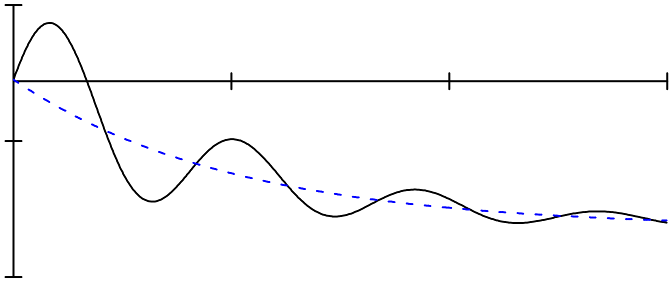
ECE 3510 Exam 1 given: Spring 20

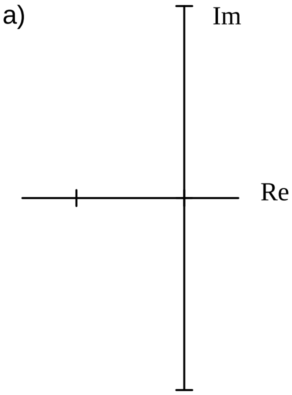
(Most of the space between problems has been removed.)

1. (12 pts) For each of the time-domain signals shown, draw the poles of the signal's Laplace transform on the axes provided. All time scales are the same. The axes below all have the same scaling. Your answers should make sense relative to one another. Clearly indicate double poles if there are any.

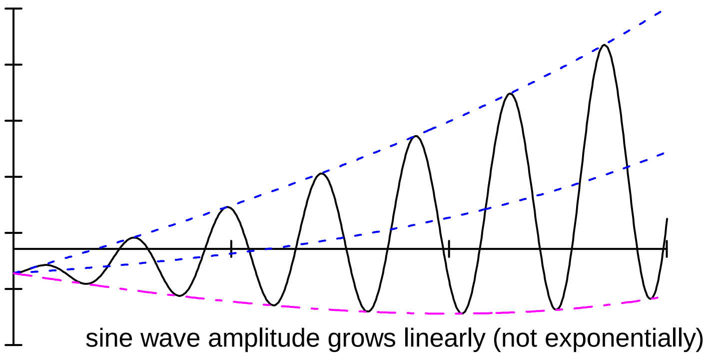
a)



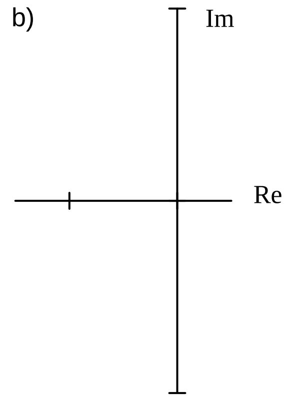
a)



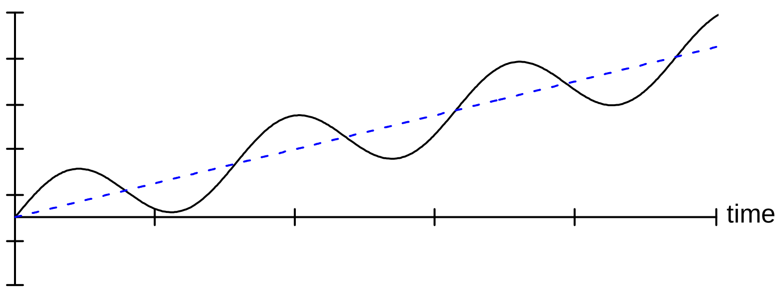
b)

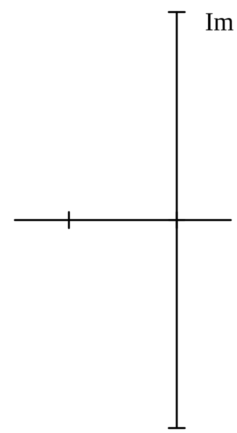


b)



2. (7 pts) a) The time-domain signal shown below is the **step response** of a **SYSTEM**, draw the poles and/or zeros of the **system's** transfer function on the axes provided.



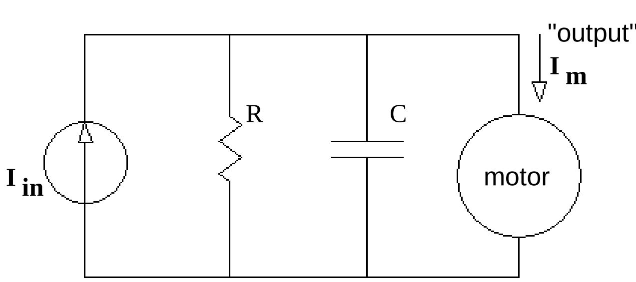


b) Is the system BIBO stable?

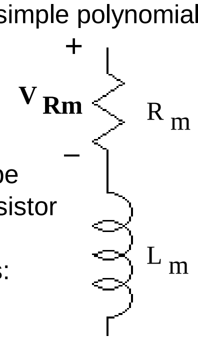
3. (19 pts) a) Find the transfer function of the circuit shown. Consider the motor current ($\mathbf{I_m}$) as the "output".

You <u>MUST</u> show work to get credit. Simplify your expression for **H(s)** so that the denominator is a simple polynomial.

$$\mathbf{H}(s) = \frac{\mathbf{I_m}(s)}{\mathbf{I_{in}}(s)} = ?$$



The motor may be modeled as a resistor in series with an inductor, like this:



b) Modify the transfer function found in part a). Now consider $\mathbf{V_{Rm}}$ as the "output".

$$\mathbf{H}_2(s) = \frac{\mathbf{V}_{\mathbf{Rm}}(s)}{\mathbf{I}_{\mathbf{in}}(s)} = ?$$

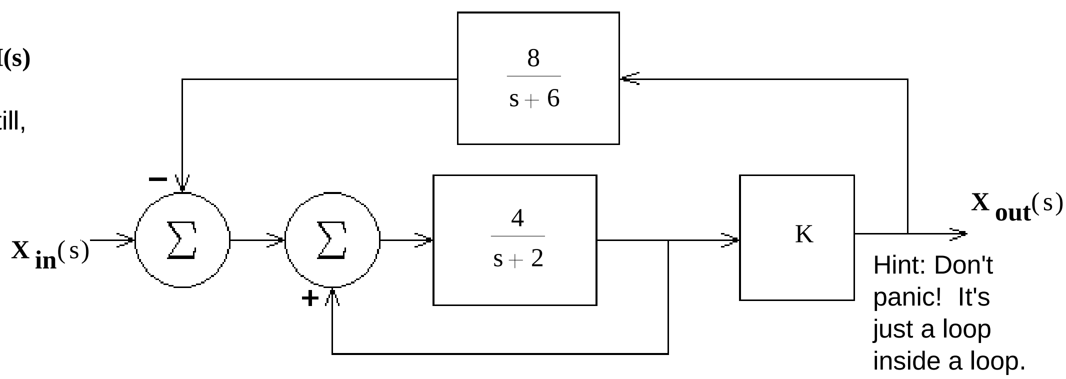
c) Transfer functions have units based on the units of the output and the units of input.
What are the units of $\mathbf{H}(s)$ from part a)

d) What are the units of $\mathbf{H}_2(s)$ from part b)

4. (18 pts) a) A feedback system is shown in the figure. What is the transfer function of the whole system, with feedback.

SHOW YOUR WORK
Simplify your expression for **H(s)** so that the denominator is a simple polynomial, or better still, in a factored form.

$$\mathbf{H}(s) = \frac{\mathbf{X}_{\mathbf{out}}(s)}{\mathbf{X}_{\mathbf{in}}(s)} = ?$$



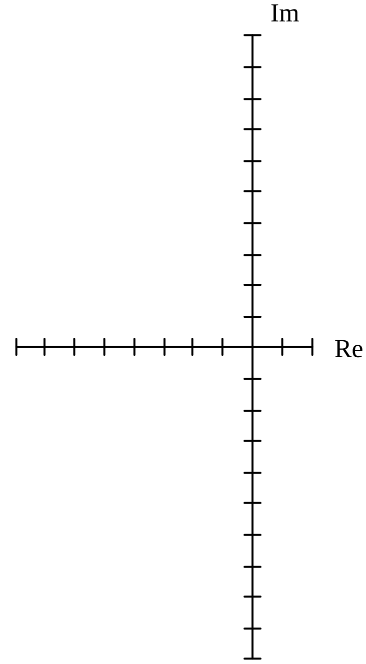
Hint: Don't panic! It's just a loop inside a loop.

b) Find the value of K to make the transfer function critically damped.

c) Does the transfer function have any zeros? Answer no or find the s value(s) of the zero(s).

5. (18 pts) A system has this transfer function: $\mathbf{H}(s) = \dfrac{2\cdot(s+39)}{s^2+6\cdot s+13}$

a) What is the steady-state response ($y_{ss}(t)$) of this system to the input: $x(t) = \left(8+5\cdot e^{-5\cdot t}\cdot\cos(7\cdot t)\right)\cdot u(t)$



b) Show **all** the poles of the output **signal** on the axis provided.
Make sure I can tell the values of the real & imaginary parts.

c) What is the natural frequency (ω_n) of this **system**?

d) What is the damping factor (ζ) of this **system**?

The poles of the output **signal**, not the system.

e) If this **system** had a **step input** instead of the input above, what % overshoot would the output have?

6. (26 pts) This system: $\mathbf{H}(s) = \dfrac{20\cdot(s+10)}{s\cdot(s+4)}$ Has this input: $x(t) = 4\cdot\sin(8\cdot t)\cdot u(t)$

a) Express the output, and separate into 4 partial fractions that you can find in the Laplace transform table without using complex numbers. Show what they are, but don't find the coefficients.

$\mathbf{Y}(s) =$

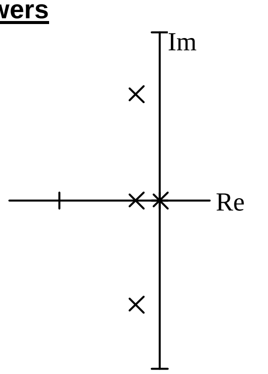
b) Continue with the partial fraction expansion just far enough to find the step and the **transient** coefficients as a numbers. DON'T find coefficients for the sinusoidal parts.

c) Use steady-state AC analysis to find the **time-domain** representation of the **sinusoidal steady-state** output.

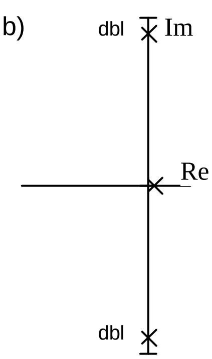
d) Now put all the pieces together to get the full time-domain output. $y(t) = ?$
Leave the sinusoidal part as you found it in part c).

Answers

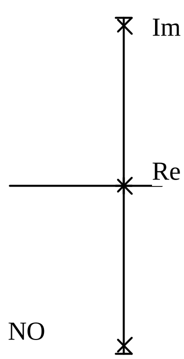
1. a)



b)



2. a)



b) NO

3 & 4 are below

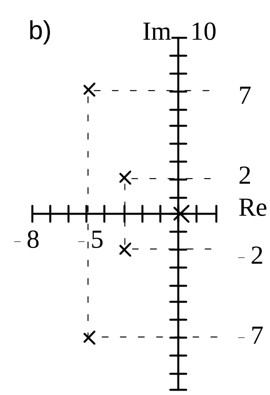
5. a) $48 \cdot u(t)$

c) 3.606

d) 0.832

e) $0.9 \cdot \%$

b)



3. a) $\dfrac{\dfrac{1}{L_m \cdot C}}{s^2 + \left(\dfrac{R_m}{L_m} + \dfrac{1}{R \cdot C}\right) \cdot s + \dfrac{1}{L_m \cdot C} \cdot \left(1 + \dfrac{R_m}{R}\right)}$

b) $\dfrac{\dfrac{R_m}{L_m \cdot C}}{s^2 + \left(\dfrac{R_m}{L_m} + \dfrac{1}{R \cdot C}\right) \cdot s + \dfrac{1}{L_m \cdot C} \cdot \left(1 + \dfrac{R_m}{R}\right)}$

c) unitless

d) Ω

4. a) $\dfrac{4 \cdot K \cdot (s+6)}{s^2 + 4 \cdot s + 32 \cdot K - 12}$

b) 0.5 c) -6

6. a) $\dfrac{A}{s} + \dfrac{B}{s+4} + \dfrac{C \cdot s}{\left(s^2+64\right)} + \dfrac{D \cdot 8}{\left(s^2+64\right)}$

b) $25 \quad -12$

c) $14.32 \cdot \cos(8 \cdot t - 204.8 \cdot deg) \cdot u(t)$ or $14.32 \cdot \cos(8 \cdot t + 155.2 \cdot deg) \cdot u(t)$

d) $\left(25 - 12 \cdot e^{-4 \cdot t} + 14.32 \cdot \cos(8 \cdot t + 155.2 \cdot deg)\right) \cdot u(t)$