ECE 3510 Exam 1 Information

A Short Table of (Unilateral) Laplace Transforms

	$f(t)$	$F(s)$	
1	$\delta(t)$	1	
2	$u(t)$	$\frac{1}{s}$	
3	$tu(t)$	$\frac{1}{s^2}$	
4	$t^n u(t)$	$\frac{n!}{s^{n+1}}$	
5	$e^{\lambda t}u(t)$ $\quad e^{-t/\tau}$	$\frac{1}{s-\lambda}$	$\frac{1}{s+\frac{1}{\tau}}$
6	$te^{\lambda t}u(t)$	$\frac{1}{(s-\lambda)^2}$	
7	$t^n e^{\lambda t}u(t)$	$\frac{n!}{(s-\lambda)^{n+1}}$	
8a	$\cos bt\, u(t)$	$\frac{s}{s^2+b^2}$	
8b	$\sin bt\, u(t)$	$\frac{b}{s^2+b^2}$	
9a	$e^{-at}\cos bt\, u(t)$	$\frac{s+a}{(s+a)^2+b^2}$	$= \frac{s+a}{s^2+2as+a^2+b^2}$
9b	$e^{-at}\sin bt\, u(t)$	$\frac{b}{(s+a)^2+b^2}$	
10a	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{(r\cos\theta)s+(ar\cos\theta-br\sin\theta)}{s^2+2as+(a^2+b^2)}$	
10b	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb}+\frac{0.5re^{-j\theta}}{s+a+jb}$	
10c	$re^{-at}\cos(bt+\theta)\, u(t)$	$\frac{As+B}{s^2+2as+c}$	
	$r=\sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}},\ \theta=\tan^{-1}\frac{Aa-B}{A\sqrt{c-a^2}}$		
	$b=\sqrt{c-a^2}$		
10d	$e^{-at}\left[A\cos bt+\frac{B-Aa}{b}\sin bt\right]u(t)$	$\frac{As+B}{s^2+2as+c}$	
	$b=\sqrt{c-a^2}$		
	$t\cdot e^{-a\cdot t}\cdot\cos(b\cdot t)$	$\frac{(s+a)^2-b^2}{\left[(s+a)^2+b^2\right]^2}$	
	$t\cdot e^{-a\cdot t}\cdot\sin(b\cdot t)$	$\frac{2\cdot b\cdot(s-a)}{\left[(s+a)^2+b^2\right]^2}$	

Laplace Transform

$$F(s) = \int_0^{\infty} f(t)\cdot e^{-s\cdot t}\,dt$$

Euler's equations

$$\cos(\omega\cdot t) = \frac{e^{j\cdot\omega\cdot t}+e^{-j\cdot\omega\cdot t}}{2} \qquad \sin(\omega\cdot t) = \frac{e^{j\cdot\omega\cdot t}-e^{-j\cdot\omega\cdot t}}{2\cdot j}$$

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The Laplace Transform Properties

Operation	$f(t)$	$F(s)$
Addition	$f_1(t) + f_2(t)$	$F_1(s) + F_2(s)$
Scalar multiplication	$kf(t)$	$kF(s)$
Time differentiation	$\frac{df}{dt}$	$sF(s) - f(0^-)$
	$\frac{d^2f}{dt^2}$	$s^2F(s) - sf(0^-) - \dot{f}(0^-)$
	$\frac{d^3f}{dt^3}$	$s^3F(s) - s^2f(0^-) - s\dot{f}(0^-) - \ddot{f}(0^-)$
Time integration	$\int_{0^-}^{t} f(\tau)\,d\tau$	$\frac{1}{s}F(s)$
	$\int_{-\infty}^{t} f(\tau)\,d\tau$	$\frac{1}{s}F(s) + \frac{1}{s}\int_{-\infty}^{0^-} f(t)\,dt$
Time shift	$f(t-t_0)u(t-t_0)$	$F(s)e^{-st_0} \quad t_0 \geq 0$
Frequency shift	$f(t)e^{s_0t}$	$F(s-s_0)$
Frequency differentiation	$-tf(t)$	$\frac{dF(s)}{ds}$
Frequency integration	$\frac{f(t)}{t}$	$\int_{s}^{\infty} F(z)\,dz$
Scaling	$f(at),\ a \geq 0$	$\frac{1}{a}F\left(\frac{s}{a}\right)$
Time convolution	$f_1(t) * f_2(t)$	$F_1(s)F_2(s)$
Frequency convolution	$f_1(t)f_2(t)$	$\frac{1}{2\pi j}F_1(s) * F_2(s)$
Initial value	$f(0^+)$	$\lim_{s\to\infty} sF(s) \quad (n > m)$
Final value	$f(\infty)$	$\lim_{s\to 0} sF(s)$ (poles of $sF(s)$ in LHP)

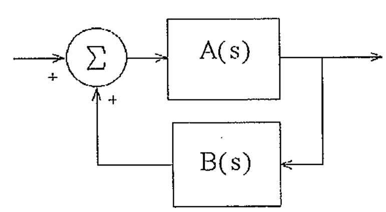
$$0 = a\cdot x^2 + b\cdot x + c \qquad x = \frac{-b + \sqrt{b^2 - 4\cdot a\cdot c}}{2\cdot a}$$

$b^2 - 4\cdot a\cdot c > 0$ overdamped

$b^2 - 4\cdot a\cdot c = 0$ critically damped

$b^2 - 4\cdot a\cdot c < 0$ under damped

Standard feedback loop transfer function



$$\frac{A(s)}{1 - A(s)\cdot B(s)}$$