ECE 3510 Exam 1 Information

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ECE 3510 Laplace Transforms (Unilateral)

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$$F(s) = \int_0^\infty f(t)\cdot e^{-s\cdot t}\,dt$$

	f(t)	F(s)		
1	$\delta(t)$	1		
2	$u(t)$	$\frac{1}{s}$		
3	$t\cdot u(t)$	$\frac{1}{s^2}$		
4	$t^n\cdot u(t)$	$\frac{n!}{s^{n+1}}$		
5a	$e^{a\cdot t}\cdot u(t)$	$\frac{1}{s-a}$		
5b	$e^{-\frac{t}{\tau}}\cdot u(t)$	$\frac{1}{s+\frac{1}{\tau}}$	$\tau = -\frac{1}{a}$ = time constant	
6	$t\cdot e^{a\cdot t}\cdot u(t)$	$\frac{1}{(s-a)^2}$		
7	$t^n\cdot e^{a\cdot t}\cdot u(t)$	$\frac{n!}{(s-a)^{n+1}}$		
8a	$\cos(b\cdot t)\cdot u(t)$	$\frac{s}{s^2+b^2}$	$b = \omega$ = radian frequency	
8b	$\sin(b\cdot t)\cdot u(t)$	$\frac{b}{s^2+b^2}$		
9a	$e^{a\cdot t}\cdot\cos(b\cdot t)\cdot u(t)$	$\frac{s-a}{(s-(a+bj))\cdot(s-(a-bj))}$	=	$\frac{s-a}{s^2-2\cdot a\cdot s+\left(a^2+b^2\right)}$
		$\frac{s-a}{(s-a)^2+b^2}$	=	$\frac{s-a}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}}$
9b	$e^{a\cdot t}\cdot\sin(b\cdot t)\cdot u(t)$	$\frac{b}{(s-(a+bj))\cdot(s-(a-bj))}$	=	$\frac{b}{s^2-2\cdot a\cdot s+\left(a^2+b^2\right)}$
		$\frac{b}{(s-a)^2+b^2}$	=	$\frac{b}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}}$
11a	$t\cdot e^{a\cdot t}\cdot\cos(b\cdot t)\cdot u(t)$	$\frac{(s-a)^2-b^2}{\left[(s-a)^2+b^2\right]^2}$	=	$\frac{(s-a)^2-b^2}{\left[s^2-2\cdot a\cdot s+\left(a^2+b^2\right)\right]^2}$
11b	$t\cdot e^{a\cdot t}\cdot\sin(b\cdot t)\cdot u(t)$	$\frac{2\cdot b\cdot(s-a)}{\left[(s-a)^2+b^2\right]^2}$	=	$\frac{2\cdot b\cdot(s-a)}{\left[s^2-2\cdot a\cdot s+\left(a^2+b^2\right)\right]^2}$

Euler's equations $\cos(\omega\cdot t) = \frac{e^{j\cdot\omega\cdot t}+e^{-j\cdot\omega\cdot t}}{2}$ $\sin(\omega\cdot t) = \frac{e^{j\cdot\omega\cdot t}-e^{-j\cdot\omega\cdot t}}{2\cdot j}$

ECE 3510 Laplace Transforms

ECE 3510 Some Laplace Properties

Operation	f(t)	F(s)
Time differentiation	$\frac{d}{dt}f(t)$	$s \cdot F(s) - f(0^-)$
	$\frac{d^2}{dt^2}f(t)$	$s^2 \cdot F(s) - s \cdot f(0^-) - \frac{d}{dt}f(0^-)$ initial slope
Time integration	$\int_{0^-}^{t} f(\tau)\, d\tau$	$\frac{1}{s} \cdot F(s)$
	$\int_{-\infty}^{t} f(\tau)\, d\tau$	$\frac{1}{s} \cdot F(s) + \frac{1}{s} \cdot \int_{-\infty}^{0^-} f(t)\, dt$
Initial value	$f(0^+)$	$\lim_{s \to \infty} s \cdot F(s)$ $n>m$ # of poles > # of zeroes
Final value	$f(\infty)$	$\lim_{s \to 0} s \cdot F(s)$ (all poles of $sF(s)$ in LHP)

Misc Information

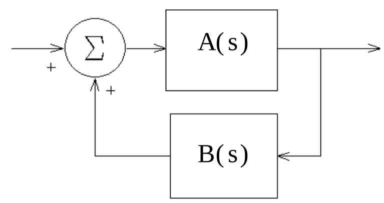
$$0 = a \cdot x^2 + b \cdot x + c \qquad x = \frac{-b + \sqrt{b^2 - 4 \cdot a \cdot c}}{2 \cdot a}$$

$b^2 - 4 \cdot a \cdot c > 0$ overdamped

$b^2 - 4 \cdot a \cdot c = 0$ critically damped

$b^2 - 4 \cdot a \cdot c < 0$ under damped

Standard feedback loop transfer function



$$\frac{A(s)}{1 - A(s) \cdot B(s)}$$

$$\mathbf{Y}(s) = \overbrace{\frac{b_2 \cdot s^2 + b_1 \cdot s + b_0}{s^2 + a_1 \cdot s + a_0}}^{\mathbf{H}(s)} \cdot \mathbf{X}(s) + \frac{s \cdot y(0) + \frac{d}{dt}y(0) + a_1 \cdot y(0) - b_2 \cdot s \cdot x(0) - b_2 \cdot \frac{d}{dt}x(0) - b_1 \cdot x(0)}{s^2 + a_1 \cdot s + a_0}$$