

ECE3510 Lab #1

First-Order Systems

Objectives

The objective of this lab is to study the characteristics of step responses of first-order systems. A brush DC motor is used for the experiments. Concepts of time constant and DC gain are introduced. Discrepancies between the idealized first-order model and the actual responses are observed. Another objective of the first lab is also to become familiar with the hardware and the software in the lab.

Introduction

The response of a brush DC motor with the voltage v (in V or volts) considered as an input and the angular velocity ω in (rad/s or s^{-1}) as an output, can be approximately described by a first-order differential equation:

$$\frac{d\omega}{dt} = -a\omega + kv \quad (8.1)$$

In the Laplace domain, the transfer function is given by

$$P(s) = \frac{\omega(s)}{v(s)} = \frac{k}{s+a} \quad (8.2)$$

The parameter a is the pole of the system and has the dimension of rad/s or s^{-1} . The parameter k has the dimension of $V^{-1} s^{-2}$. The *time constant* of the system is the parameter T given by

$$T = 1/a \quad (8.3)$$

T has the units of seconds. The DC gain of the system is given by

$$P(0) = \frac{k}{a} \quad (8.4)$$

and has the units of $V^{-1}s^{-1}$ or (rad/s)/V. In practice, a common unit is rpm/V, where rpm refers to revolutions-per-minute.

The step response of the system is the response for a step of input voltage

$$v(t) = v_0 \quad t \geq 0 \quad (8.5)$$

For a step input, the output of the system is given by

$$\omega(t) = \frac{k}{a} v_0 (1 - e^{-t/T}). \quad (8.6)$$

The steady-state value of the output is given by $(k/a) \cdot v_0$, i.e., the DC gain multiplied by the steady-state value of the input. Therefore, the DC gain indicates how much voltage is needed to reach a certain speed.

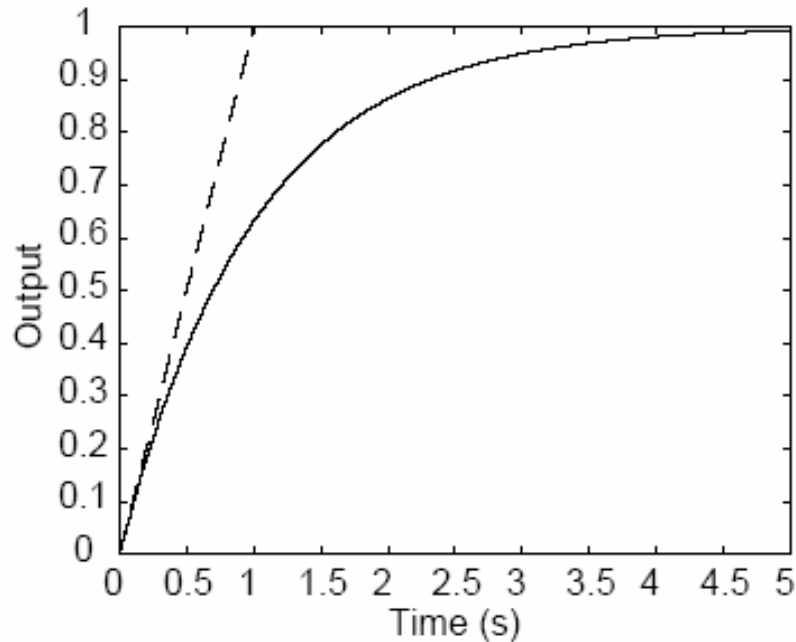


Figure 8.22: Step response of a first-order system

The step response of a system with transfer function $P(s) = 1/(s+1)$ is sketched in Fig. 8.22. On the graph, one finds that the tangent to the output at $t=0$ reaches the steady-state value of the step response for $t=T=1$ sec. It also turns out that, for $t=T$, the output reaches 63% of its final value ($1 - e^{-1} \approx 0.63$).

Pre-lab

Using partial fraction expansions, verify that the step response of the first-order system (8.2) is indeed given by equation (8.6) and also prove the property that the tangent at $t=0$ reaches the steady-state value for $t=T$.

Experiments

Equipment needed:

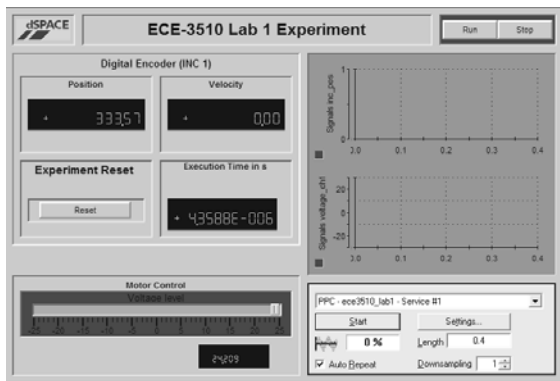
1. Brush DC motor
2. Dual power amplifier
3. Voltmeter
4. Cable rack
5. DSpace kit which includes an encoder cable and I/O breakout box.
6. Small and large wheels with two hex wrenches to change the amount of inertia on the motor shaft.

Preliminary testing

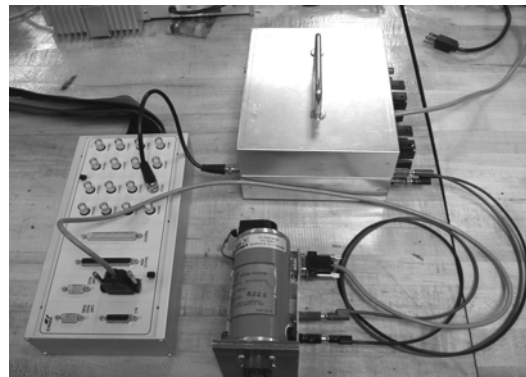
Be sure to read the dSpace tutorial for basic hardware and software setup.

- Use the tutorial instructions to load the V_control_DC_motor experiment from the Documents folder of your lab computer.
- Connect the encoder input cable from the I/O breakout box port “INC1” to the encoder plug on the motor support bracket. Check the operation of the encoder by manually spinning the motor noting a change in encoder position in the layout window
- Connect a BNC cable from DACH1 to a channel on the dual linear amplifier. Set the slider in the layout for +10 V, click run in the layout and measure the resulting voltage at the output of the amplifier. It should be ~10 V. Click stop to reset the system.
- Move the slider to zero the Voltage to the motor
- Finish hooking up the system by attaching red and black leads from the motor to the amplifier
- Click run and move the slider around to get a feel for how the system drives the motor. Also, note the values and graphs of the interface.

The system should look as follows when setup:



ControlDesk Software Interface



Physical Hardware setup

Step response

The experiment $V_control_DC_motor$ will allow you to apply voltages to the motor by moving the slider in the layout window. Note that the slider value in this program specifies the voltage to be applied to the motor and accounts for the value of the amplifier gain. The program stores samples at 500 Hz or one sample every 2 msec.

To capture the data needed for this lab use the procedure for *Working with data* in the dSpace tutorial to save data using *autoname*. This will allow you to gather separate data for each run of the system. A good name might be *vars*.

To start capturing, *Run* the experiment until the plots are completed and then *Stop* it. Each time the experiment is run then stopped a new file *varsXXX.mat* (where XXX is the number of the dataset you have captured) will be created to contain the captured data.

The data is available in Matlab® by setting the current working directory to the location of the saved files and by typing *mat_unpack*. The script file, *mat_unpack.m* provides an automated method for unpacking the .mat file created by dSpace. Follow the prompts to store the position, voltage, and time variables in variables *pos*, *vol* and, *t* which have the units of radians, volts, and seconds respectively.

Do the following:

- Set the slider for 25 V and click *Run* to capture the step response then click *Stop*
- Obtain the 25 V step response after removing the coupler and mounting the small wheel on the shaft of the motor, and then again with the large wheel (get the appropriate screwdriver from the TA or lab attendant).

- Reinstall the coupler.

Computations to be done using the data sets:

- In Matlab®, observe the position data then reconstruct numerically the velocity as a function of time. A good numerical approximation would be to note the formula

$$\omega(t) = (\text{pos}(t) - \text{pos}(t - T_s)) / T_s \quad (8.7)$$

where T_s is the sampling period (2 msec). The data is not store with time as the index so the equation will need to be adapted.

- Determine the steady-state velocity by taking the mean of the velocity over a period of time where it is (approximately) constant.
- Plot the response over 0.05 sec, and compare it to the response shown in Fig. 1 (the scales will be different, but the shape should be comparable).
- Obtain an estimate of the time constant based on the time it takes for the output to reach 63% of its steady-state value.
- From the results, calculate the values of the DC gain and of the parameters k and a of the system. Also, give the DC gain in rad/sec and rpm/V.
- These simple experiments should demonstrate the validity of the first-order model for the DC motor, as well as its limitations. Taking a close look at the step response around $t = 0$ should reveal that the response is not linear with respect to time, contrary to Fig. 8.22. A more detailed model of the DC motor would account for the inductance of the motor, and for the time that it takes for the current to change. A second-order model, rather than first-order, would result and would more accurately reflect the behavior of the motor. In practice however, any model, no matter how high order or how detailed it is, is only an approximation of the real system.

Report at a glance

Be sure to include:

- Pre-lab calculations.
- Plots of the three different step responses (preferably on the same graph for comparison).
- Estimated values for the steady-state velocity, the time constant, the DC gain, a and k .
Give the units for all quantities, and report the DC gain both in rad/s and in rpm/V .
- Discuss the effects of added inertia on the motor parameters.
- Comments.