ECE 3510 Bode Plot Notes

A. Stolp
3/15/06,
3/11/07

Steps to make Bode Plots

Sample transfer function: $$P(s) = K\cdot\frac{(s+z_1)\cdot(s+z_2)\cdot(s+z_3)}{s^2\cdot(s+p_1)\cdot(s+p_2)\cdot(s^2+2\cdot\zeta\cdot\omega_n+\omega_n^2)}$$

if complex pole is expressed: $\left[(s+a)^2+b^2\right]$ then: $\omega_n = \sqrt{a^2+b^2}$

1. a) Rewrite, replacing all s's with blanks:

$$P(\_) = K\cdot\frac{(\_+z_1)\cdot(\_+z_2)\cdot(\_+z_3)}{\_^2\cdot(\_+p_1)\cdot(\_+p_2)\cdot(\_+\omega_n)\cdot(\_+\omega_n)}$$

notice that you also simplify the complex poles and/or zeros for now

2. a) Scale your frequency axis to start at some frequency less than your smallest pole or zero.

   b) Plug this starting frequency in for any poles or zeros at the origin (write it in the blank as $j\omega$).

$$P(j\omega,\_) = K\cdot\frac{(\_+z_1)\cdot(\_+z_2)\cdot(\_+z_3)}{(j\cdot\omega_{start})^2\cdot(\_+p_1)\cdot(\_+p_2)\cdot(\_+\omega_n)\cdot(\_+\omega_n)}$$

   c) Ignore all the other blanks and calculate your initial magnitude, initial slope and the initial phase angle.

$$\text{Initial magnitude} = K\cdot\frac{(z_1)\cdot(z_2)\cdot(z_3)}{(j\cdot\omega_{start})^2\cdot(p_1)\cdot(p_2)\cdot(\omega_n)\cdot(\omega_n)}$$

Initial slope

ω's in the numerator --> $+20$dB/decade each

ω's in the denominator --> -20dB/decade each

Initial phase

numerator, j => 90º, **-** => +180º

denominator, j => -90º, **-** => **-**180º

3. a) Extend the line to the first pole or zero.

   b) Replace that pole or zero with $j\omega$ and cross out the value of the pole or zero:

$$K\cdot\frac{(j\omega+\cancel{z_1})\cdot(\_+z_2)\cdot(\_+z_3)}{(j\omega)^2\cdot(\_+p_1)\cdot(\_+p_2)\cdot(\_+\omega_n)\cdot(\_+\omega_n)}$$

OR

$$K\cdot\frac{(\_+z_1)\cdot(\_+z_2)\cdot(\_+z_3)}{(j\omega)^2\cdot(j\omega+\cancel{p_1})\cdot(\_+p_2)\cdot(\_+\omega_n)\cdot(\_+\omega_n)}$$

   c) Use this to find the new slope and phase angle.

   Unless you replaced what was once a $-s$ or crossed out a negative value:

   zeros turn up the slope --> $+20$dB/decade

   poles turn down the slope --> -20dB/decade

   zeros increase the phase angle --> $+90$deg

   poles decrease the phase angle --> -90deg

4. Repeat step 3 for each successive pole or zero.

   After the last one you may want to check the magnitude or slope and phase again.

5. Draw a smooth line through the bode plots to estimate the actual magnitude and phase.

   a) At poles and zeroes that are NOT complex

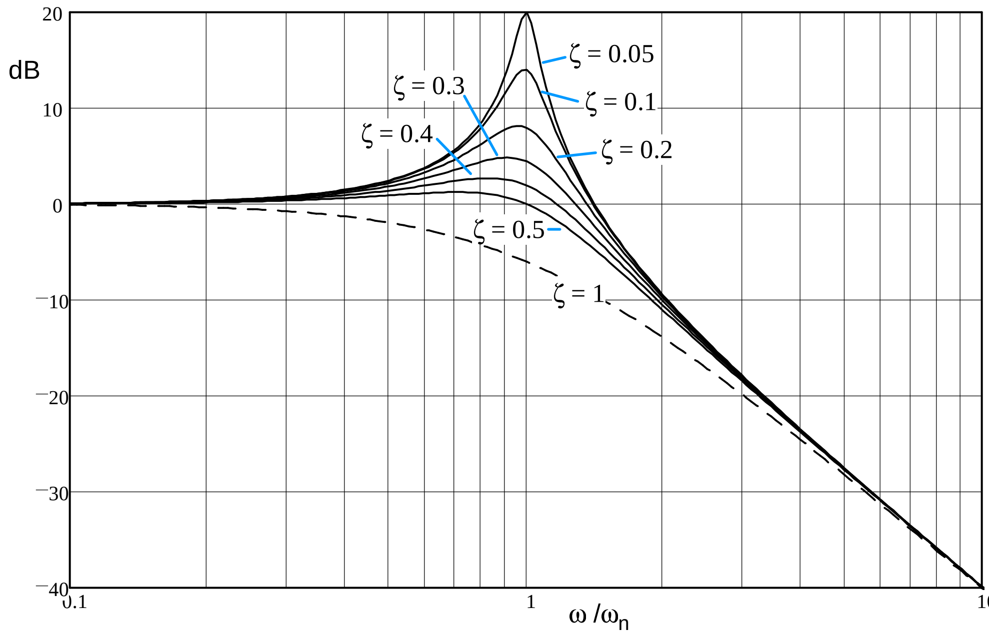
Actual magnitude:	-3dB at single poles +3dB at single zeros	-6dB at double poles +6dB at double zeros	etc..	Magnitude effects extend about 1 decade fore and aft.

   Angle effects extend about 1.5 decade fore and aft.

5. b) At poles and zeroes that ARE complex

Correct the smooth line at the complex poles by scaling the plots on this page.

You may have to use a mirror image of these plots



The damping factor comes from the complex pole expression in one of two ways:

$$\left(s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^{\,2}\right)$$

natural frequency $\omega_n = \sqrt{\omega_n^{\,2}}$

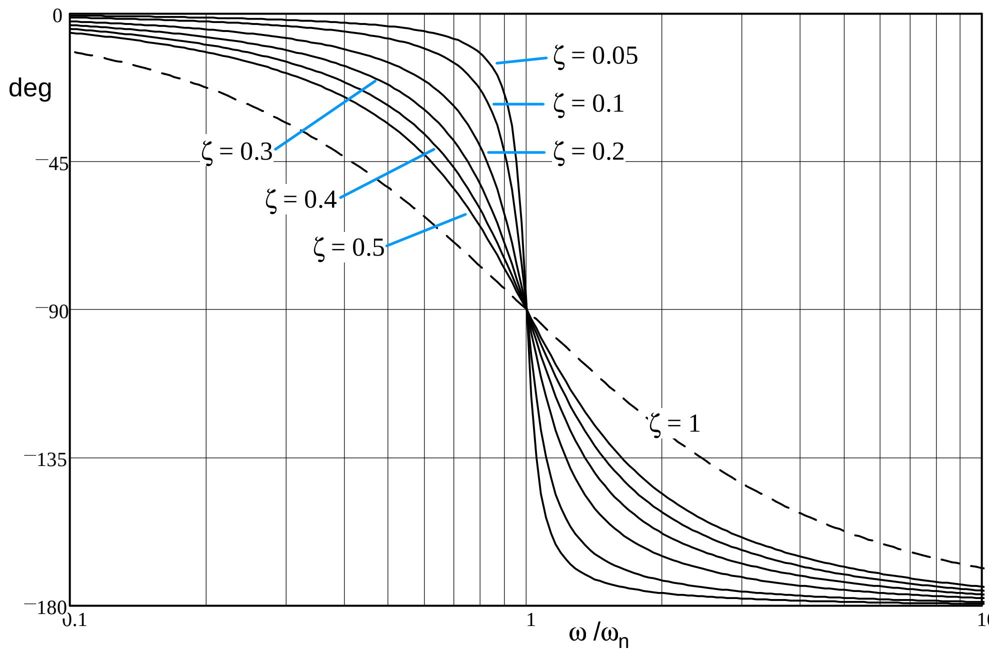
damping factor: $\zeta = \dfrac{2\cdot\zeta\cdot\omega_n}{2\cdot\omega_n}$

if complex pole is expressed:

$$\left[(s+a)^2 + b^2\right]$$

natural frequency $\omega_n = \sqrt{a^2 + b^2}$

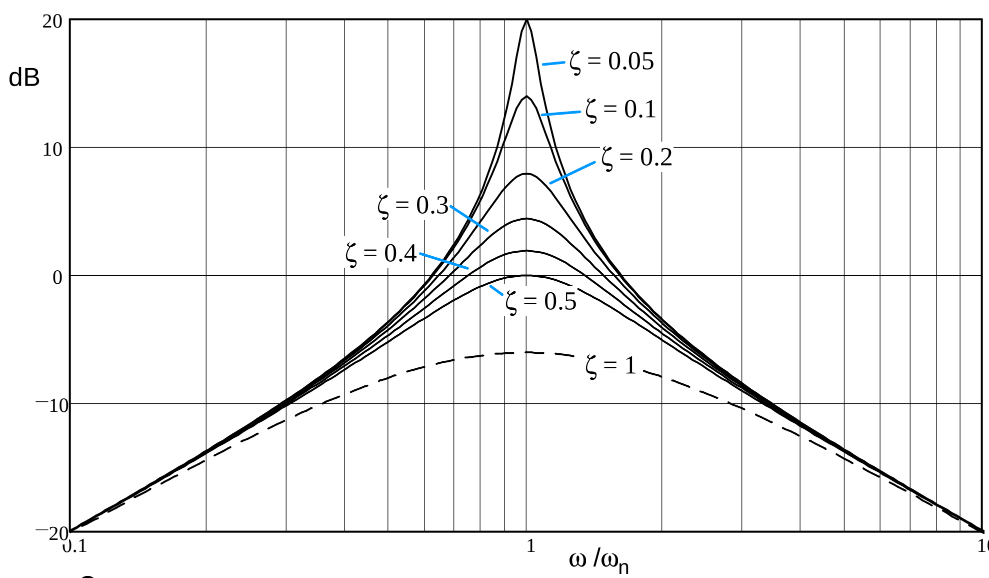
damping factor $\zeta = \dfrac{a}{\omega_n}$



At ω_n the actual magnitude is:

$\dfrac{1}{2\cdot\zeta}$ in dB $20\cdot\log\left(\dfrac{1}{2\cdot\zeta}\right)$

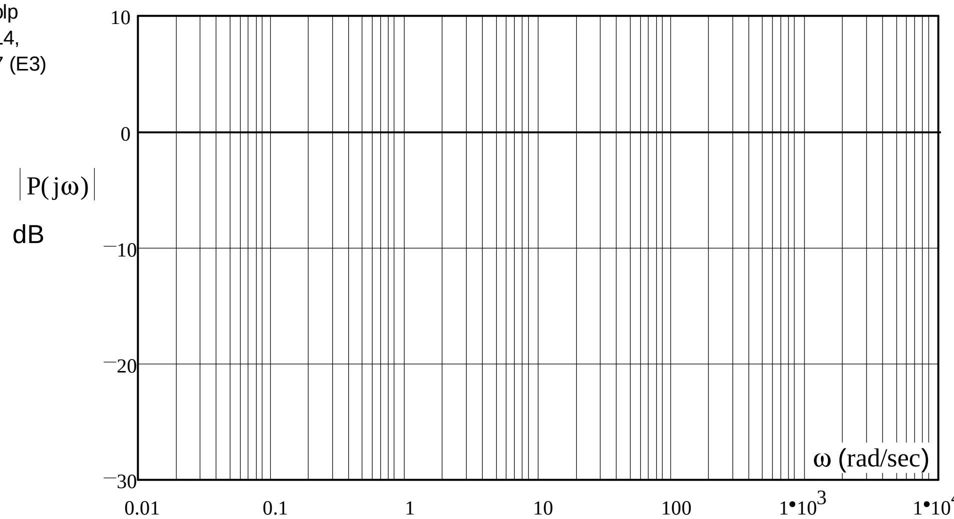
To correct the plot at complex zeros, use these plots upside-down

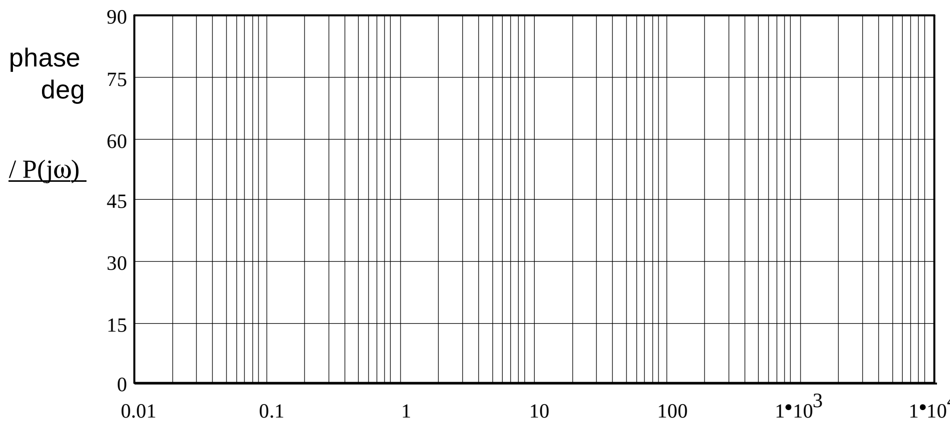


ECE 3510
Bode Plot Examples

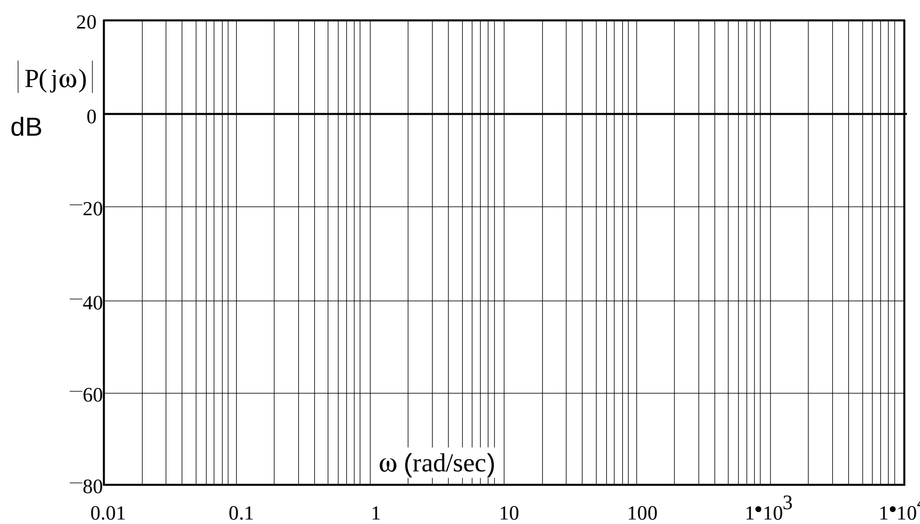
A. Stolp
3/27/14,
4/4/17 (E3)

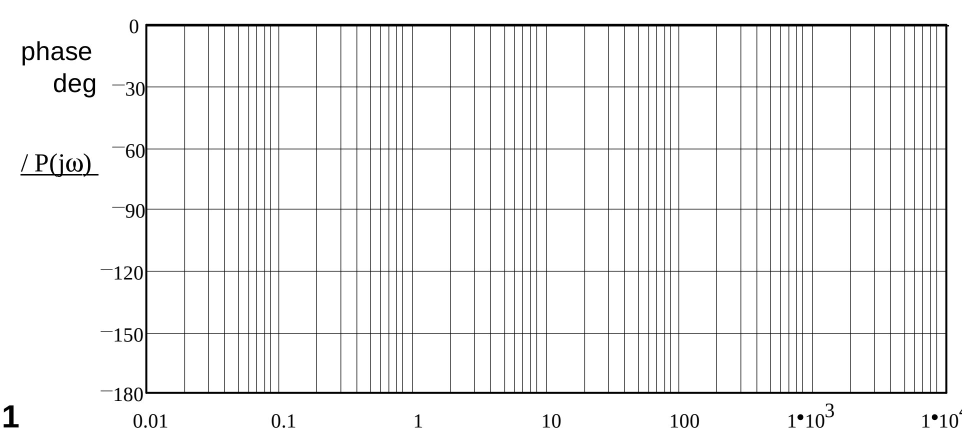
Ex. 1 $P(s) = \dfrac{2\cdot(s+10)}{s+100}$





Ex. 2 $P(s) = \dfrac{s+20}{4\cdot(s+1)^2}$





ECE 3510 Bode Examples p.2

Ex. 3 $P_3(s) := \dfrac{20000\cdot(-s+0.1)}{(s+4)\cdot(s+1000)}$

clear the - in front of the s $= \dfrac{-20000\cdot(s-0.1)}{(s+4)\cdot(s+1000)}$

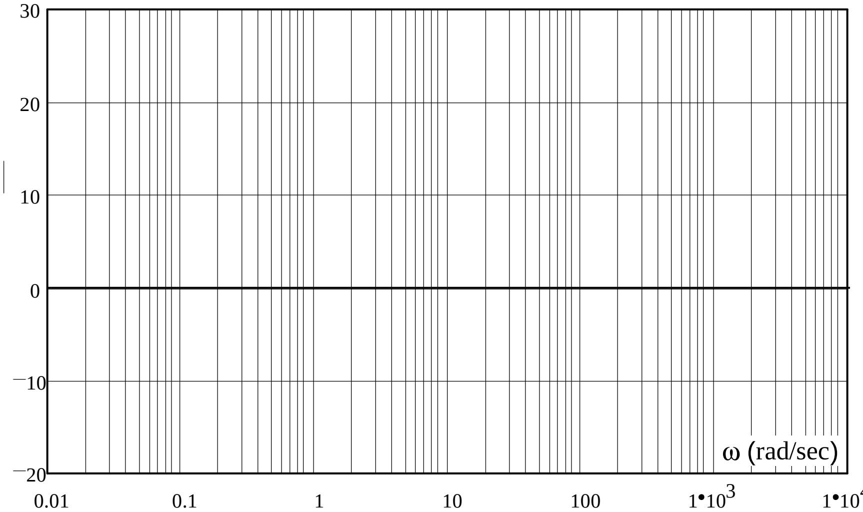
$\omega < 0.1$

180º+180º=360º

$\dfrac{-20000\cdot(\_-0.1)}{(\_+4)\cdot(\_+1000)} = 0.5$ $\quad -6\cdot dB$

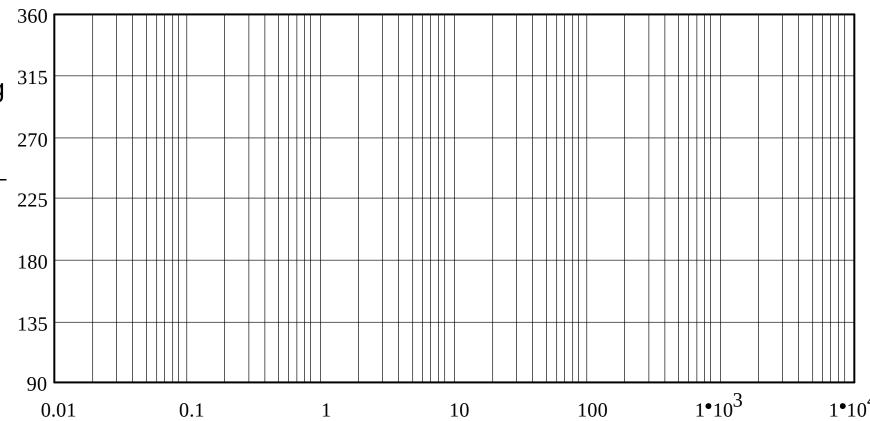
$\underline{/\ 360^o}$

$|P(j\omega)|$
dB



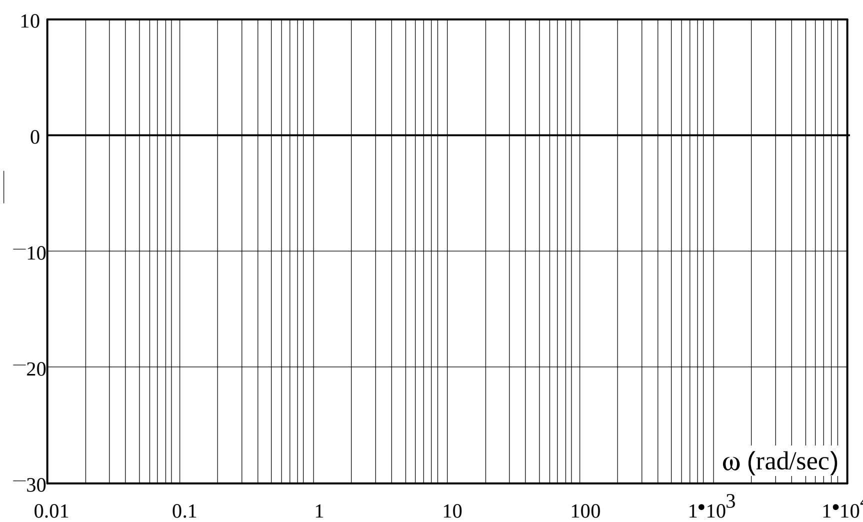
phase
deg

$\underline{/\ P(j\omega)}$



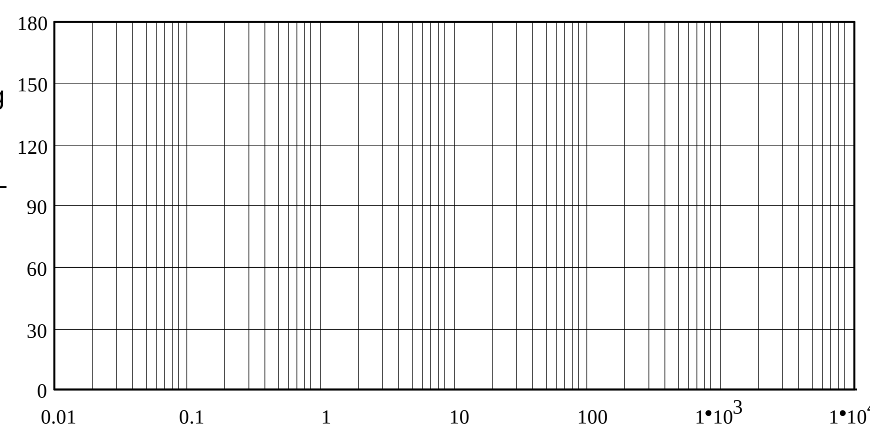
Ex. 4 $P_4(s) := \dfrac{0.5\cdot(s+1)\cdot(s-20)}{s\cdot(s+100)}$

$|P(j\omega)|$
dB



phase
deg

$\underline{/\ P(j\omega)}$

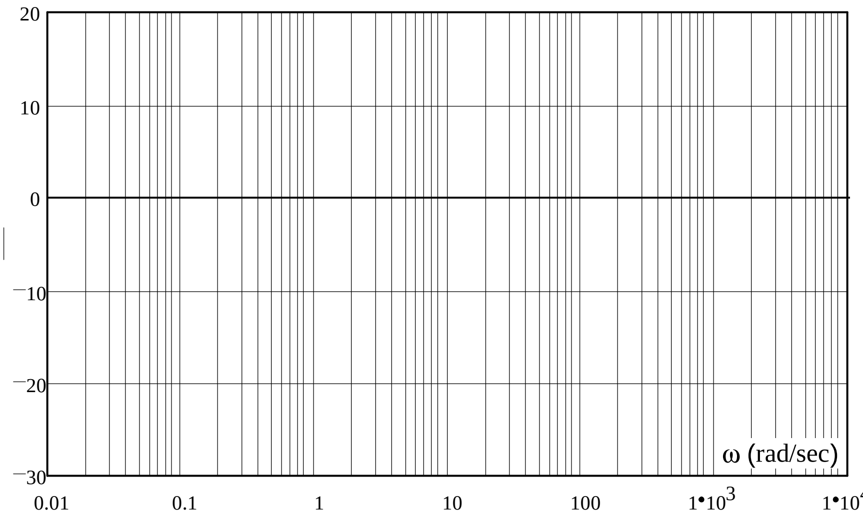


Ex. 5

$$P_5(s) := \frac{5000 \cdot s \cdot (s-4)}{(s+0.2)\cdot(s+20)\cdot(s+1000)}$$

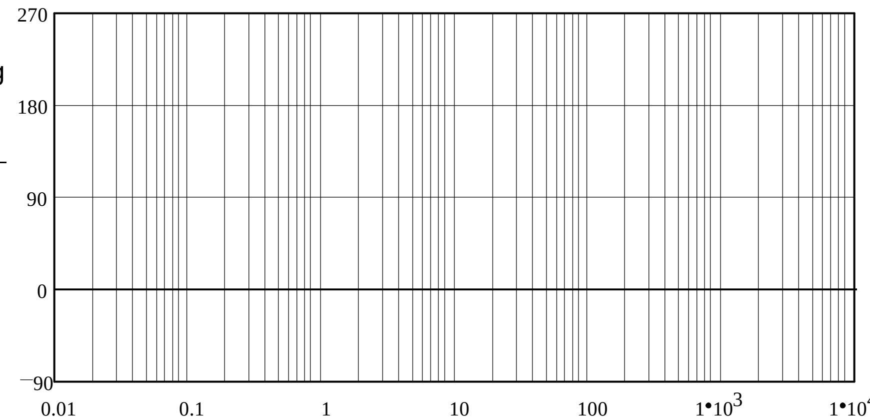
$|P(j\omega)|$

dB



phase
deg

$\angle P(j\omega)$



Ex. 6

$$P_6(s) := \frac{20000 \cdot (s+0.1)}{(s+2)\cdot\left(s^2+10\cdot s+10000\right)}$$

$$s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^{\,2}$$

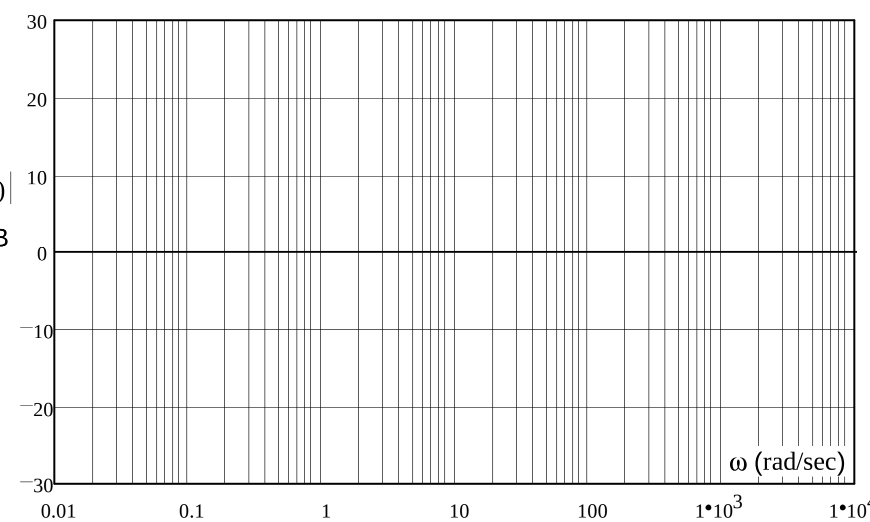
natural frequency $\omega_n = \sqrt{\omega_n^{\,2}} \quad =$

damping factor: $\zeta = \dfrac{2\cdot\zeta\cdot\omega_n}{2\cdot\omega_n} \quad =$

dB peak: $20\cdot\log\left(\dfrac{1}{2\cdot.05}\right) = 20$

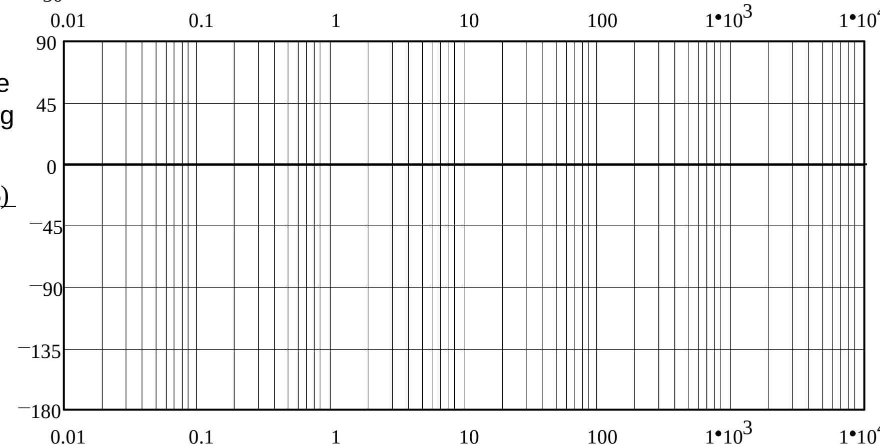
$|P(j\omega)|$

dB



phase
deg

$\angle P(j\omega)$



Ex. 7

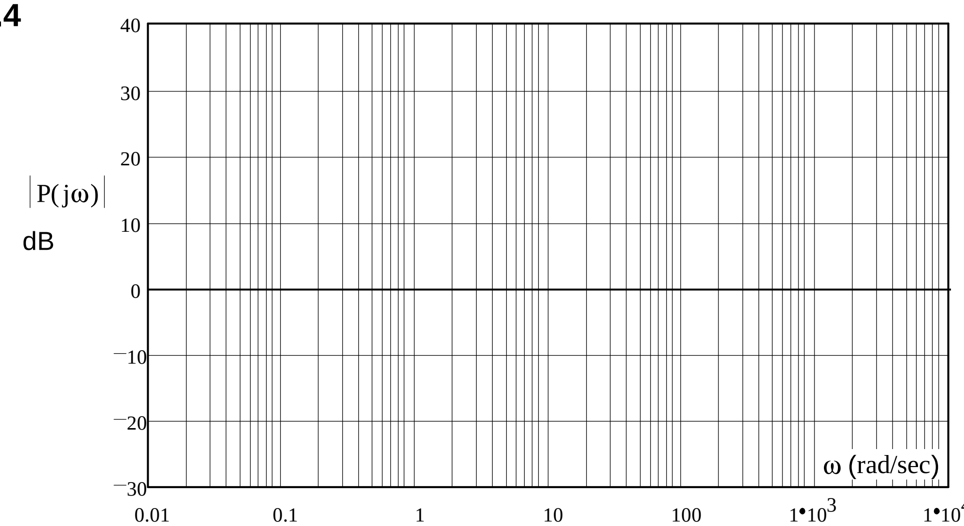
$$P_7(s) := \frac{400\cdot(s+0.1)\cdot(s+100)}{\left[(s+0.4)^2+15.84\right]\cdot(s+1000)}$$

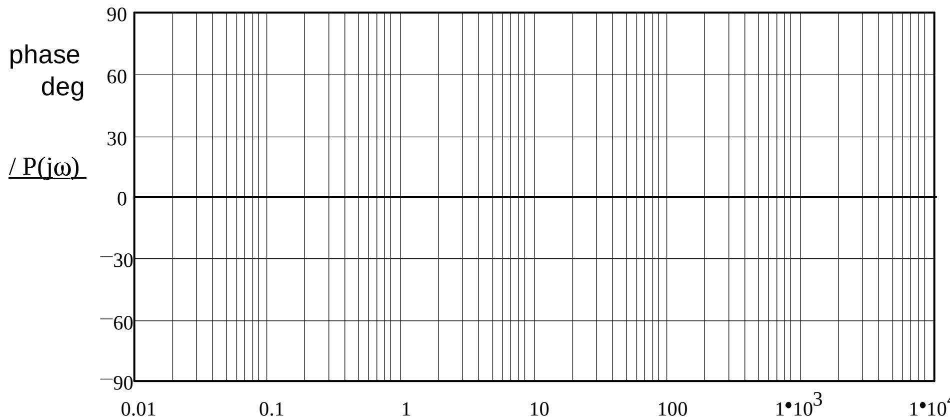
natural freq. $\quad (s+a)^2+b^2$

$\omega_n = \sqrt{a^2+b^2} \quad =$

damping factor: $\quad \zeta = \dfrac{a}{\omega_n} \quad =$

peak $\quad \dfrac{1}{2\cdot\zeta} =$





Ex. 8 $\quad (s+a)^2+b^2$

$$P_8(s) := \frac{25\cdot\left[(s+10)^2+9900\right]}{\left(s^2+s+4\right)\cdot(s+2000)}$$

$s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}$

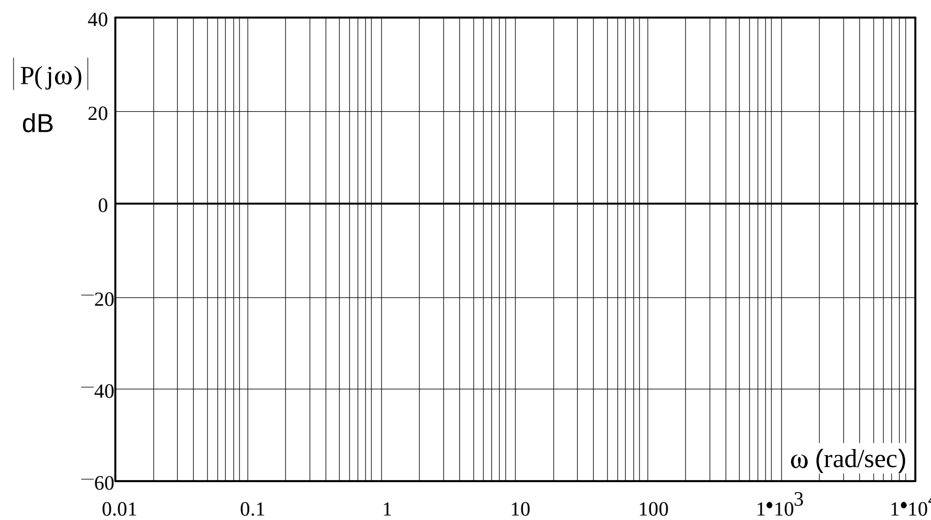
natural frequency $\quad \omega_{n1} = \sqrt{\omega_{n1}^{\,2}} \quad =$

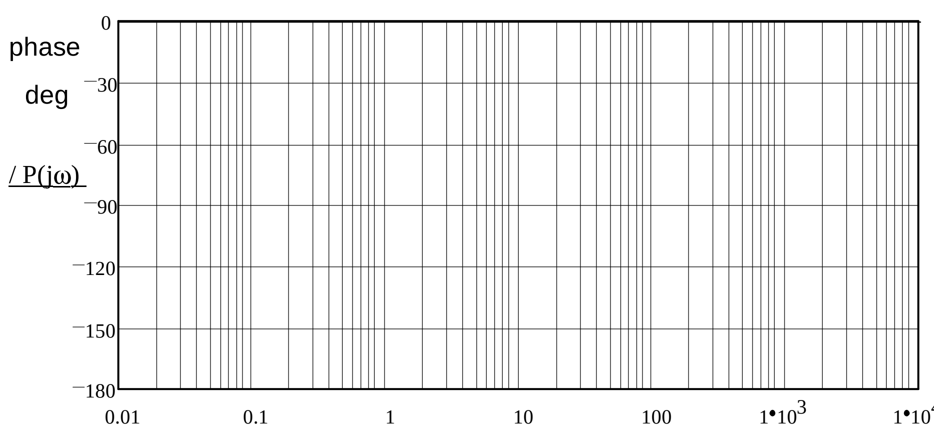
damping factor: $\quad \zeta = \dfrac{2\cdot\zeta\cdot\omega_{n1}}{2\cdot\omega_{n1}} =$

natural freq.
$\omega_{n2} = \sqrt{a^2+b^2} \quad =$

damping factor: $\quad \zeta = \dfrac{a}{\omega_n} \quad =$

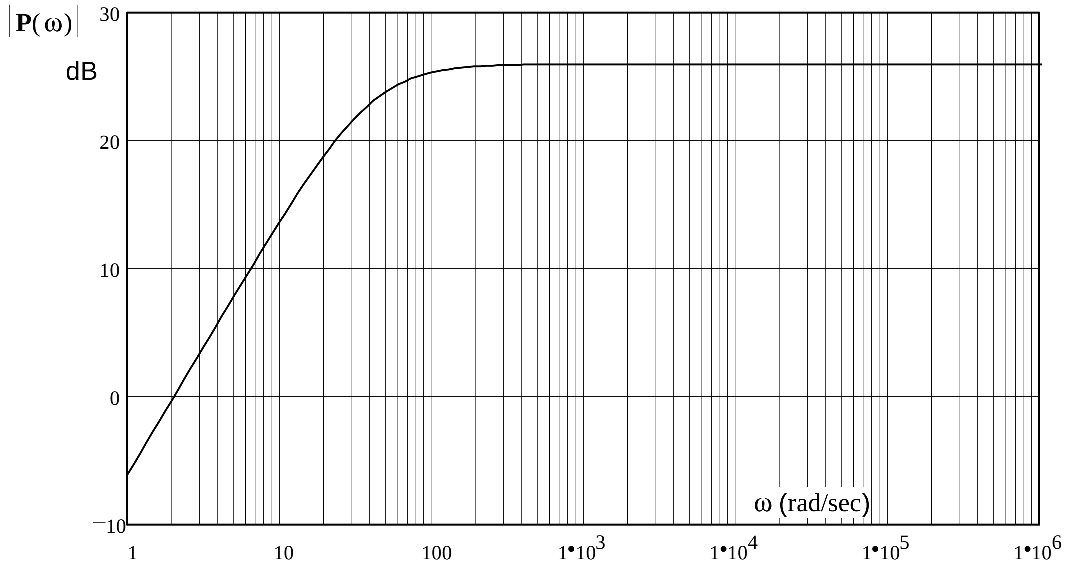
peak $\quad \dfrac{1}{2\cdot\zeta} =$

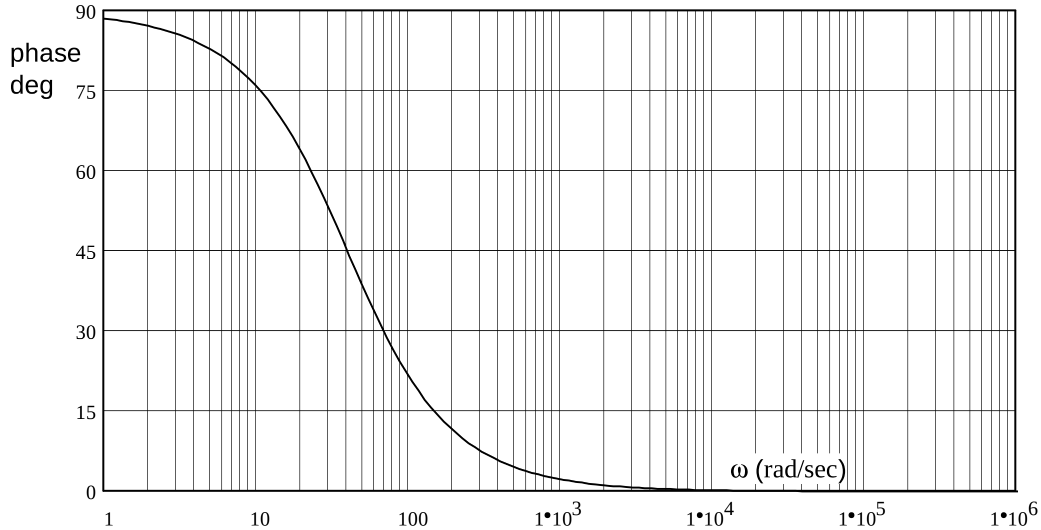




A. Stolp
4/6/14, 4/4/17

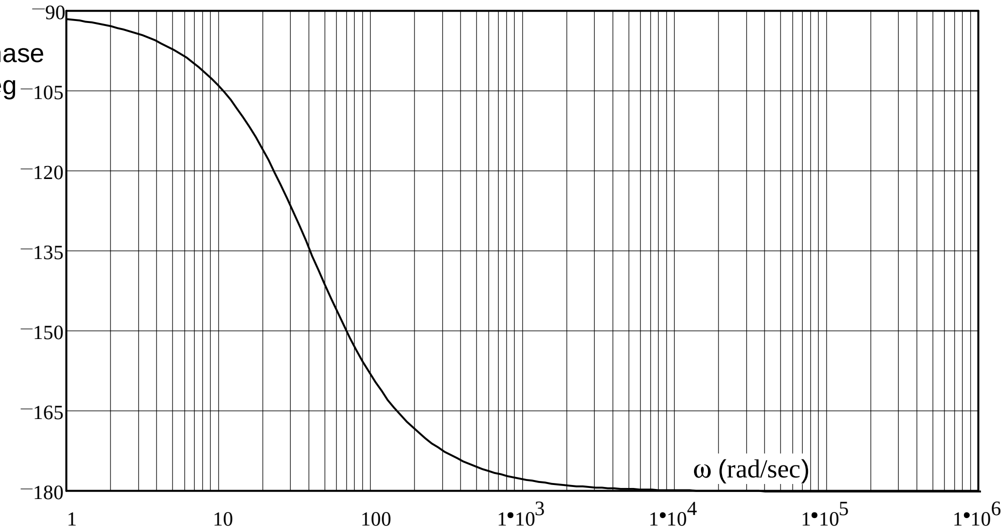
Ex. 1 $\mathbf{P}(s) = ?$



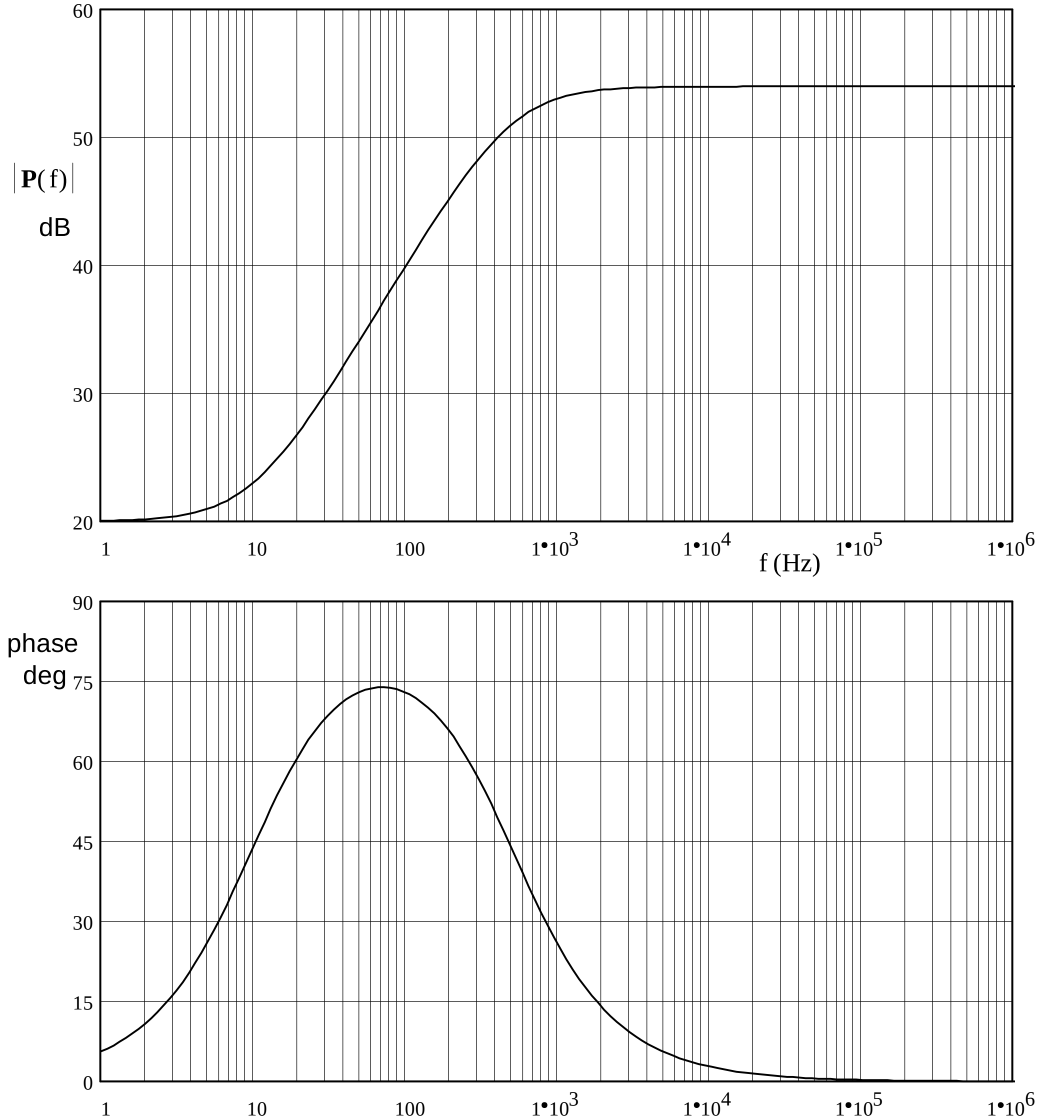


Ex. 2 What if the phase plot was:

$\mathbf{P}(s) = ?$

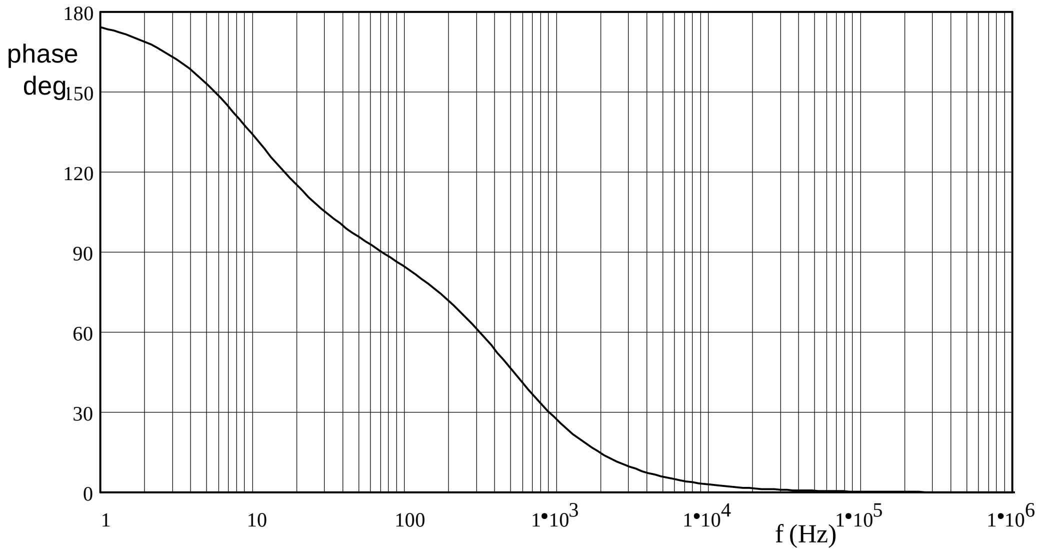


Ex. 3 $\mathbf{P}(s) = ?$

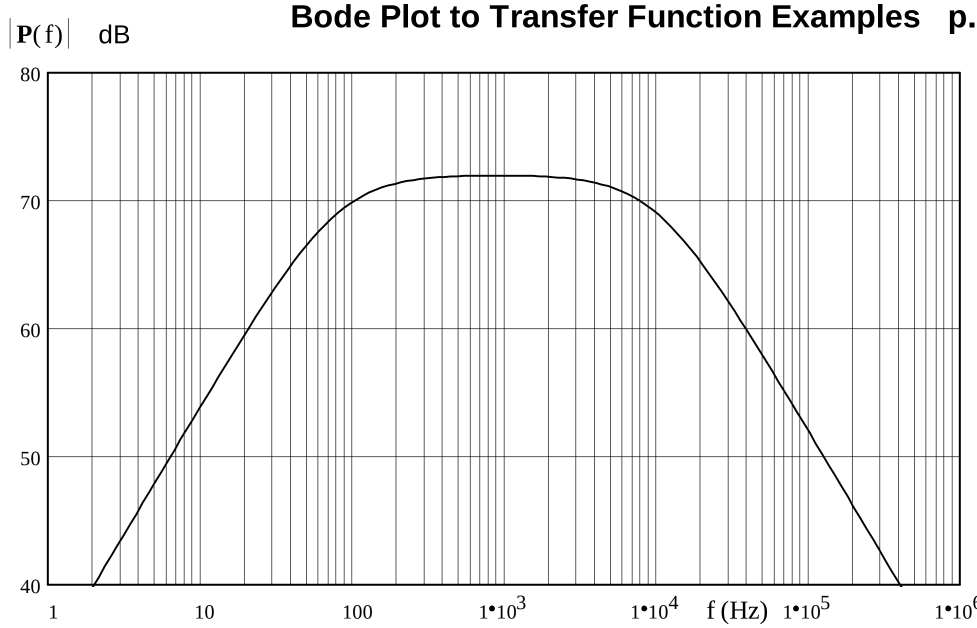


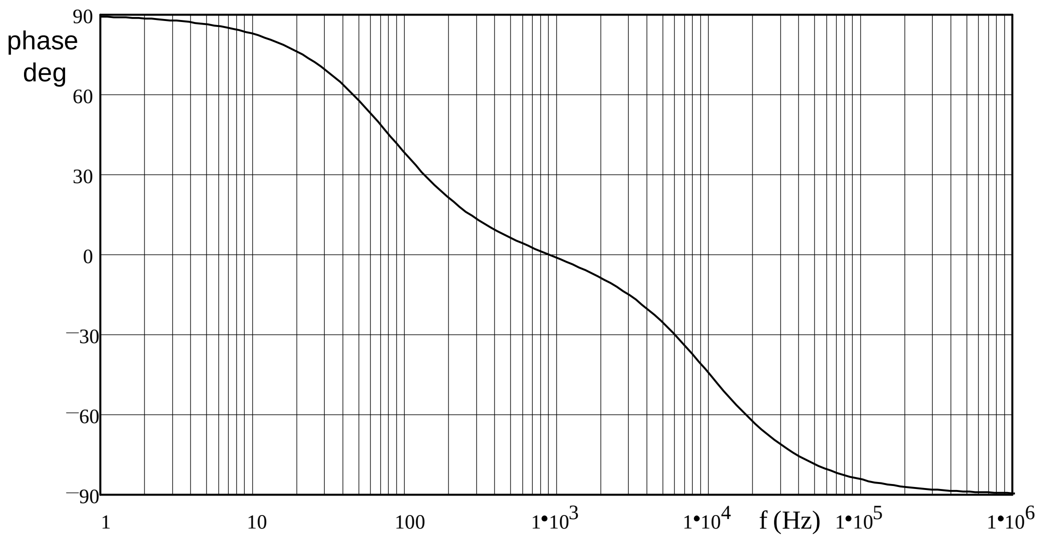
Ex. 4 What if the phase plot was:

$\mathbf{P}(s) = ?$



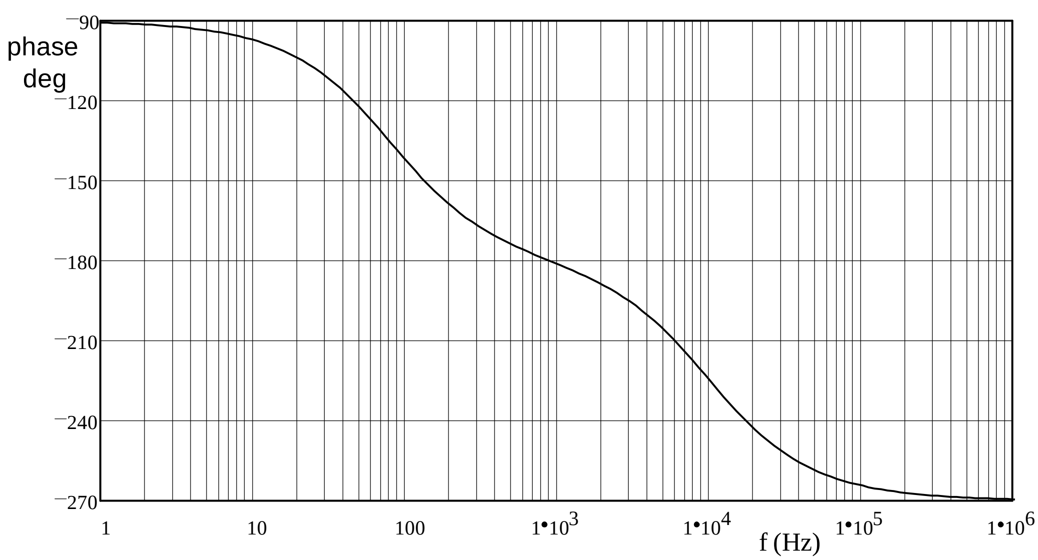
Ex. 5 $\mathbf{P}(s) = ?$



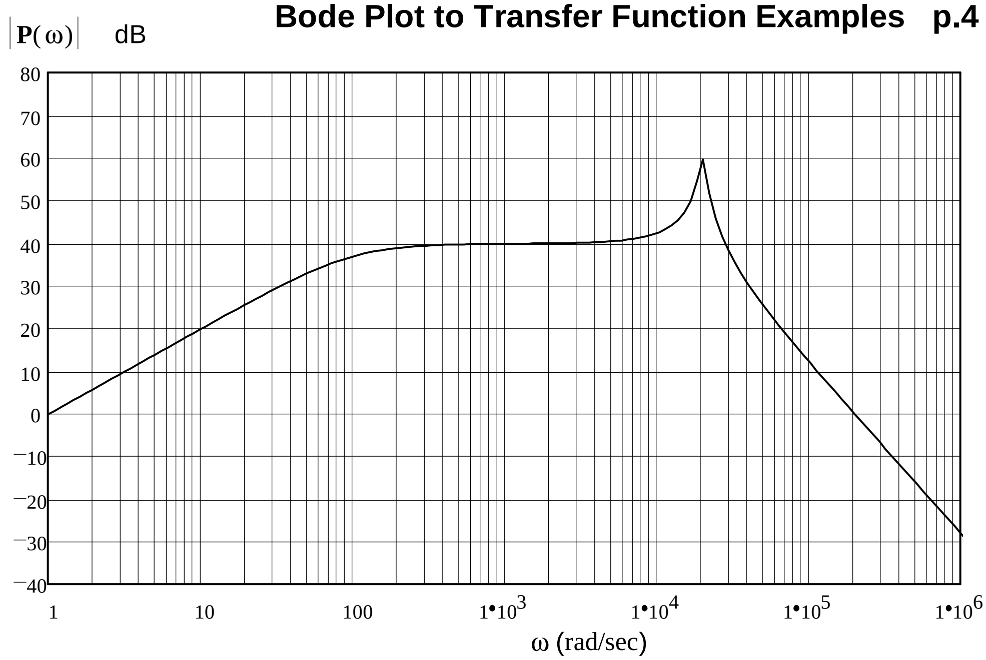


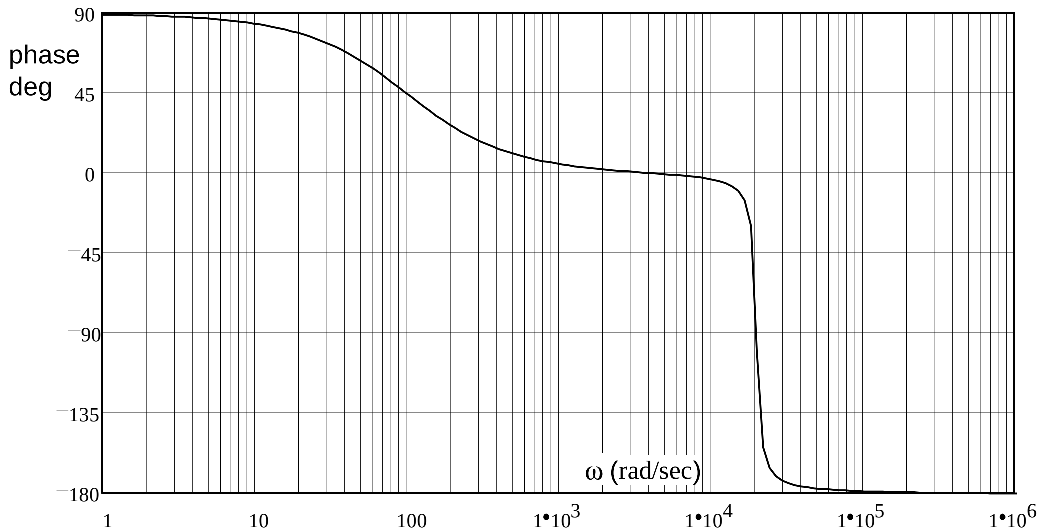
Ex. 6 What if the phase plot was:

$\mathbf{P}(s) = ?$



Ex. 7 $\mathbf{P}(s) = ?$





Note: A somewhat more involved method is outlined in Nise section 10.13 (p.660 in 3rd ed., 665 in 4th). That method involves estimating only one pole or zero at a time and then subtracting the effect from original to more clearly see the others. This can work much better with real experimental data. Real data always has delay effects and other non-linearities which make the process much harder.

ECE 3510 Gain, Phase, and Delay margins

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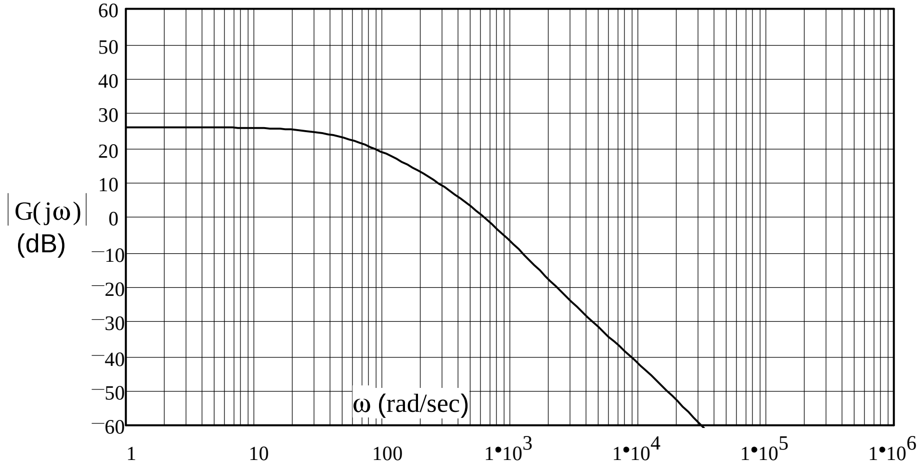
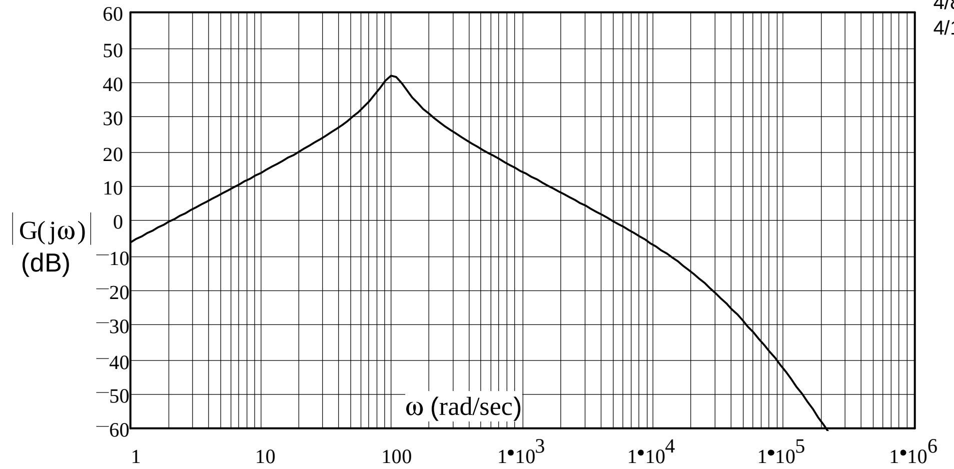
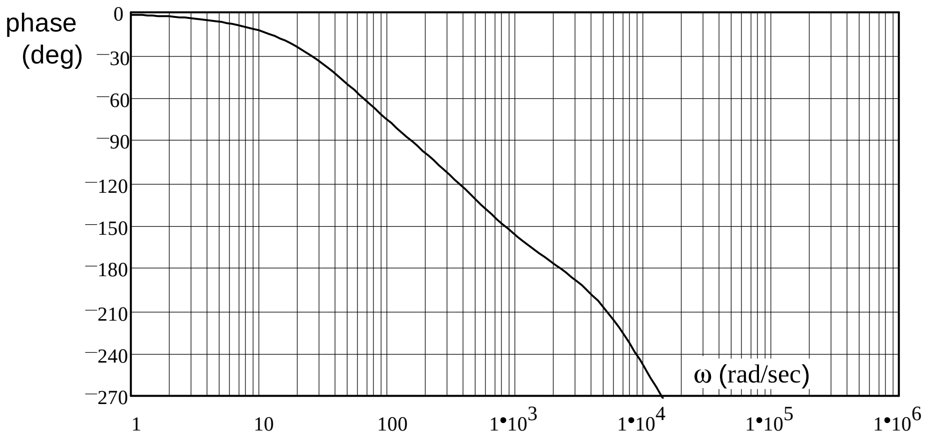
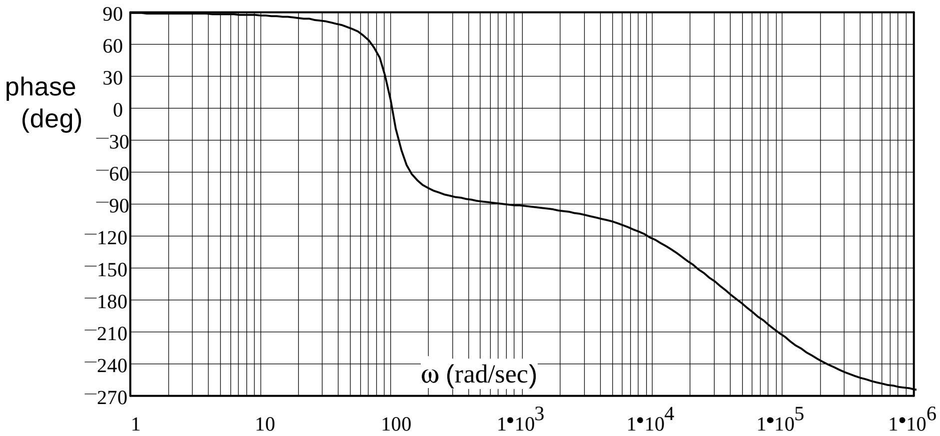
4/8/14
4/13/20









Gain Margin

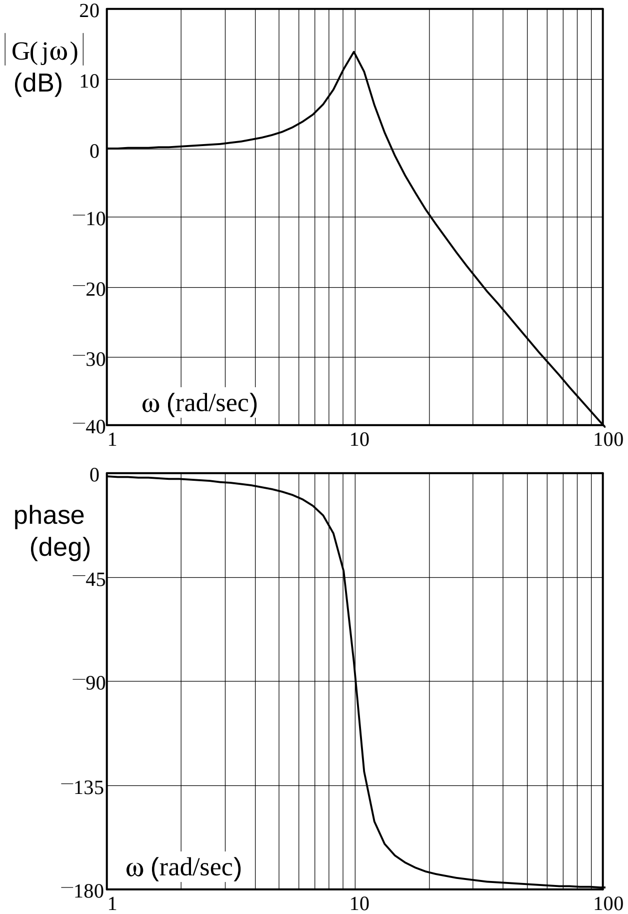
Phase Margin

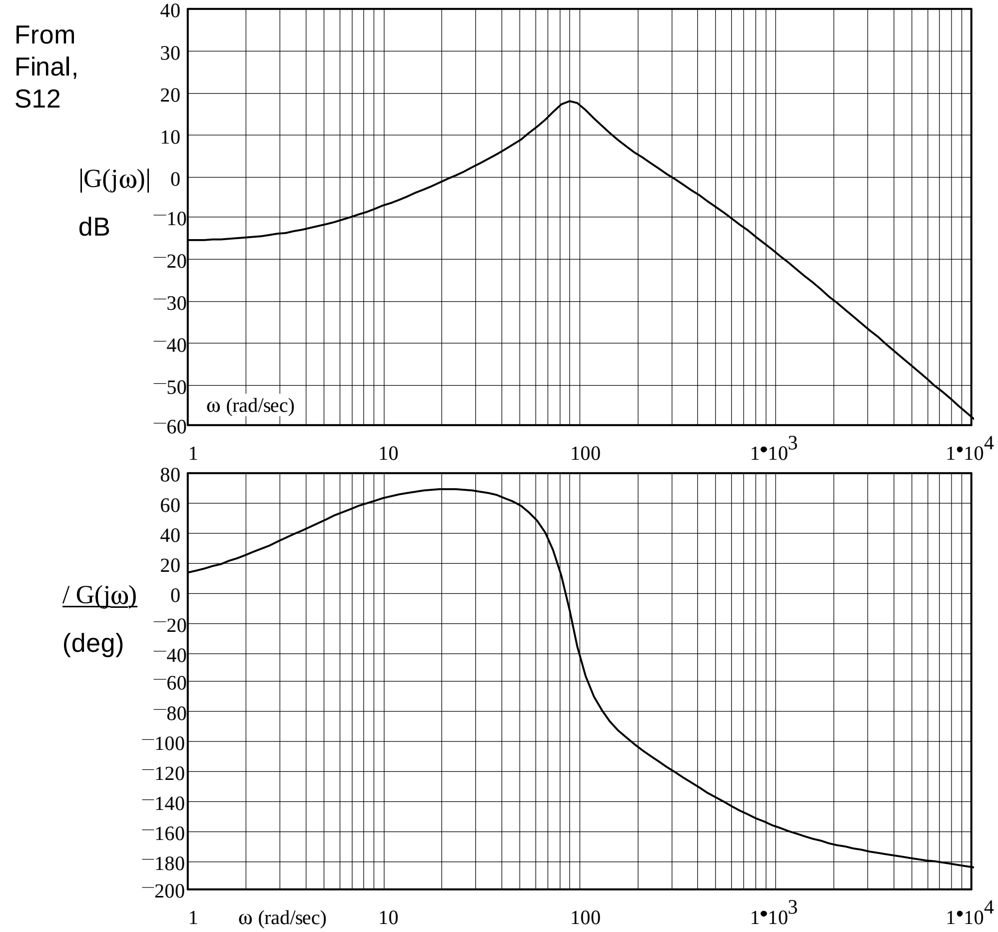
Delay Margin

Gain Margin

Phase Margin

Delay Margin





Gain Margin

Phase Margin

Delay Margin

Gain Margin

Phase Margin

Delay Margin

ECE 3510 Gain, Phase, and Delay margins, p2

Using Frequency-domain (Bode Plot) Design for the Double Integrator

A. Stolp
4/14/20

Also the Basis of problem 3 in homework BP3

Double Integrator

A very common system and a difficult design problem.

It's Newton's fault: $F = m \cdot a = m \cdot \frac{d^2}{dt^2} x$ $\qquad x = \frac{1}{m} \cdot \left(\int \int F \, dt \, dt \right)$ $\qquad \mathbf{X}(s) = \mathbf{F}(s) \cdot \frac{1}{m \cdot s^2}$

Same for angular motion: $T = J \cdot \alpha = J \cdot \frac{d^2}{dt^2} \theta$ $\qquad$ & $\mathbf{P}(s) = \frac{1}{m \cdot s^2}$

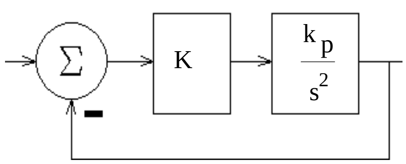
This problem arises anytime force is the input and position is the output.

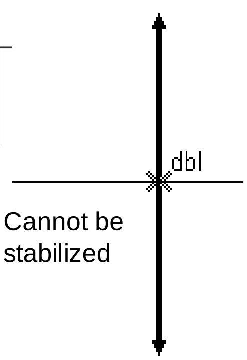
Force is the ONLY way to get the motion of any object to change, so yes, this is a common problem.

In the Inverted Pendulum lab, the movement of the base was simplified to a first-order system to avoid the difficulties that come from this very issue.

The example used in section 5.3.9 is a VERY REAL example.

in general: $\mathbf{P}(s) = \frac{k_p}{s^2}$



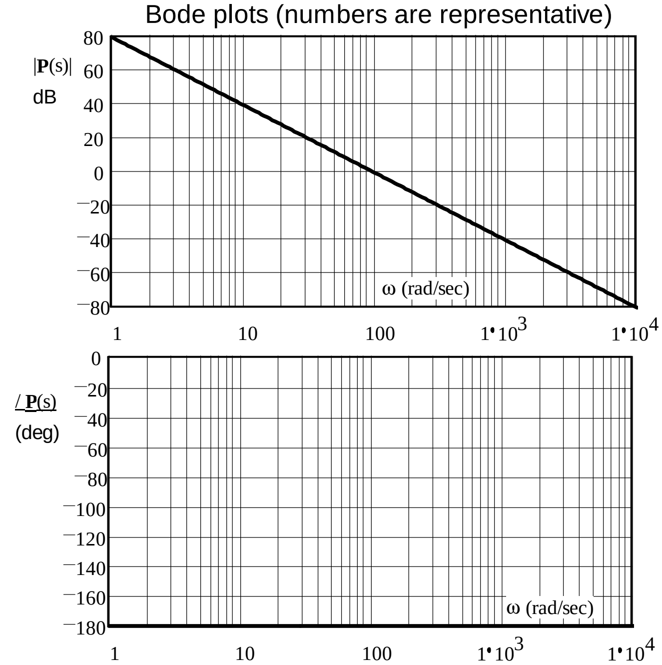


Root-locus for the double-integrator in a closed-loop (CL) feedback system.

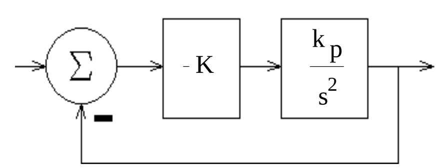
Cannot be stabilized

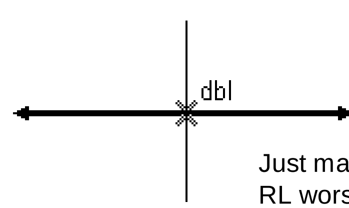
MUST use a compensator.

Bode plots (numbers are representative)



If the angle is always 180, then wouldn't positive feedback work? (make the gain negative)





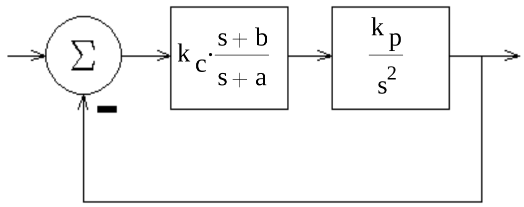
Just makes the RL worse.

Given the issues with a PD (the differentiator). lets use a Lead controller.

Lead controller

See section 5.3.9

$$\mathbf{C}(s) = k_c \cdot \frac{s+b}{s+a}$$



Put the two together,

$$\mathbf{G}(s) \;=\; k_c \cdot \frac{s+b}{s+a} \cdot \frac{k_p}{s^2} \;=\; k_p \cdot k_c \cdot \frac{s+b}{s^2 \cdot (s+a)}$$

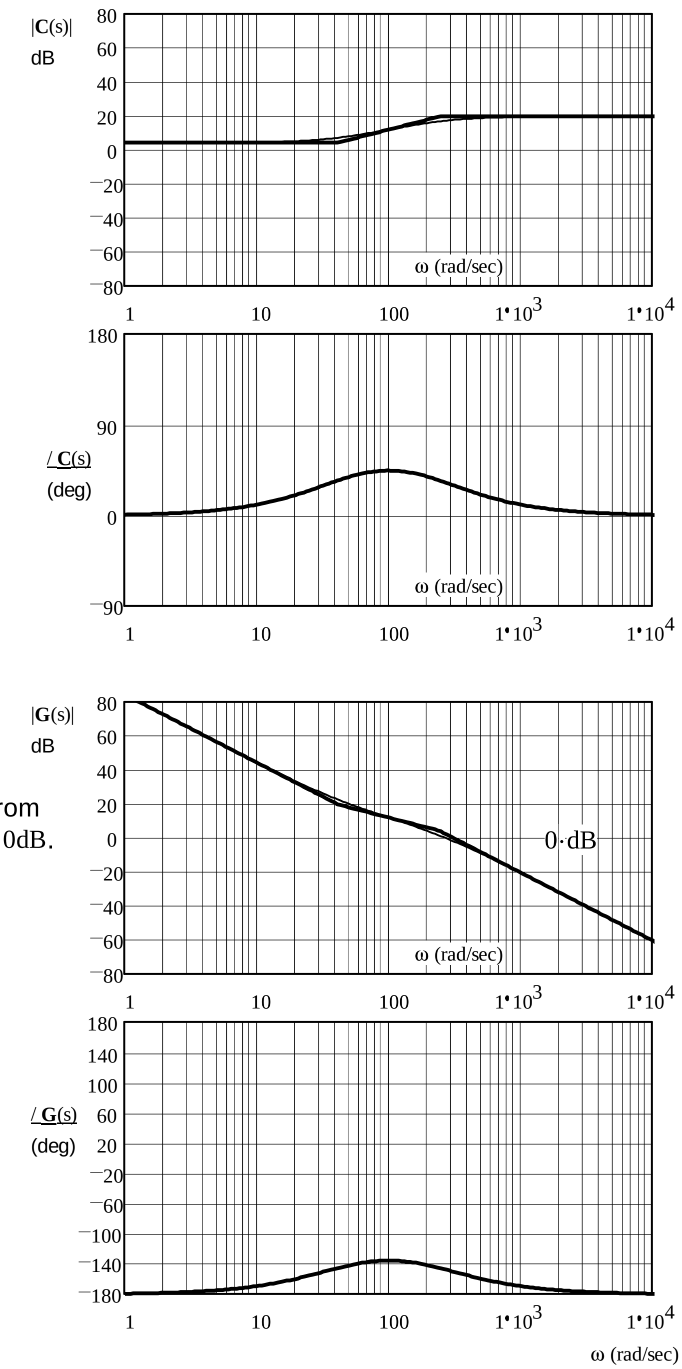
But now the maximum phase angle difference from 180 doesn't occur where the magnitude crosses 0dB.

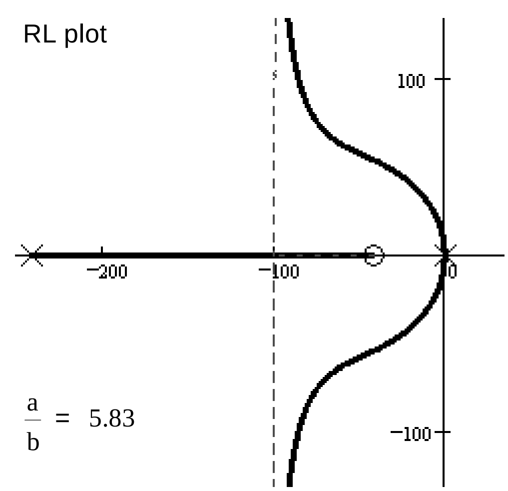
This problem is resolved in the math shown in the book, which makes:

$$\omega_c \;=\; \omega_p$$

freq. of maximum phase difference = freq. where $\mathbf{G}(s)$ crosses 0dB.

Bode plots (numbers are representative)





The Bottom Line

I've combined information from the table on page 155 with table on page 152.

For double integrator problem

approximation from simpler system of section 5.3.7

1. Select your a/b ratio, use this ratio as a single number in following equations.

$\left(\frac{a}{b}\right)$	$\phi_p = PM$	ζ	$\%OS = PO$ = percent overshoot based on ζ approx.
5.83	45^o	0.44	$20.5\cdot\%$
9	53.1^o	0.55	$14\cdot\%$
13.9	60^o	0.6	$9.5\cdot\%$

use $\left(\frac{a}{b}\right)$ as a single number

Or use eq. 5.67

Extension of table using approximate relationship between PM and overshoot developed in section 5.3.7

2. Use eq. 5.69 to relate ω_c to k_p and k_c.

$$\frac{k_p \cdot k_c}{\omega_c^2}\cdot\sqrt{\frac{b}{a}} = 1$$

OR, rearranged:

$$\omega_p = \omega_c = \sqrt{k_p \cdot k_c \cdot \sqrt{\frac{b}{a}}}$$

Note: $\frac{b}{a} = \frac{1}{\left(\frac{a}{b}\right)}$

Depending on your knowns and unknowns, other rearrangements may be useful:

$$k_p \cdot k_c = \omega_c^2 \cdot \sqrt{\frac{a}{b}} \qquad k_p = \frac{\omega_c^2}{k_c}\cdot\sqrt{\frac{a}{b}} \qquad k_c = \frac{\omega_c^2}{k_p}\cdot\sqrt{\frac{a}{b}}$$

To get answers and plots for BD5, prob.3, I arbitrarily used:

$\omega_c := 10$ $\quad k_p := 1$ and found k_c from the eq.

3. Find:

$$a = \omega_c \cdot \sqrt{\frac{a}{b}} = \omega_p \cdot \sqrt{\frac{a}{b}} \qquad b = \omega_c \cdot \sqrt{\frac{b}{a}} = \omega_p \cdot \sqrt{\frac{b}{a}}$$

the pole location of $\mathbf{C}(s)$ the zero location of $\mathbf{C}(s)$

Why Bode Plots?

1. Provides a method to find the approximate transfer function as used in the Flexible Beam lab.
2. Terms GM and PM are in wide use and you need to know what they mean.
3. Sometimes used for design method as in the Flexible Beam lab.

   You will find from BD5, prob.3, that the approximations of overshoot given in the table above are not very good (off by about a factor of 2), but, those predicted by the second-order approximation are even worse (b/c of zero close to origin).

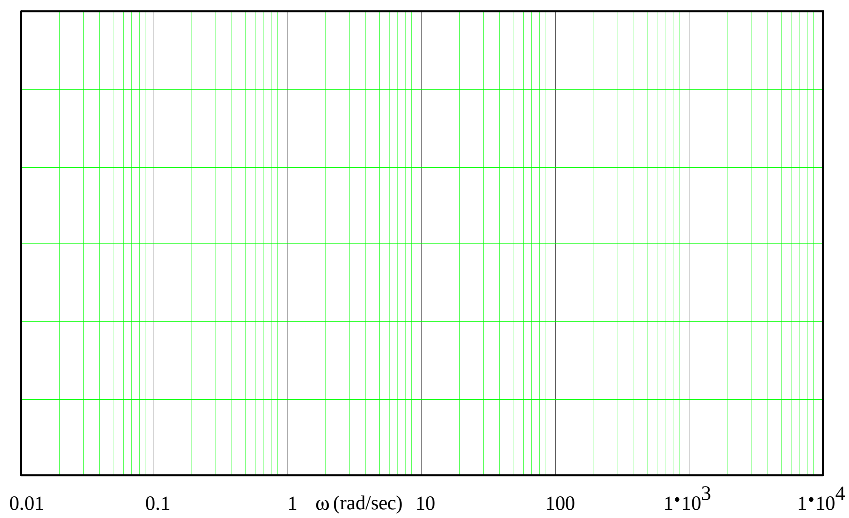
Name: ____________________ ECE 3510 homework # BP1 Due: Wed, 4/7/21

A.Stolp d

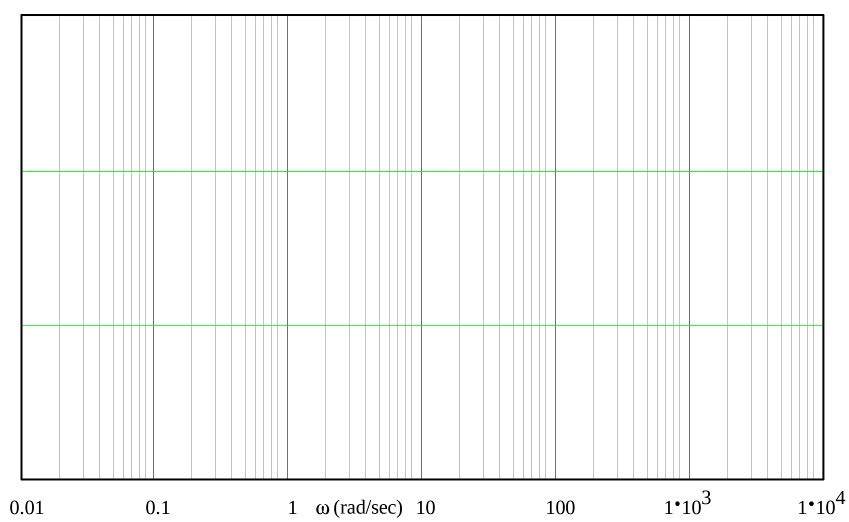
1. Sketch the Bode plots for the following transfer functions.
   Label the graphs, give the slopes of the lines in the magnitude plot and draw the "smooth" lines.

   a) $P(s) = \dfrac{s+10}{(s+1)\cdot(s+100)}$

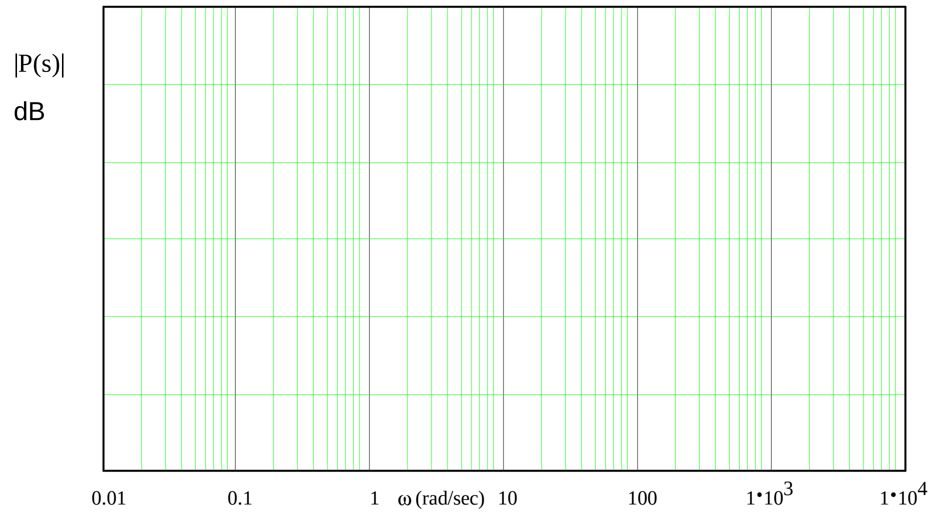
|P(s)|
dB

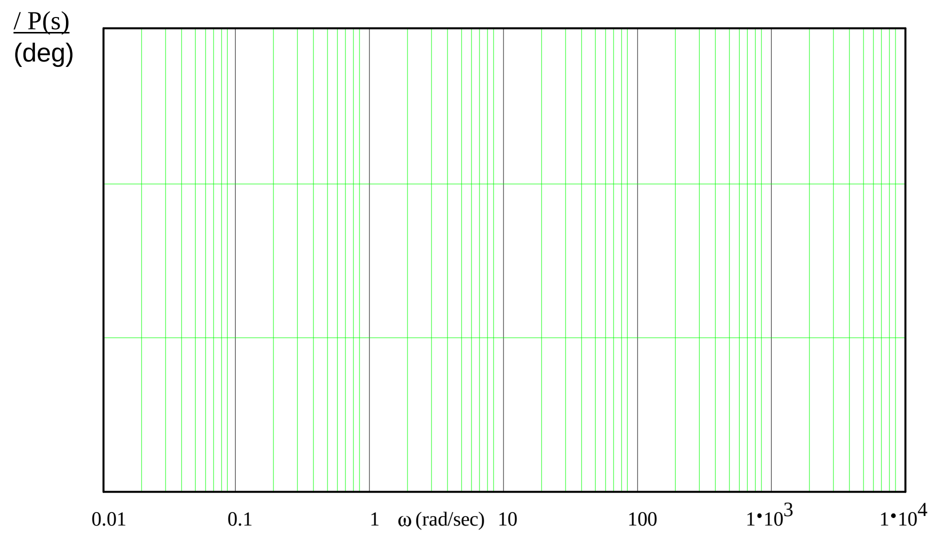


/ P(s)
(deg)

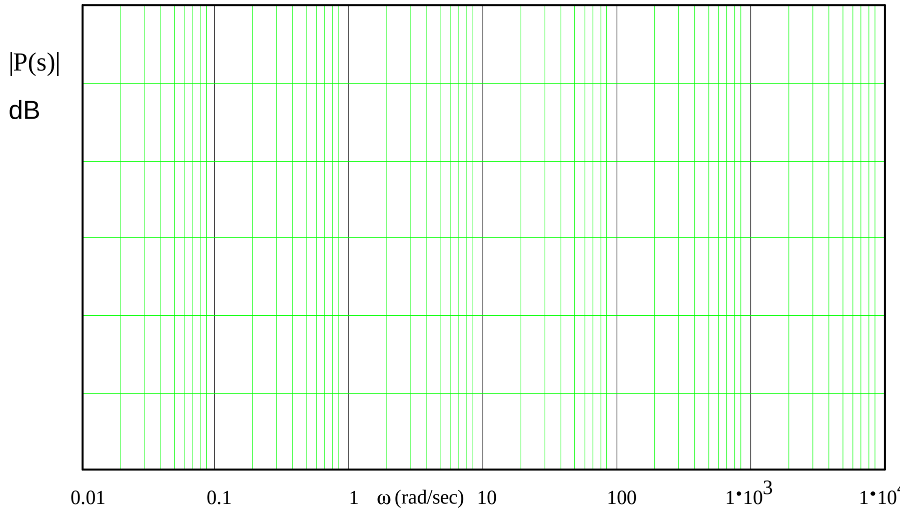


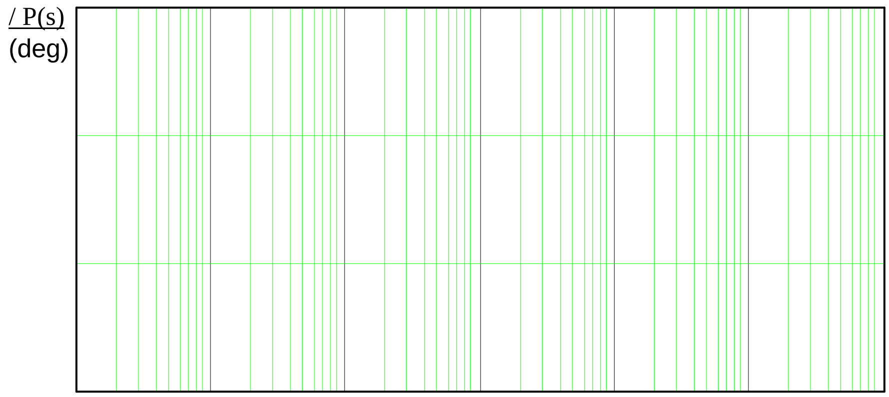
b) $P(s) = \dfrac{s - 0.4}{s \cdot (s + 400)}$



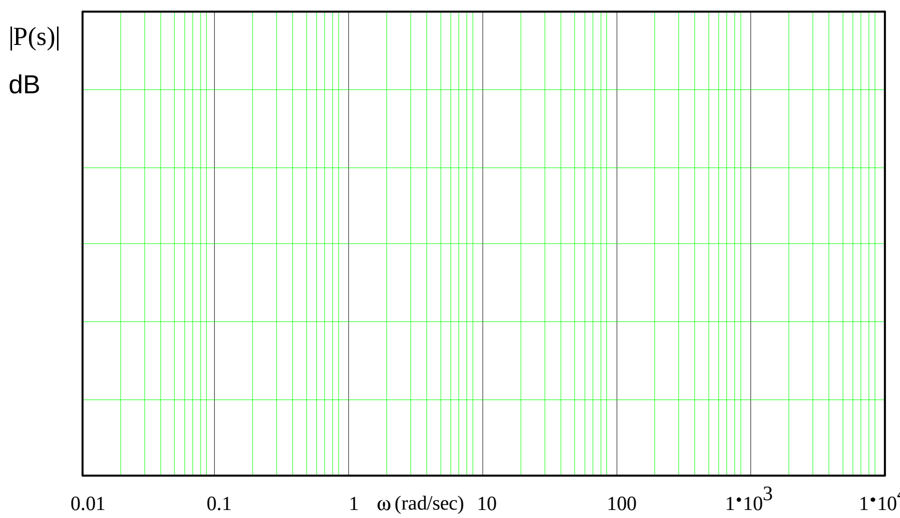


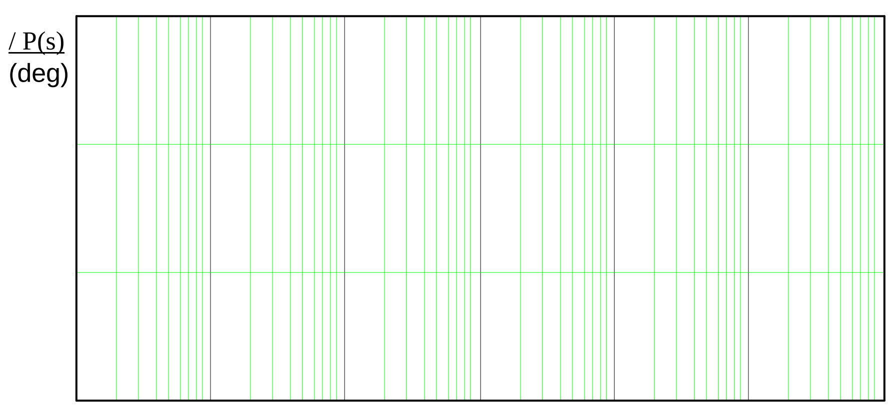
c) $P(s) = \dfrac{800 \cdot (s - 10) \cdot (s + 10)}{(s + 0.1) \cdot (s + 400)^2}$ plot on next page





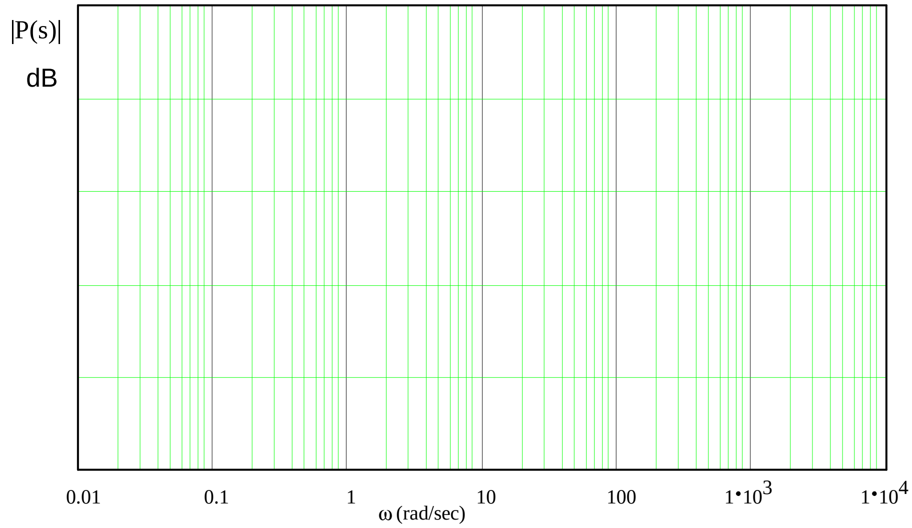
d) $P(s) = \dfrac{900}{(s+30)^2 \cdot (s+1)}$

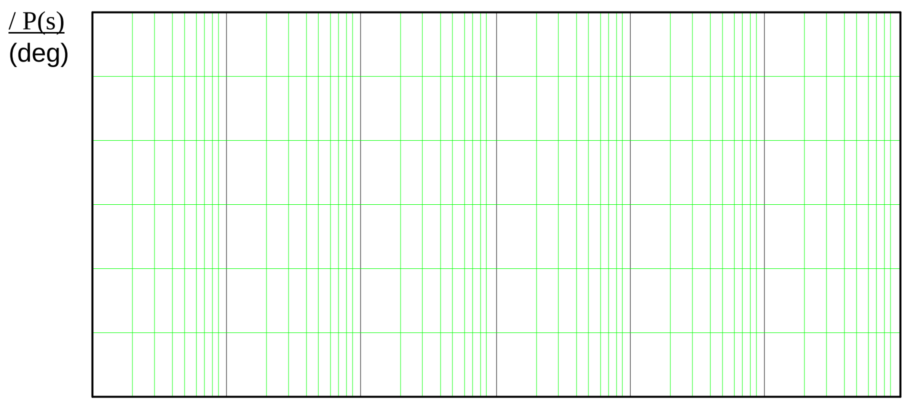




1. Sketch the Bode plots for the following transfer functions. Label the graphs, give the slopes of the lines in the magnitude plot and draw the "smooth" lines.

   a) $P(s) = \dfrac{s + 50}{s^2 + 0.4 \cdot s + 4}$

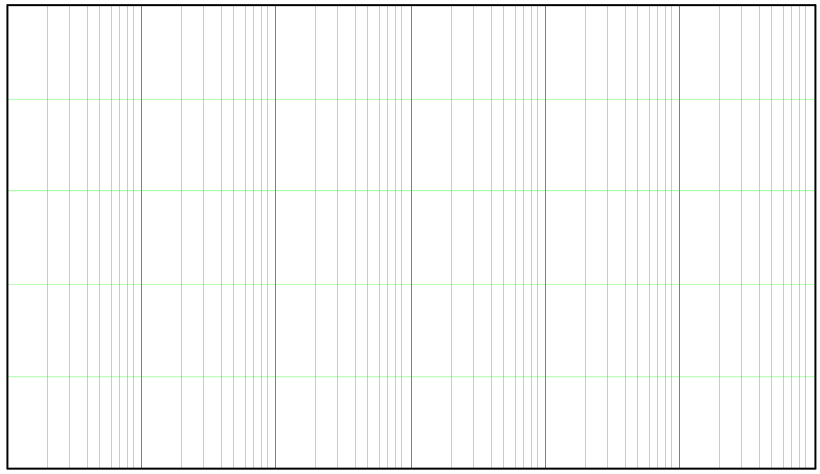




   b) $P(s) = \dfrac{s^2 + 2 \cdot s + 100}{s^2}$ plot on next page

|P(s)|

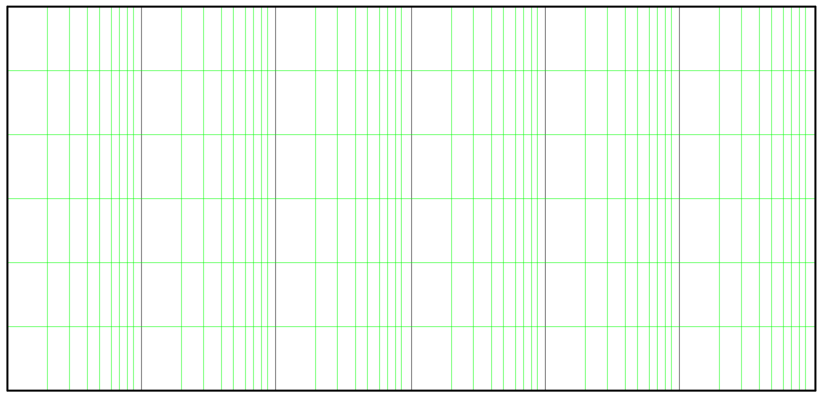
dB



0.01 0.1 1 ω (rad/sec) 10 100 $1\cdot10^3$ $1\cdot10^4$

/ P(s)

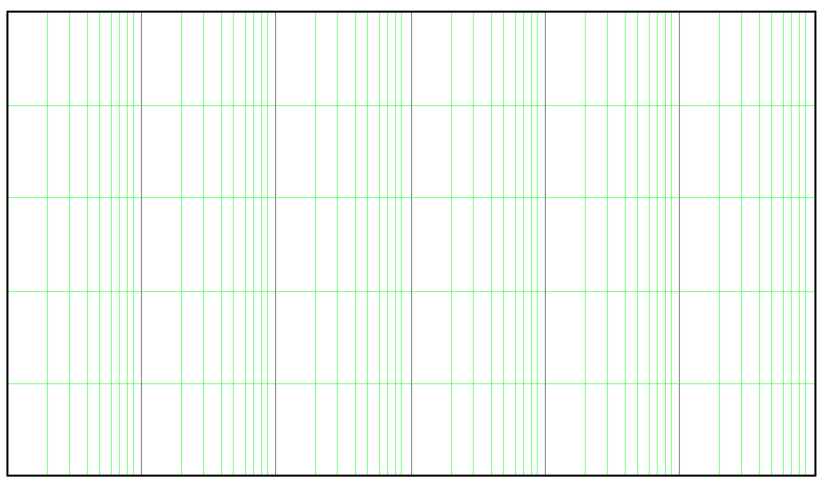
(deg)



c) $P_2(s) := \dfrac{(s+4)\cdot(s+2000)}{s^2+2\cdot s+1600}$ plot below

|P(s)|

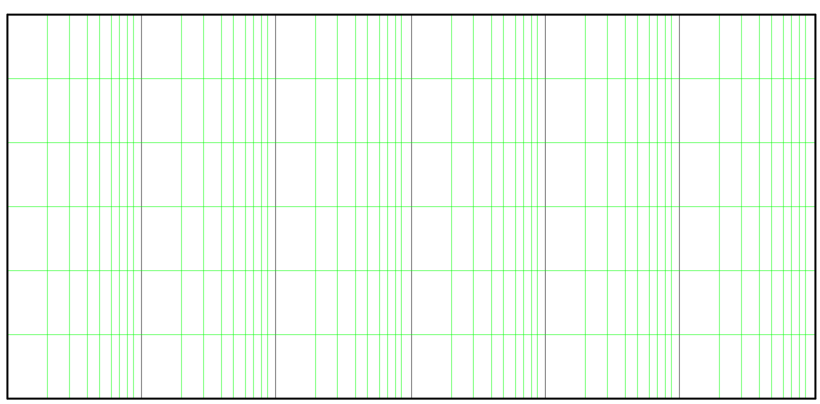
dB



0.01 0.1 1 ω (rad/sec) 10 100 $1\cdot10^3$ $1\cdot10^4$

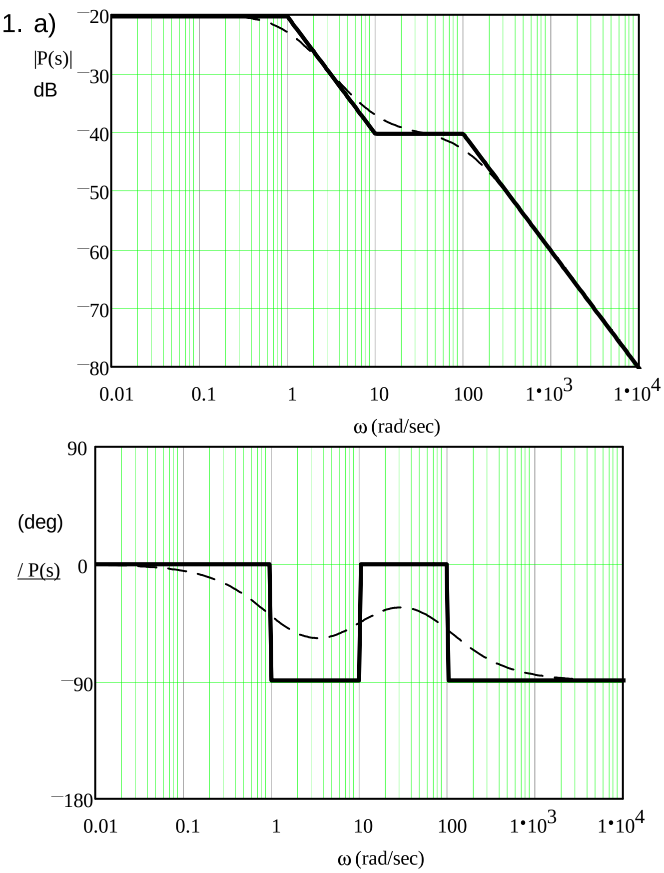
/ P(s)

(deg)

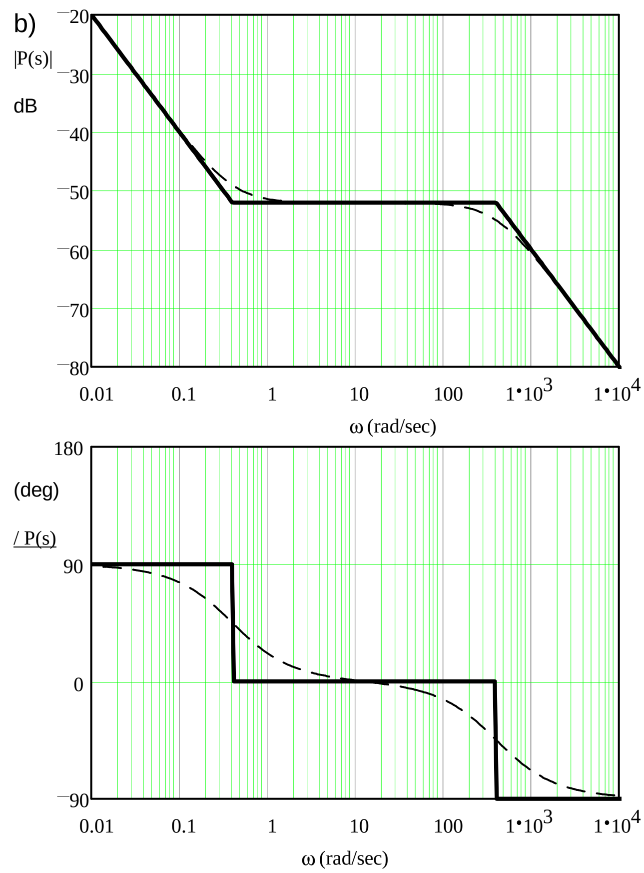


Answers

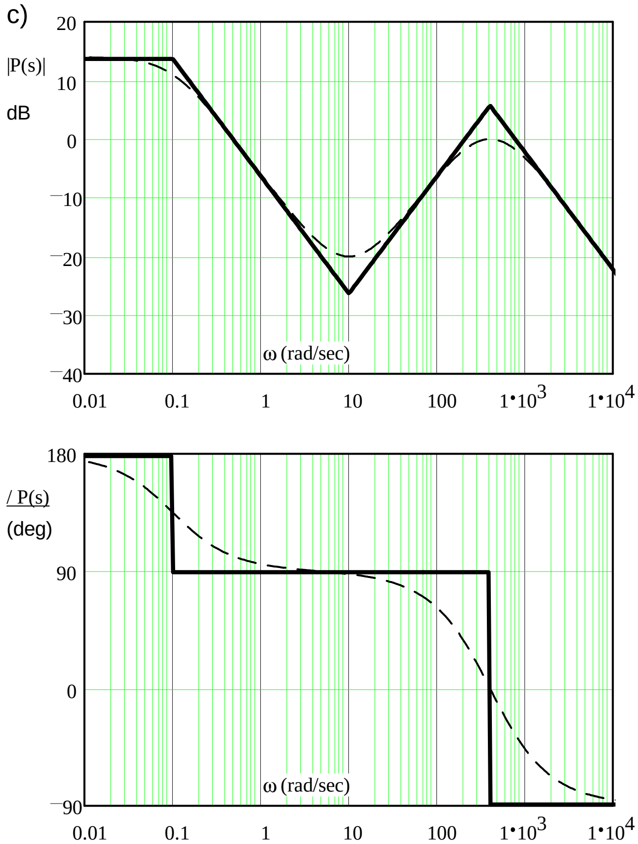
1. a)



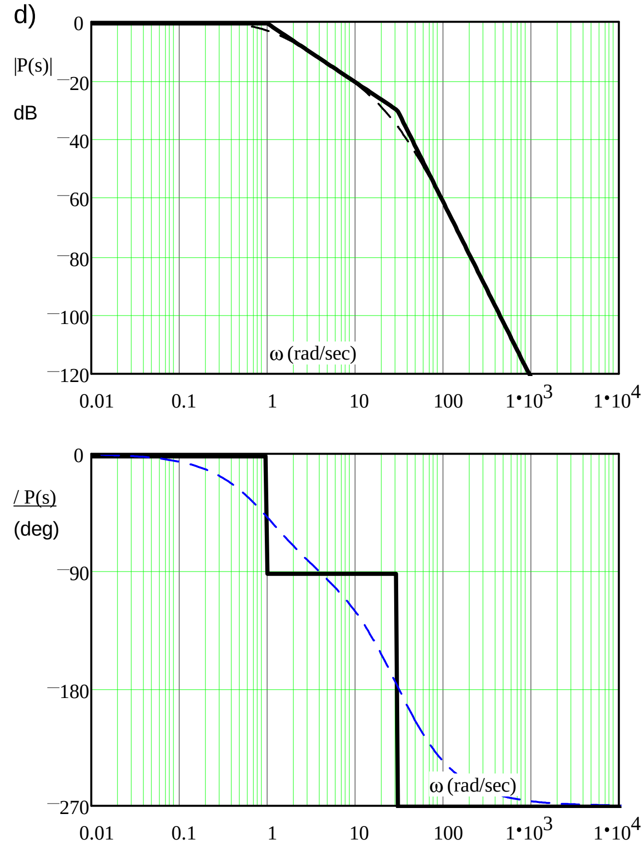
b)



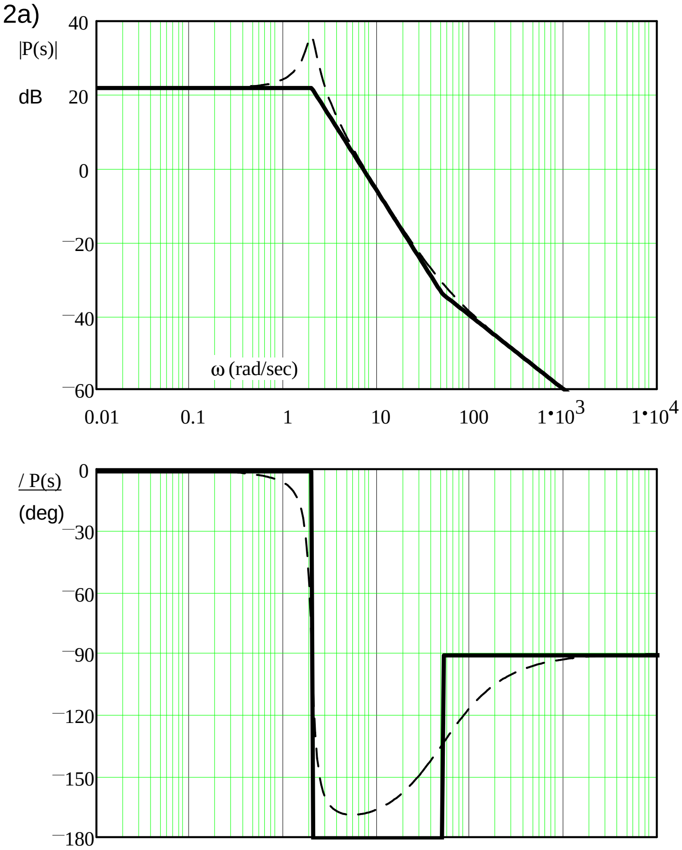
c)



d)

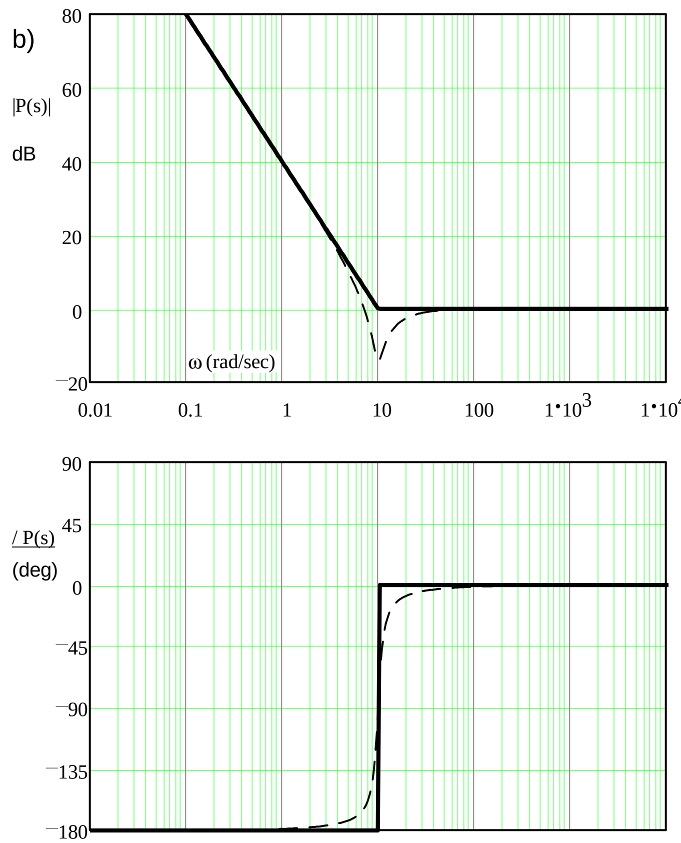


2a)



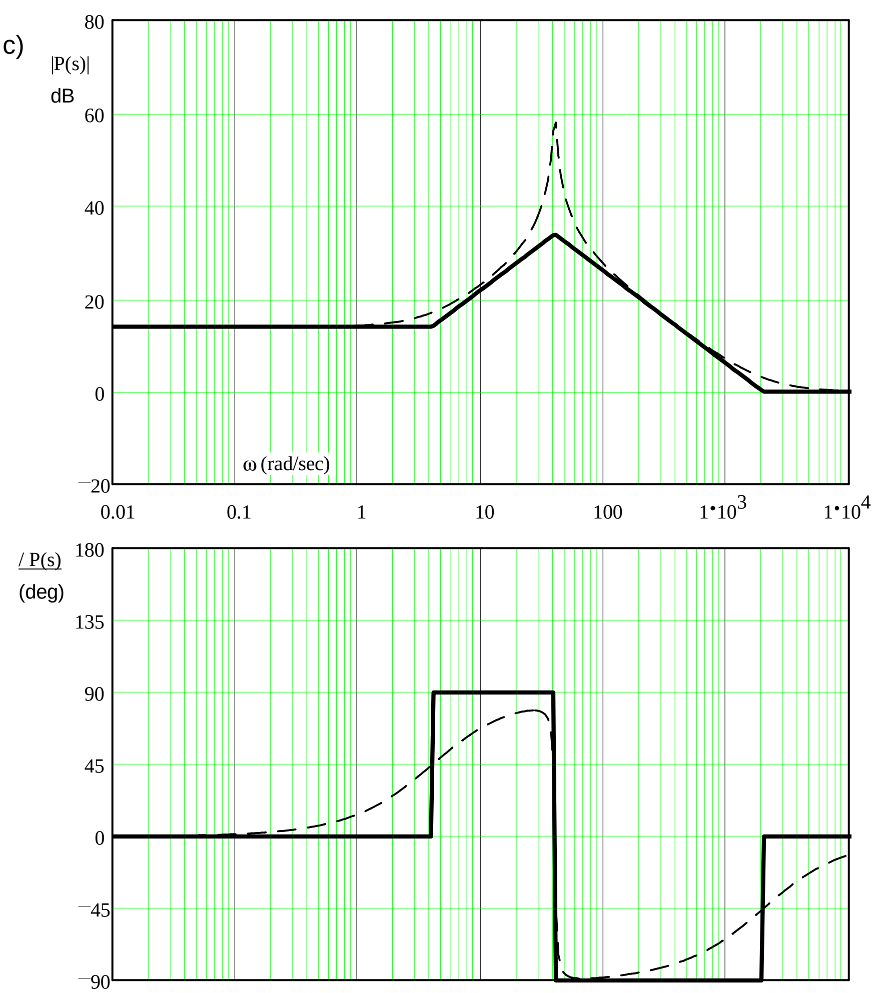
|P(s)|
dB
ω (rad/sec)
0.01
0.1
1
10
100
1•10^3
1•10^4
/ P(s)
(deg)

b)



|P(s)|
dB
ω (rad/sec)
0.01
0.1
1
10
100
1•10^3
1•10^4
/ P(s)
(deg)

c)

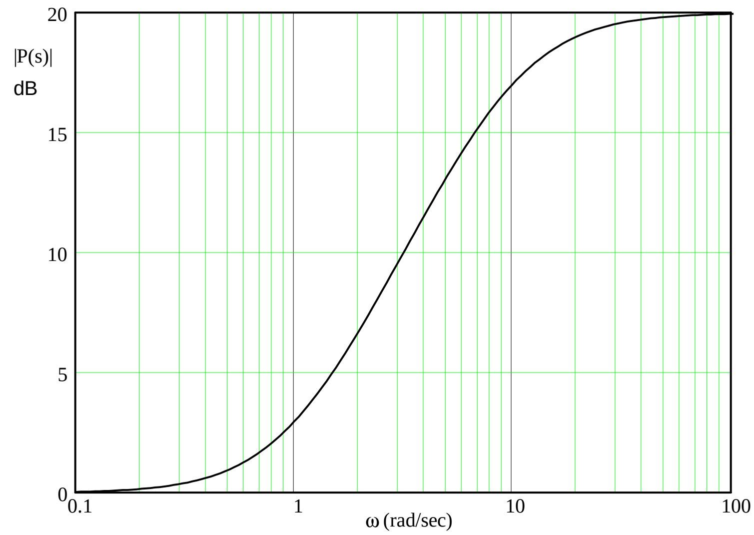


|P(s)|
dB
ω (rad/sec)
0.01
0.1
1
10
100
1•10^3
1•10^4
/ P(s)
(deg)

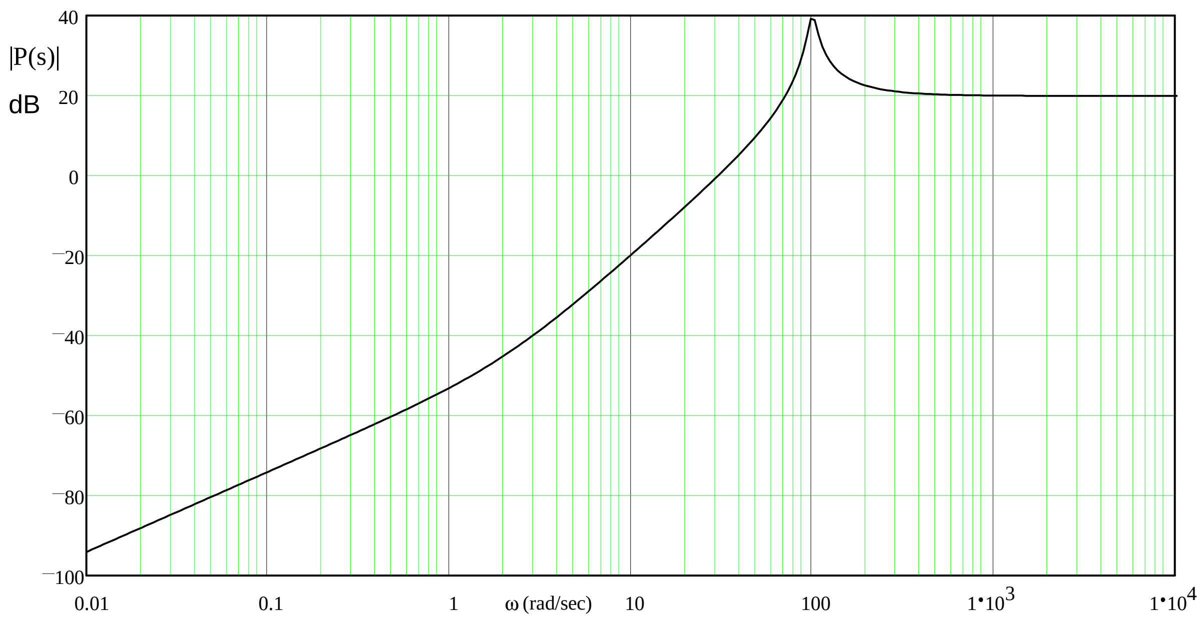
Name: ____________________ Homework # BP2 Due: Sat, 4/10/21 c

You must show the work needed to get the answers below. Add your own paper if necessary.

1. (a & c are from Problem 5.2 in Bodson text.)
   a) The magnitude Bode plot of a system is shown below. What are the possible transfer functions of stable systems having this Bode plot?

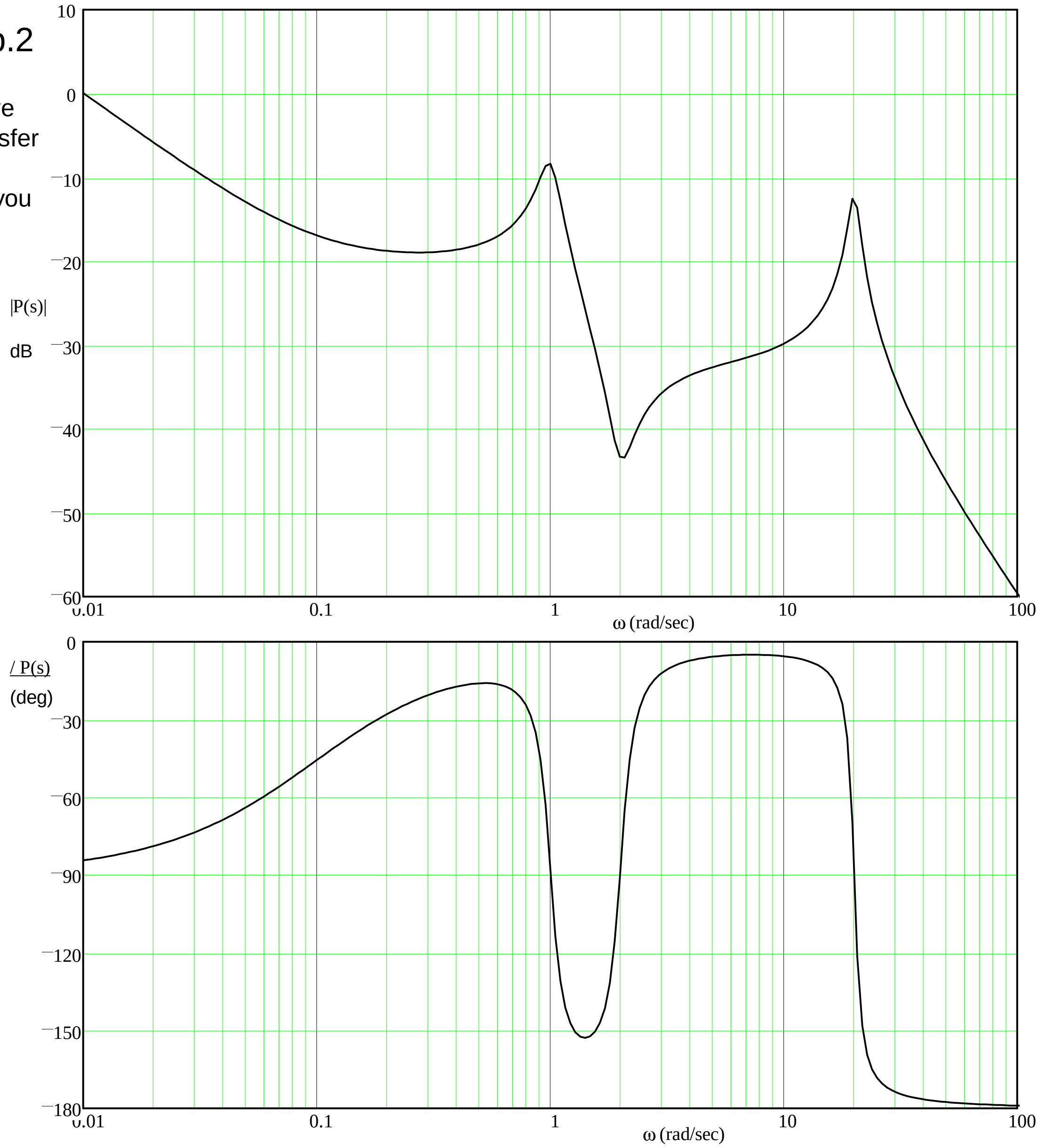


b) A Bode plot is shown below, estimate of the transfer function of the system. Assume no negative signs in the transfer function (all poles and zeros in LHP). Show your work (how you made your estimate).

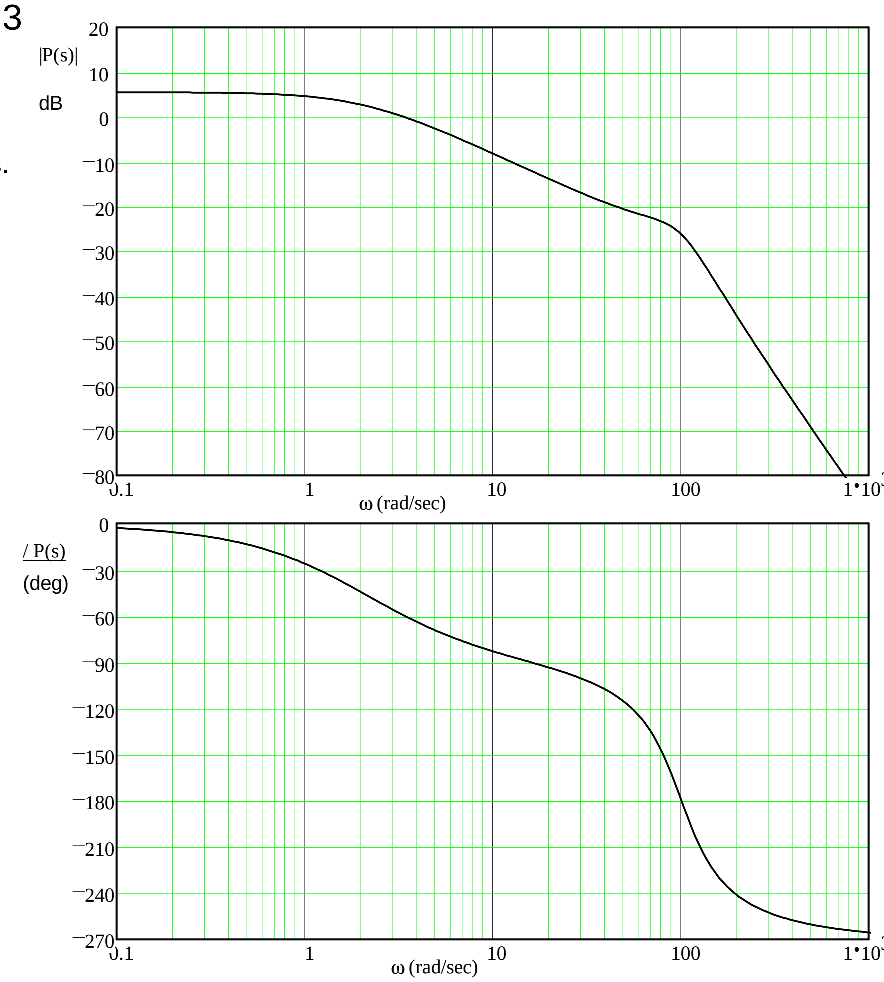


ECE 3510
Homework BP2   p.2

c) The Bode plots of a system are shown. Give an estimate of the transfer function of the system. Show your work (how you made your estimate).

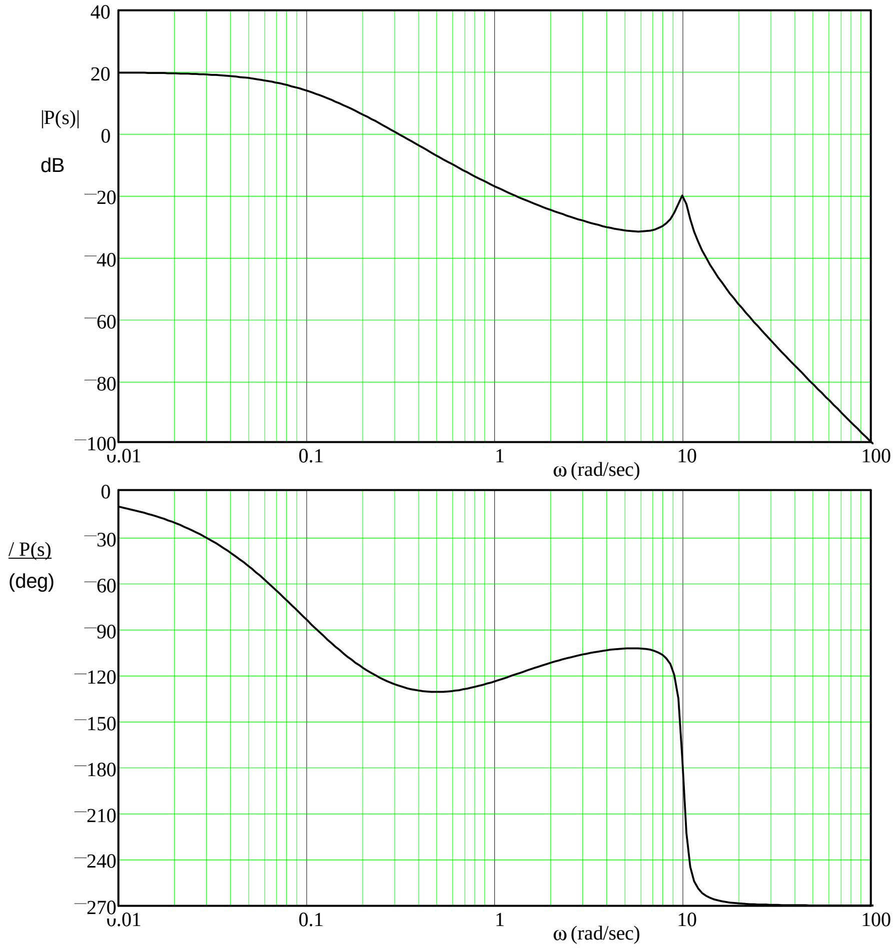


2. The system whose Bode plots are given at right is stable in closed-loop. Find its gain margin, phase margin, and delay margin. Show your work on the drawings.



3. Problem 5.3 in the text.

a) The system whose Bode plots are given at left is stable in closed-loop. Find its gain, phase, and delay margins. Show your work on the drawings.



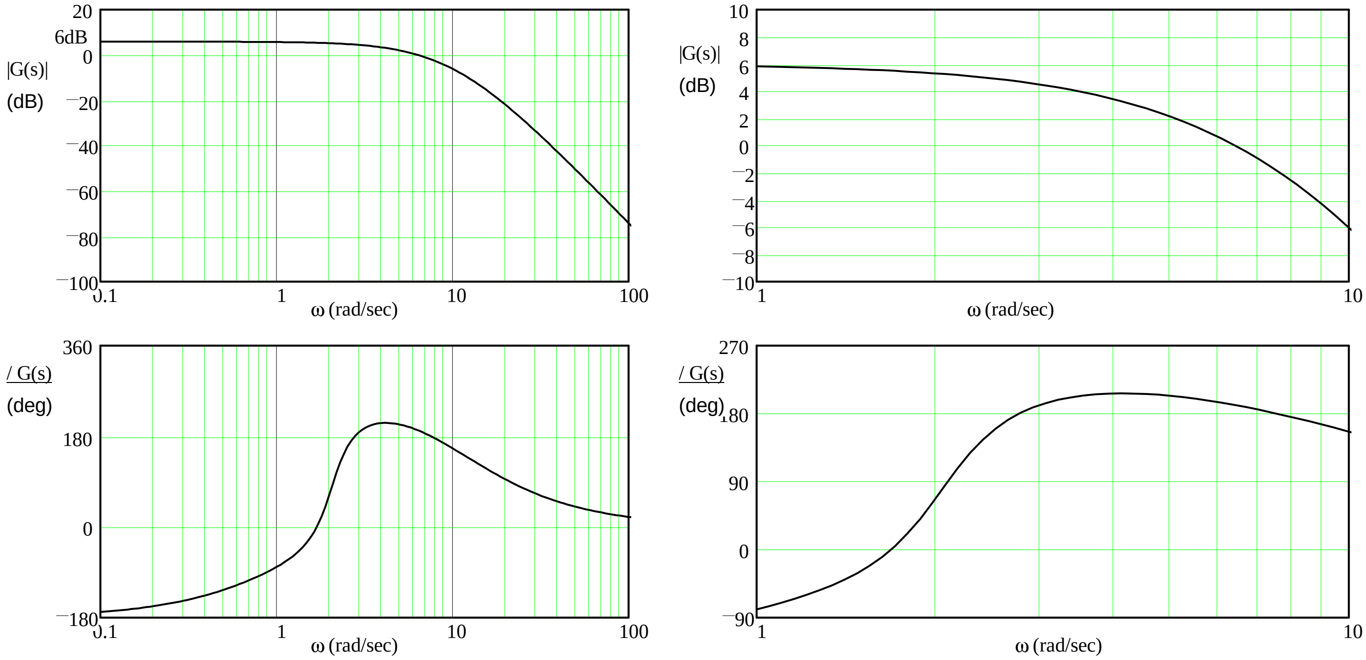
b) Describe the behavior of the closed-loop system of part (a) if the open-loop gain is increased to a value close to the maximum value given by the gain margin. In particular, what can you say about the locations of the poles of the closed-loop system?

c) Consider an open-loop stable system which is such that the magnitude of its frequency response, including the gain factor k, is less than 1 for all ω ($|kG(s)| < 1$). Can you determine whether the closed-loop system is stable with only that information? If yes, show how.

4. 4. Problem 5.13 b & new c & d (p.149) in the text.

b) Bode plots of the open-loop transfer function of a feedback system are shown below, with the detail from 1 to 10 rad/sec shown on the left. For this system:

- How much can the open-loop gain be changed (increased and/or decreased) before the closed-loop system becomes unstable ?
- What is a rough estimate of the phase margin of the feedback system? Show on the graph how the results are obtained. The numerical results do not have to be precise.
- How much time delay can there be in feedback system before the phase margin disappears.

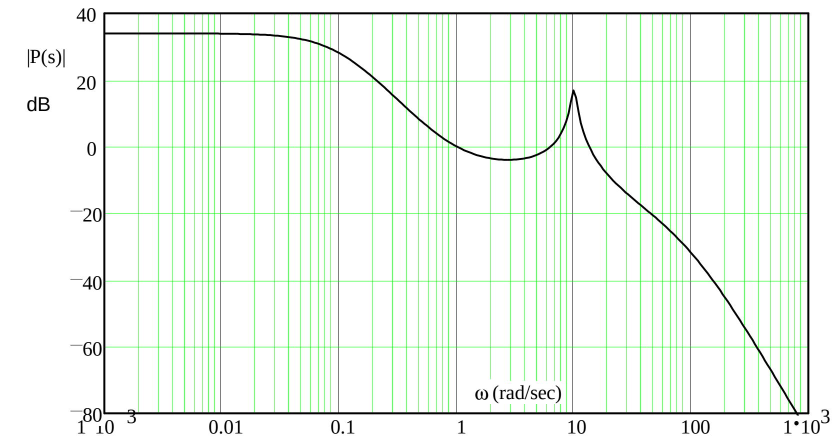


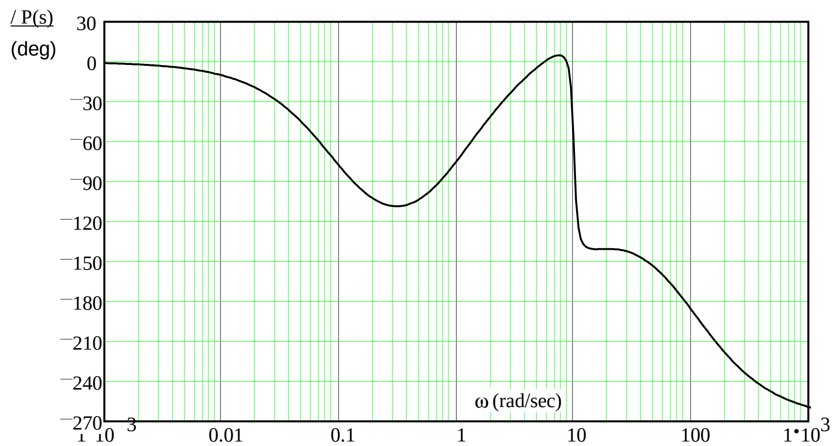
c) For the system of part (a), give the steady-state response of the open-loop system an input $x(t) = 4\cos(10t)$. express the answer in the time-domain.

d) Give the steady-state response of the closed-loop system for the same input. Hint: closed loop output is: $\text{input}\cdot\frac{G(10\cdot j)}{1+G(10\cdot j)}$

5. Like problem 5.9a (p.147) in the Bodson text.

   a) Give the gain margin, the phase margin and the delay margin of the system whose Bode plots are shown at right (the plots are for the open-loop transfer function and the closed-loop transfer function is assumed to be stable).





Answers

The rest are NOT stable

1.a) $P(s) = 10\cdot\frac{s+1}{s+10}$ $\quad 10\cdot\frac{s-1}{s+10}$ $\quad -10\cdot\frac{s+1}{s+10}$ $\quad -10\cdot\frac{s-1}{s+10}$ $\qquad 10\cdot\frac{s+1}{s-10}$ $\quad -10\cdot\frac{s-1}{s-10}$ $\quad -10\cdot\frac{s-1}{s-10}$ $\quad -10\cdot\frac{s+1}{s-10}$

b) $P(s) = \frac{10\cdot s\cdot(s+2)}{\left(s^2+10\cdot s+10000\right)}$ $\qquad$ c) $P(s) = \frac{10\cdot(s+0.1)\cdot\left(s^2+0.4\cdot s+4\right)}{s\cdot\left(s^2+0.2\cdot s+1\right)\cdot\left(s^2+2\cdot s+400\right)}$

2. $GM \simeq 25\cdot dB$ $\qquad PM \simeq 120\cdot deg$ $\qquad DM := 600\cdot ms$

3. a) $GM \simeq 21\cdot dB$ $\qquad PM \simeq 50\cdot deg$ $\qquad DM := 2.6\cdot sec$

   b) The system will have a transient ring at about 10 rad/sec. Two poles of the closed loop system will be close to $\pm$ 10j.

   c) Yes, it must be stable. Prove by closed-loop transfer function, Bode gain margin or Nyquist: N=0, P=0, Z=0

4. b) Gain may be increased by ~ 2dB and reduced by ~ 4.4dB. $\qquad PM \simeq 13\cdot deg$ $\qquad DM \simeq 36\cdot ms$

   c) $2\cdot\cos(10\cdot t+158\cdot deg)$ $\qquad$ d) $3.5\cdot\cos(10\cdot t+140\cdot deg)$

5. a) $GM \simeq 30\cdot dB$ $\qquad PM \simeq 40\cdot deg$ $\qquad DM \simeq 50\cdot ms$

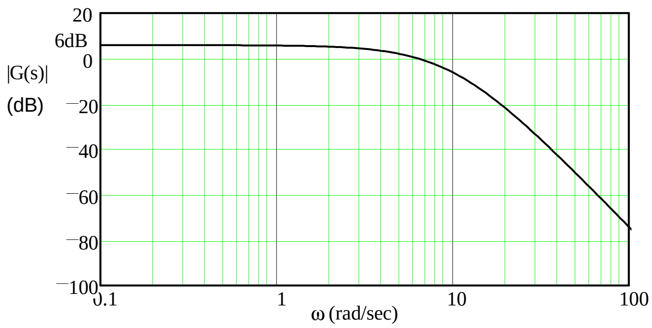
1. Problem 5.13 b & new c & d in the text.

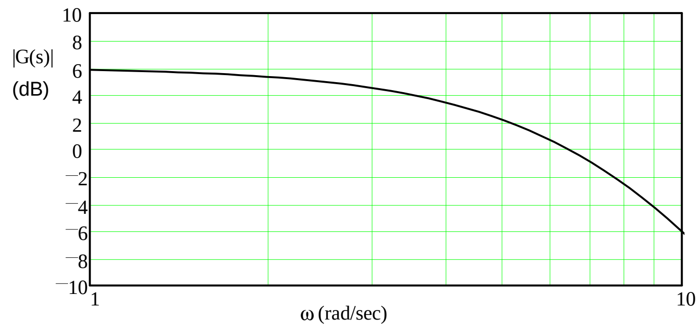
b) Bode plots of the open-loop transfer function of a feedback system are shown below, with the detail from 1 to 10 rad/sec shown on the left. For this system:

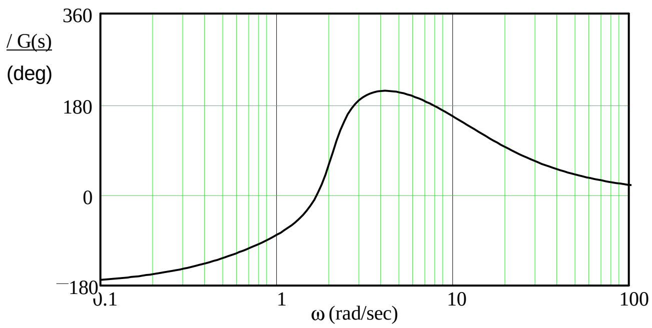
• How much can the open-loop gain be changed (increased and/or decreased) before the closed-loop system becomes unstable ?

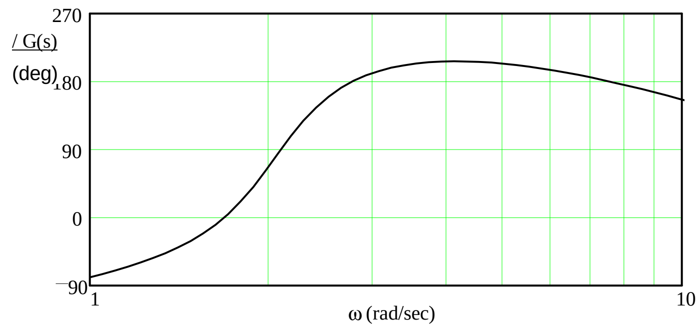
• What is a rough estimate of the phase margin of the feedback system? Show on the graph how the results are obtained. The numerical results do not have to be precise.

• How much time delay can there be in feedback system before the phase margin disappears.







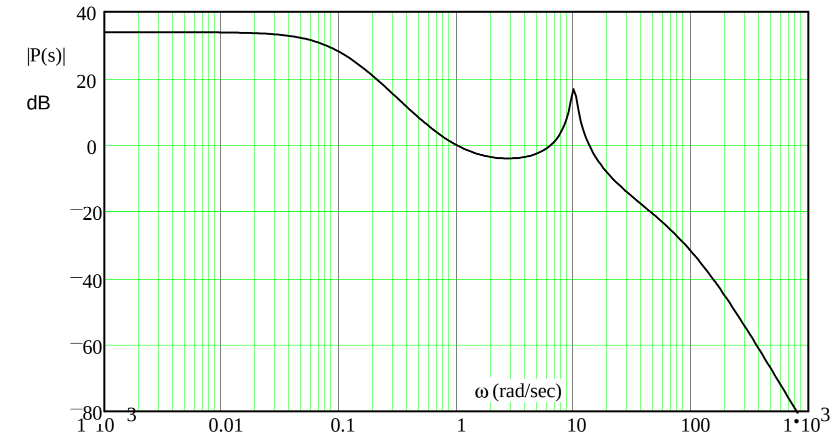


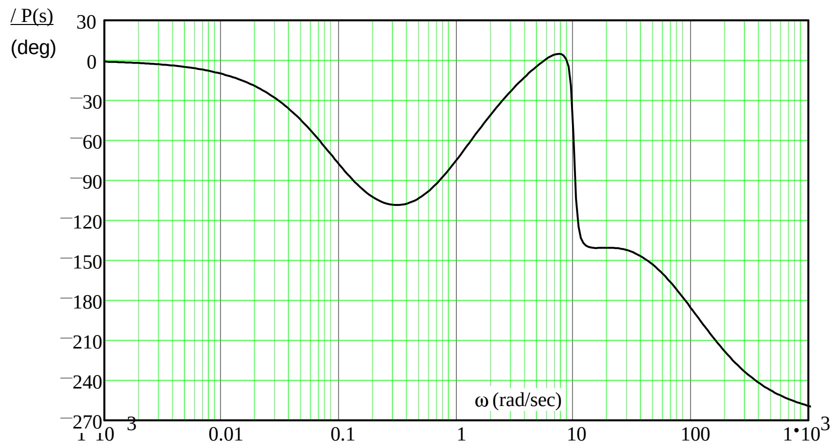
c) For the system of part (a), give the steady-state response of the open-loop system an input $x(t) = 4\cos(10t)$. express the answer in the time-domain.

d) Give the steady-state response of the closed-loop system for the same input. Hint: closed loop output is: $\text{input}\cdot\dfrac{G(10\cdot j)}{1+G(10\cdot j)}$

2. Like problem 5.9a in the Bodson text.

   a) Give the gain margin, the phase margin and the delay margin of the system whose Bode plots are shown at right (the plots are for the open-loop transfer function and the closed-loop transfer function is assumed to be stable).





Add another sheet of paper the following:

3. A system has a delay of $D := 0.01 \cdot sec$ How many degrees of phase does this represent at:

   a) $f := 1 \cdot Hz$ $\quad f := 10 \cdot Hz$ $\quad f := 100 \cdot Hz$ $\quad f := 1 \cdot kHz$

   b) $\omega := 1 \cdot \frac{rad}{sec}$ $\quad \omega := 10 \cdot \frac{rad}{sec}$ $\quad \omega := 100 \cdot \frac{rad}{sec}$ $\quad \omega := 1000 \cdot \frac{rad}{sec}$

4. a) If the phase response of a pure time delay were plotted on linear phase vs. linear frequency plot, what would be the shape of the curve?

   b) If the phase response of a pure time delay were plotted on linear phase vs. logarithmic frequency plot, what would be the shape of the curve?

5. Ch. 10, prob.30 in Nise, 3rd Ed., page 677. Similar to example 10.17 on page 658 (in section 10.12)
Ch. 10, prob.30 in Nise, 4th Ed., page 682. Similar to example 10.17 on page 663 (in section 10.12)
Ch. 10, prob.30 in Nise, 5th Ed., page ___. Similar to example 10.17 on page ___ (in section 10.12)
Ch. 10, prob.30 in Nise, 6th Ed., page 614. Similar to example 10.17 on page 600 (in section 10.12)

Given a unity feedback system with a forward-path transfer function $G(s) = \frac{K}{s\cdot(s+3)\cdot(s+12)}$

and a delay of 0.5 second, make a second-order approximation and estimate the percent overshoot if $K := 40$

Use Bode Plots and frequency response techniques or you may calculate where $|G(j\omega)| = 1$ to find ω and then the the phase margin. You may also use Matlab.

You may use the estimate from equation in Bodson text: $\zeta \simeq \frac{PM}{100\cdot deg}$ (PM in degrees and may include delay effects)

Also helpful: $\%OS = 100\%\cdot e^{\frac{-\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}$

6. In section 5.3.9 of Bodson's book, he discusses using a lead controller to stabilize a system (plant) represented by a double integrator. Give one or more examples of real systems that are essentially double integrators.

7. Problem 5.14 in the text.

a) Consider the lead controller for the double integrator. For the design that makes the crossover frequency equal to ω_C, obtain the polynomial that specifies the closed-loop poles (as a function of a/b and ω_C). Show that one closed-loop pole is at s = - ω_C no matter what a/b is.

Hints: Find: $G(s) = P(s)\cdot C(s)$

Find the denominator of the closed-loop transfer function: $D_G + N_G$

Substitute in a, b, and k_c like eq. 5.71 in book.

Use polynomial division to show that $D_G + N_G$ can be divided by $(s + \omega_c)$ with no remainder.

b) Compute the other closed-loop poles, as functions of ω_C, when $a/b = 5.83$, 9, and 13.9.

Hint: The "other" roots are the roots of the quotient.

c) Use Matlab SISO or other software of your choice to confirm the results of part c) and the % overshoot figures expected from the phase margins. ($20\cdot\%$ $14\cdot\%$ $9.5\cdot\%$)

Answers

1. b) Gain may be increased by ~ 2dB and reduced by ~ 4.4dB. $PM \simeq 13\cdot deg$ $DM \simeq 36\cdot ms$
c) $2\cdot\cos(10\cdot t + 158\cdot deg)$ d) $3.5\cdot\cos(10\cdot t + 140\cdot deg)$

2. a) $GM \simeq 30\cdot dB$ $PM \simeq 40\cdot deg$ $DM \simeq 50\cdot ms$

3. a) $3.6\cdot deg$ $36\cdot deg$ $360\cdot deg$ $3600\cdot deg$ b) $0.573\cdot deg$ $5.73\cdot deg$ $57.3\cdot deg$ $573\cdot deg$

4. a) A straight line of negative slope, ωD, where D is the time delay.
b) A negative sloping line with a slope of ωD. Since the frequency increases by a factor of 10 each decade, so would the downward slope of the line.

5. Calculated 0dB freq: $\omega := 1.045$ expect about 30% overshoot

6. Any system with mass where a force is the input and position is the "output". $F = m\cdot a = m\cdot\frac{d^2}{dt^2}x$

7. b) $a/b = 5.83$: $(-0.7071 - 0.7071\cdot j)\cdot\omega_c$ & $(-0.7071 + 0.7071\cdot j)\cdot\omega_c$

$a/b = 9$: $-\omega_c$ & $-\omega_c$ (making 3 all at ω_c)

$a/b = 13.9$: $-0.436\cdot\omega_c$ & $-2.292\cdot\omega_c$

c) Poles are confirmed with $\omega_c := 10$ $k_p := 1$ gain = 1

Overshoots are about double expected values $35\cdot\%$ $27\cdot\%$ $21\cdot\%$