# ECE 3510 Bode Plot Notes

A. Stolp
3/15/06,
3/11/07

Steps to make Bode Plots

Sample transfer function: $P(s) = K \cdot \frac{(s+z_1)\cdot(s+z_2)\cdot(s+z_3)}{s^2\cdot(s+p_1)\cdot(s+p_2)\cdot(s^2+2\cdot\zeta\cdot\omega_n+\omega_n^2)}$

if complex pole is expressed: $\left[(s+a)^2+b^2\right]$ then: $\omega_n = \sqrt{a^2+b^2}$

1. a) Rewrite, replacing all $s$'s with blanks:

$$P(\_) = K\cdot\frac{(\_+z_1)\cdot(\_+z_2)\cdot(\_+z_3)}{\_\_^2\cdot(\_+p_1)\cdot(\_+p_2)\cdot(\_+\omega_n)\cdot(\_+\omega_n)}$$

notice that you also simplify the complex poles and/or zeros for now

2. a) Scale your frequency axis to start at some frequency less than your smallest pole or zero.

b) Plug this starting frequency in for any poles or zeros at the origin (write it in the blank as $j\omega$).

$$P(j\omega,\_) = K\cdot\frac{(\_+z_1)\cdot(\_+z_2)\cdot(\_+z_3)}{(j\cdot\omega_{start})^2\cdot(\_+p_1)\cdot(\_+p_2)\cdot(\_+\omega_n)\cdot(\_+\omega_n)}$$

c) Ignore all the other blanks and calculate your initial magnitude, initial slope and the initial phase angle.

$$\text{Initial magnitude} = K\cdot\frac{(z_1)\cdot(z_2)\cdot(z_3)}{(j\cdot\omega_{start})^2\cdot(p_1)\cdot(p_2)\cdot(\omega_n)\cdot(\omega_n)}$$

Initial slope

$\omega$'s in the numerator --> + 20dB/decade each

$\omega$'s in the denominator --> - 20dB/decade each

Initial phase

numerator, j => 90º, **-** => +180º

denominator, j => -90º, **-** => **-**180º

3. a) Extend the line to the first pole or zero.

b) Replace that pole or zero with $j\omega$ and cross out the value of the pole or zero:

$$K\cdot\frac{(j\omega+\cancel{z_1})\cdot(\_+z_2)\cdot(\_+z_3)}{(j\omega)^2\cdot(\_+p_1)\cdot(\_+p_2)\cdot(\_+\omega_n)\cdot(\_+\omega_n)} \quad \text{OR} \quad K\cdot\frac{(\_+z_1)\cdot(\_+z_2)\cdot(\_+z_3)}{(j\omega)^2\cdot(j\omega+\cancel{p_1})\cdot(\_+p_2)\cdot(\_+\omega_n)\cdot(\_+\omega_n)}$$

c) Use this to find the new slope and phase angle.

Unless you replaced what was once a -s or crossed out a negative value:

zeros turn up the slope --> + 20dB/decade — zeros increase the phase angle --> + 90deg

poles turn down the slope --> - 20dB/decade — poles decrease the phase angle --> - 90deg

4. Repeat step 3 for each successive pole or zero.

After the last one you may want to check the magnitude or slope and phase again.

5. Draw a smooth line through the bode plots to estimate the actual magnitude and phase.

a) At poles and zeroes that are NOT complex

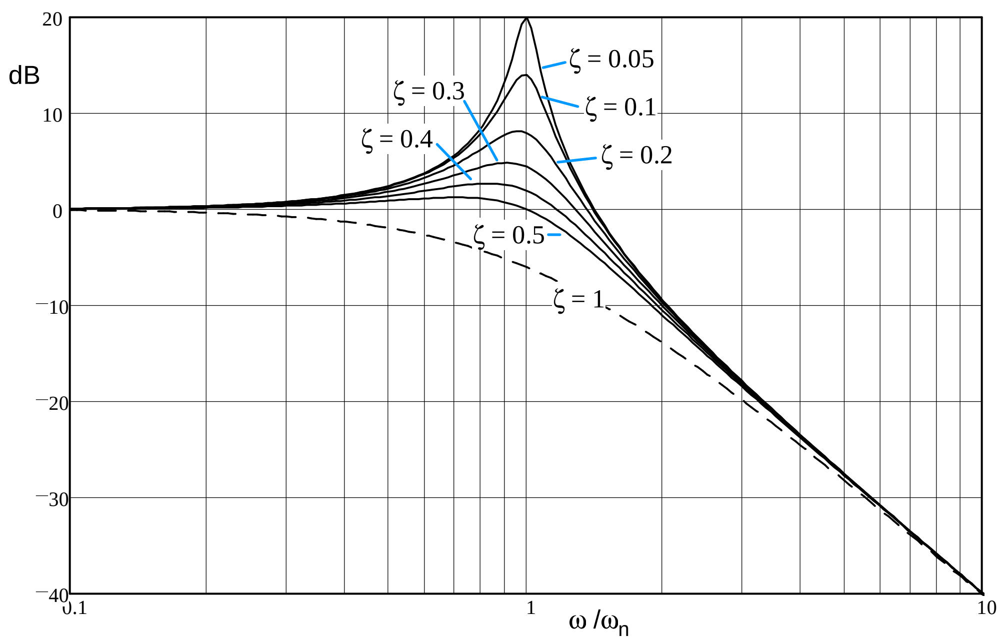
| | | | | |
|---|---|---|---|---|
| Actual magnitude: | -3dB at single poles<br>+3dB at single zeros | -6dB at double poles<br>+6dB at double zeros | etc.. | Magnitude effects extend about 1 decade fore and aft. |

Angle effects extend about 1.5 decade fore and aft.

5. b) At poles and zeroes that ARE complex

Correct the smooth line at the complex poles by scaling the plots on this page.

You may have to use a mirror image of these plots

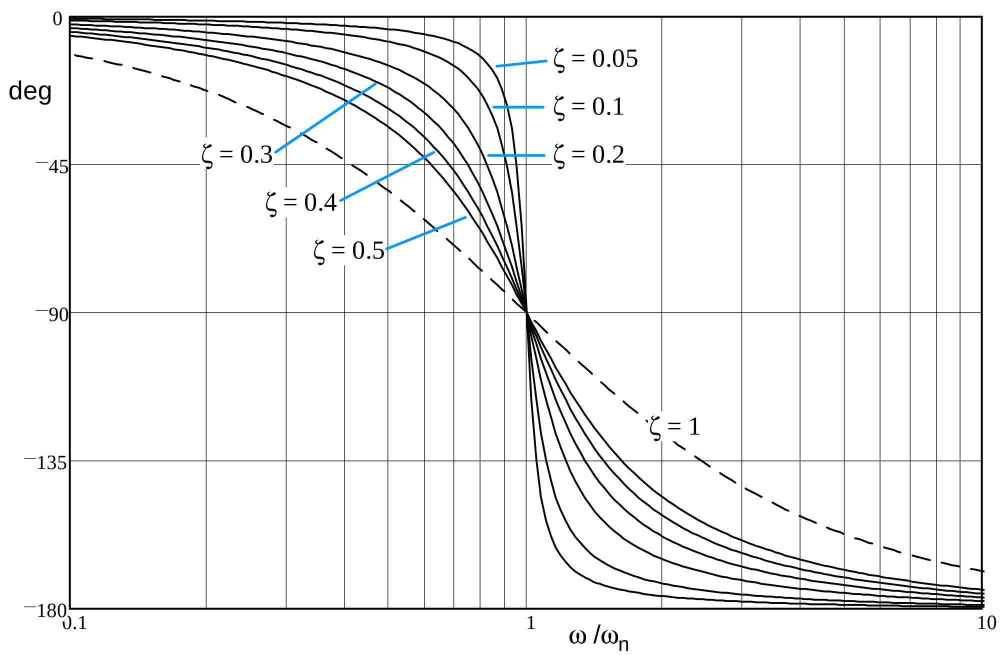



The damping factor comes from the complex pole expression in one of two ways:

$$\left(s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^{\,2}\right)$$

natural frequency $\omega_n = \sqrt{\omega_n^{\,2}}$

damping factor: $\zeta = \dfrac{2\cdot\zeta\cdot\omega_n}{2\cdot\omega_n}$




if complex pole is expressed:

$$\left[(s + a)^2 + b^2\right]$$

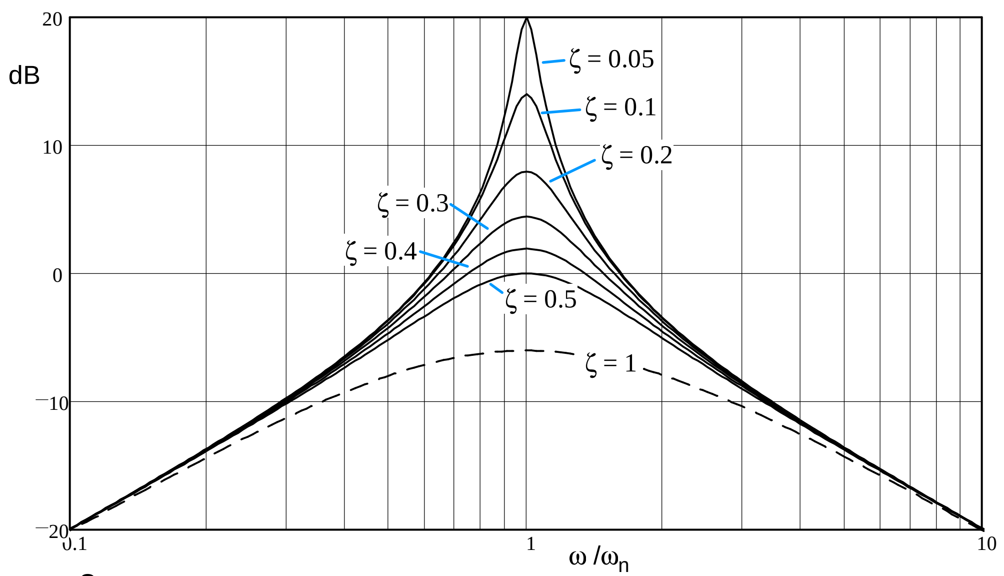
natural frequency $\omega_n = \sqrt{a^2 + b^2}$

damping factor $\zeta = \dfrac{a}{\omega_n}$

At $\omega_n$ the actual magnitude is:

$\dfrac{1}{2\cdot\zeta}$ in dB $20\cdot\log\left(\dfrac{1}{2\cdot\zeta}\right)$

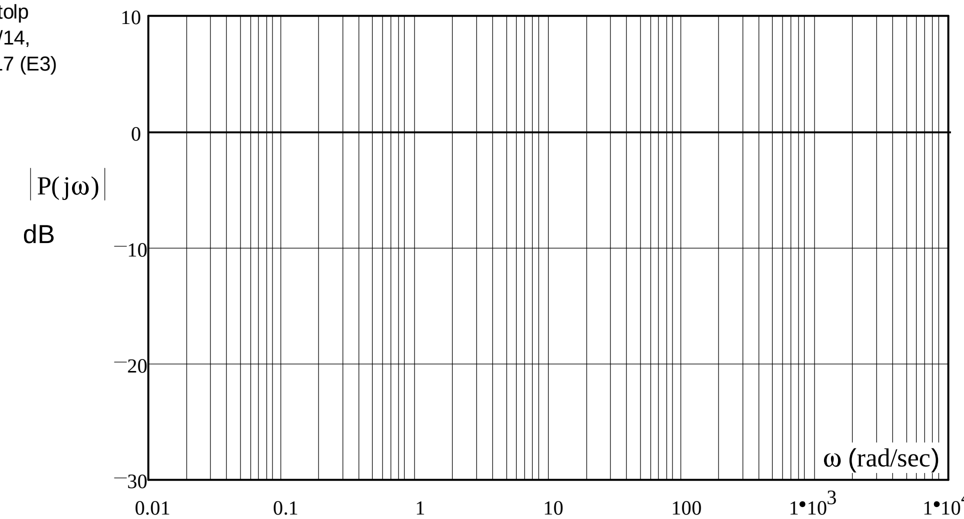
To correct the plot at complex zeros, use these plots upside-down

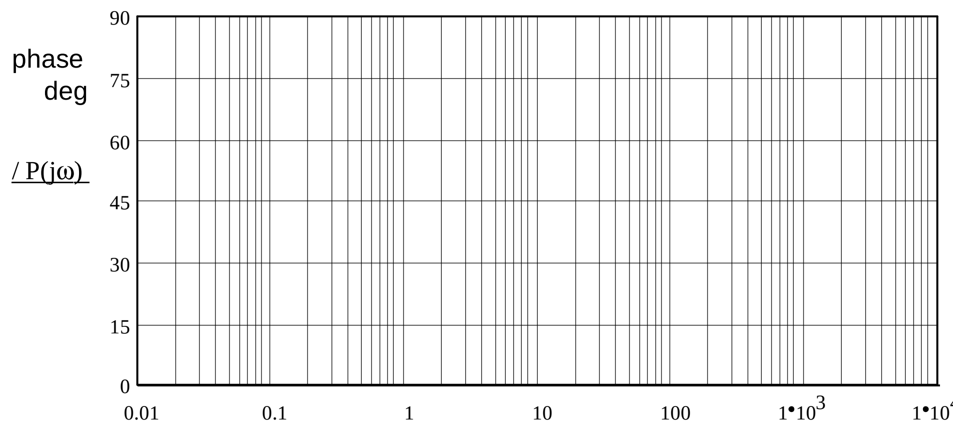


# ECE 3510
# Bode Plot Examples

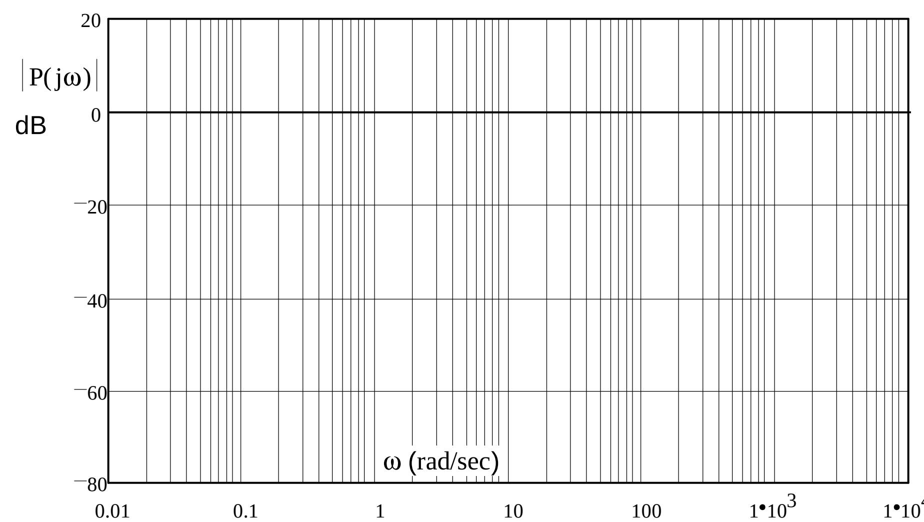
A. Stolp
3/27/14,
4/4/17 (E3)

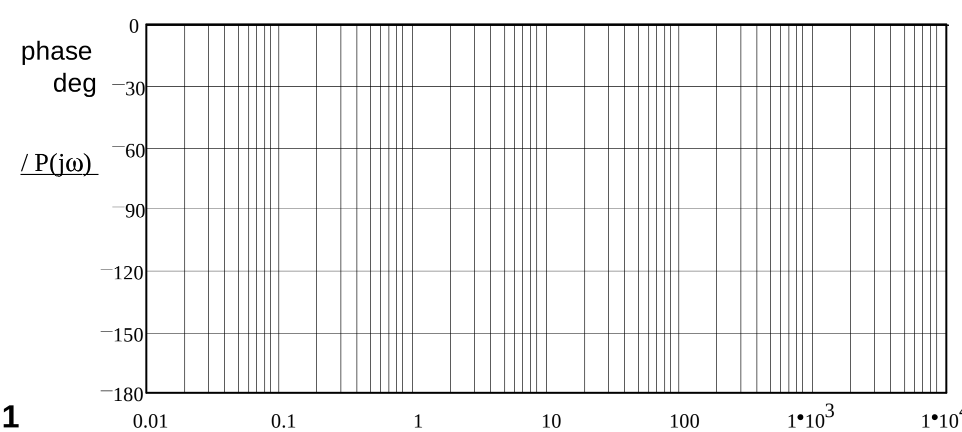
**Ex. 1** $P(s) = \dfrac{2\cdot(s+10)}{s+100}$




**Ex. 2** $P(s) = \dfrac{s+20}{4\cdot(s+1)^2}$



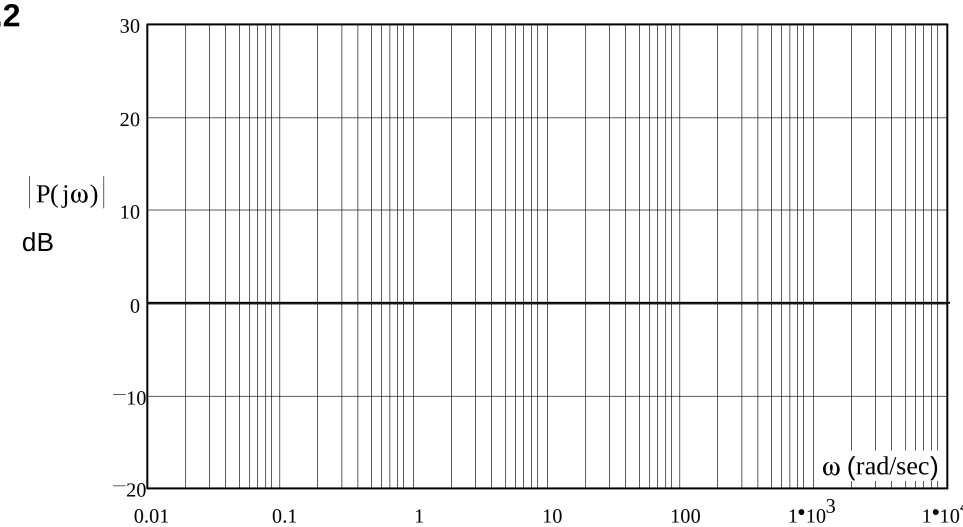
**Ex. 3** $P_3(s) := \dfrac{20000\cdot(-s+0.1)}{(s+4)\cdot(s+1000)}$

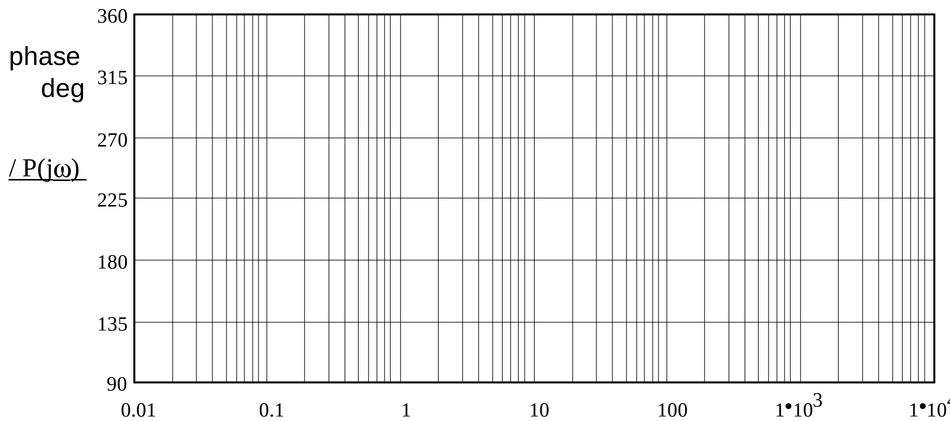
clear the - in front of the s $= \dfrac{-20000\cdot(s-0.1)}{(s+4)\cdot(s+1000)}$

$\omega < 0.1$

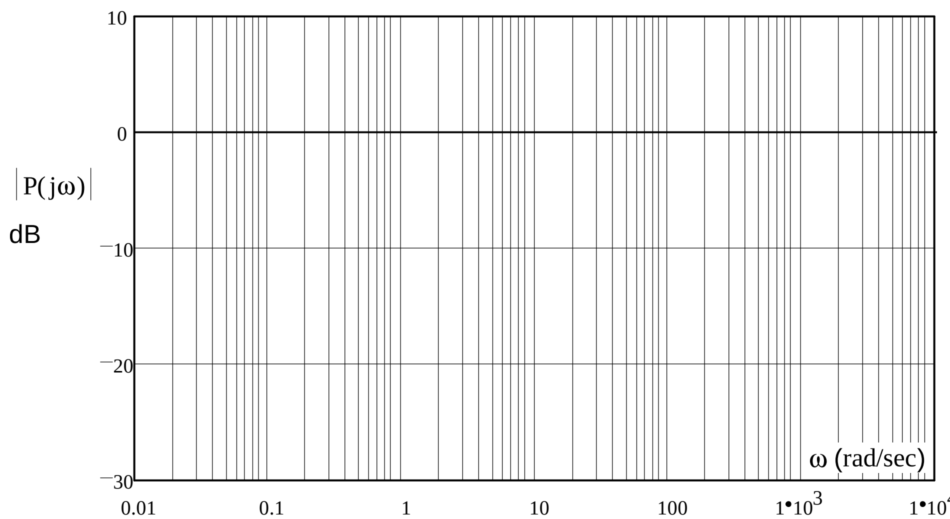
180º+180º=360º

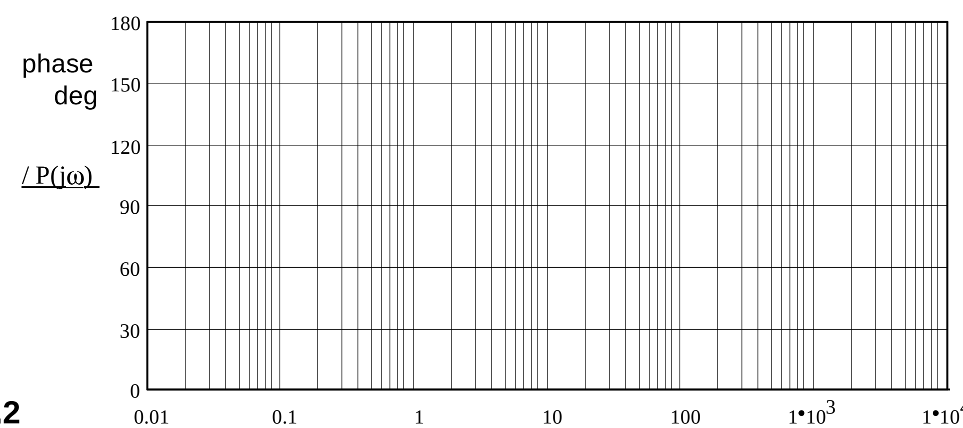
$\dfrac{-20000\cdot(\_-0.1)}{(\_+4)\cdot(\_+1000)} = 0.5$ $-6\cdot dB$ / 360º



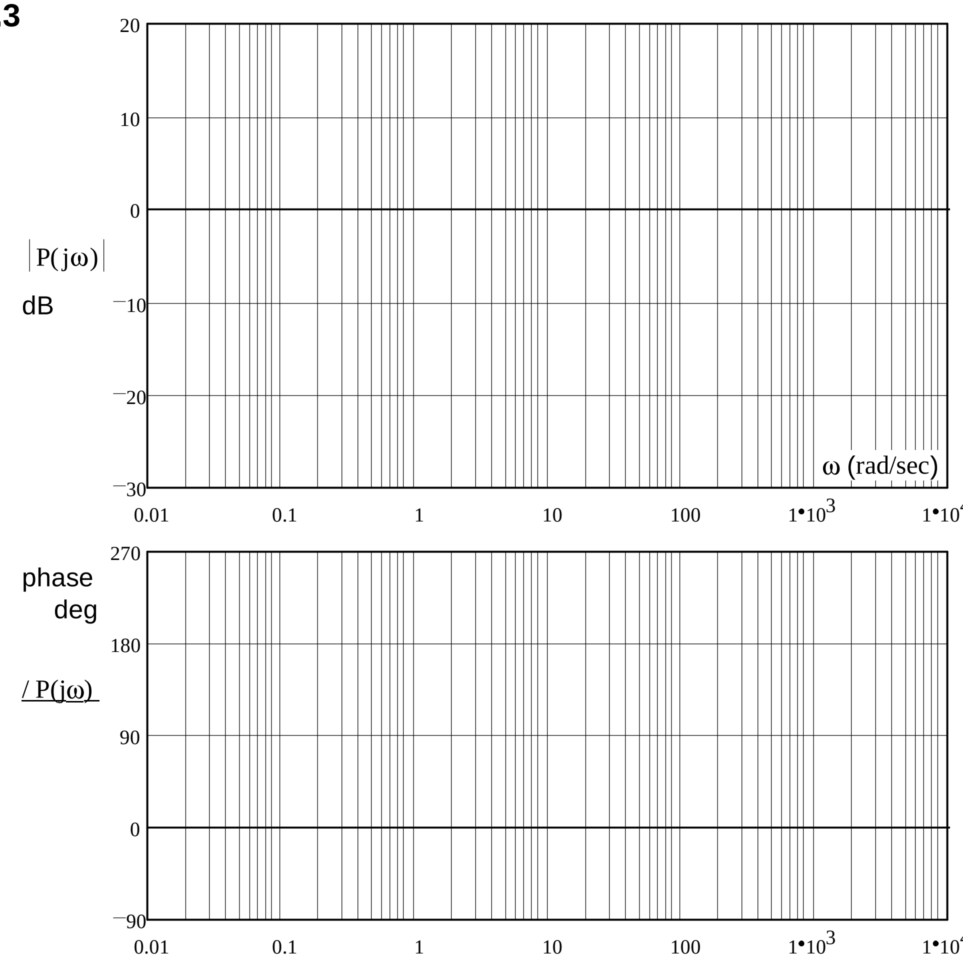

**Ex. 4** $P_4(s) := \dfrac{0.5\cdot(s+1)\cdot(s-20)}{s\cdot(s+100)}$



**Ex. 5**

$$P_5(s) := \frac{5000 \cdot s \cdot (s-4)}{(s+0.2)\cdot(s+20)\cdot(s+1000)}$$




**Ex. 6**

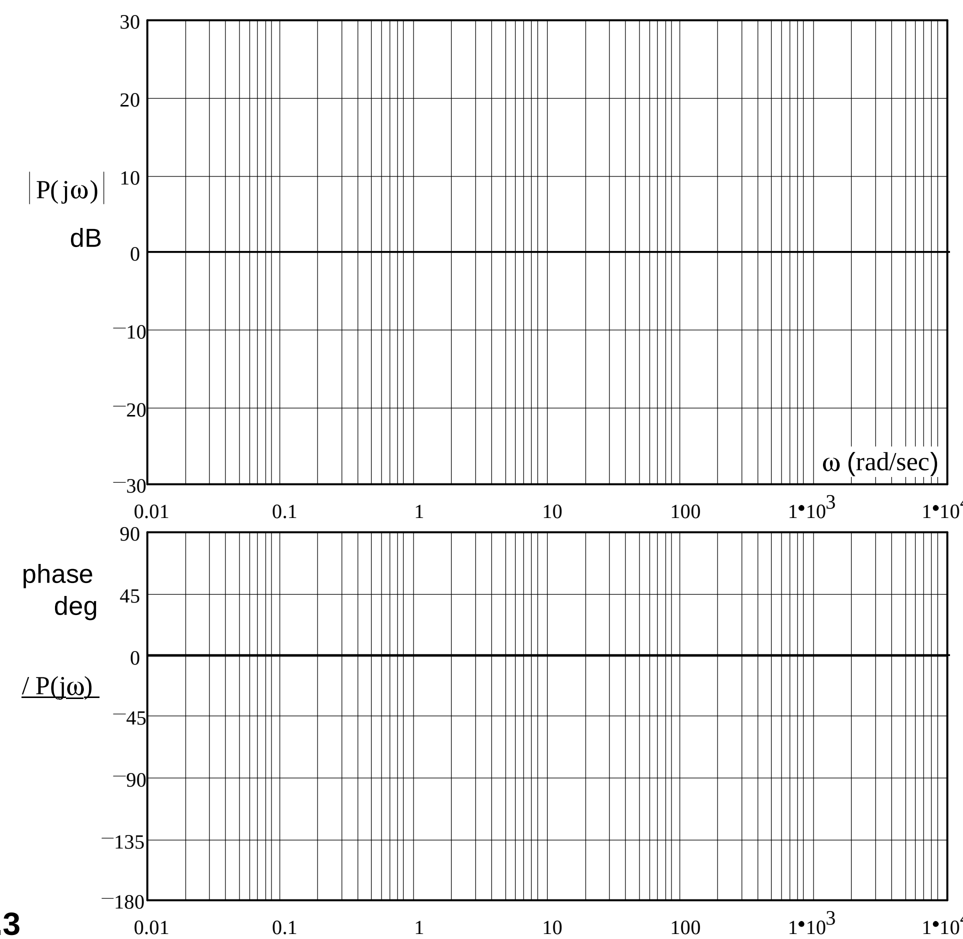
$$P_6(s) := \frac{20000 \cdot (s+0.1)}{(s+2)\cdot\left(s^2+10\cdot s+10000\right)}$$

$$s^2 + 2\cdot\zeta\cdot\omega_n\cdot s + \omega_n^{\,2}$$

natural frequency $\omega_n = \sqrt{\omega_n^{\,2}} \quad =$

damping factor: $\zeta = \dfrac{2\cdot\zeta\cdot\omega_n}{2\cdot\omega_n} \quad =$

dB peak: $20\cdot\log\left(\dfrac{1}{2\cdot .05}\right) = 20$



**Ex. 7**

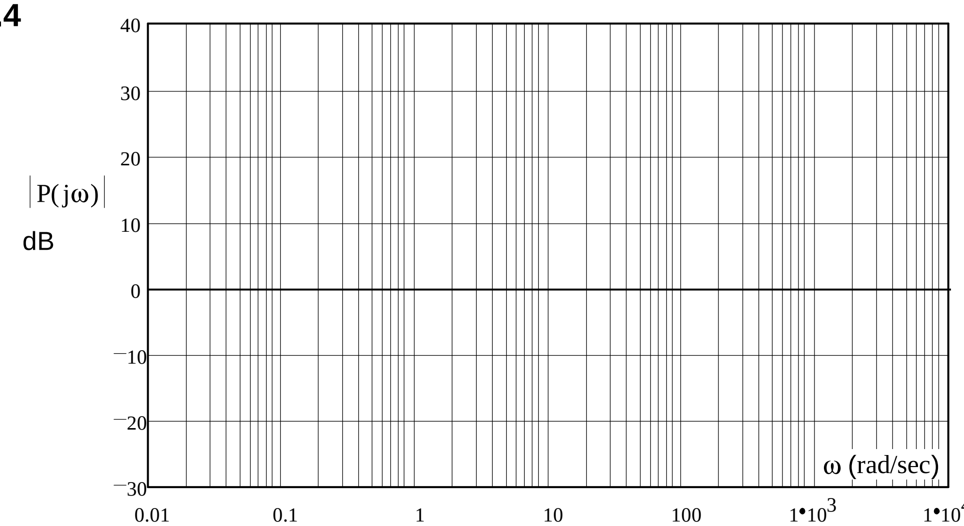
$$P_7(s) := \frac{400\cdot(s+0.1)\cdot(s+100)}{\left[(s+0.4)^2+15.84\right]\cdot(s+1000)}$$

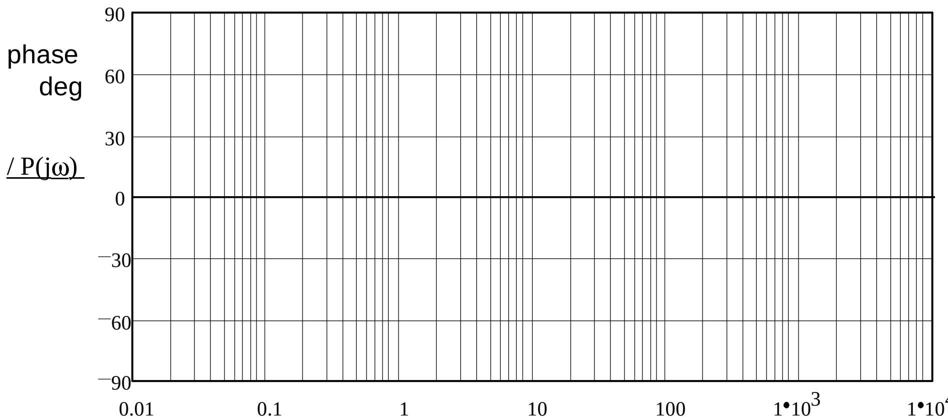
natural freq. $(s+a)^2+b^2$

$\omega_n = \sqrt{a^2+b^2} \quad =$

damping factor: $\zeta = \dfrac{a}{\omega_n} \quad =$

peak $\dfrac{1}{2\cdot\zeta} =$




**Ex. 8** $(s+a)^2+b^2$

$$P_8(s) := \frac{25\cdot\left[(s+10)^2+9900\right]}{\left(s^2+s+4\right)\cdot(s+2000)}$$

$s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\ 2}$

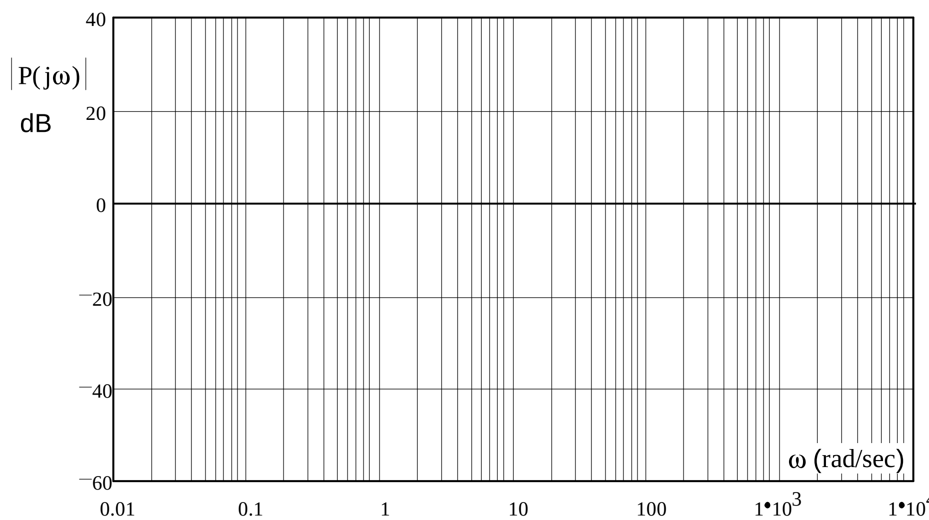
natural frequency $\omega_{n1} = \sqrt{\omega_{n1}^{\ 2}} \quad =$

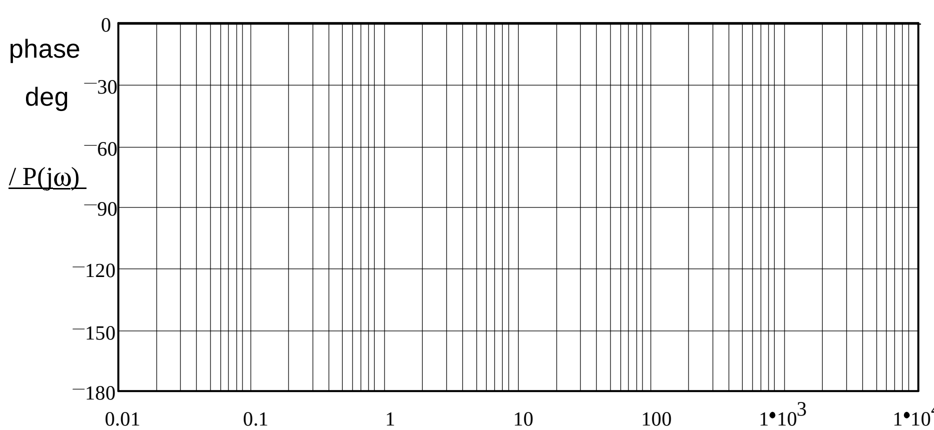
damping factor: $\zeta = \dfrac{2\cdot\zeta\cdot\omega_{n1}}{2\cdot\omega_{n1}} =$

natural freq. $\omega_{n2} = \sqrt{a^2+b^2} \quad =$

damping factor: $\zeta = \dfrac{a}{\omega_n} \quad =$

peak $\dfrac{1}{2\cdot\zeta} =$

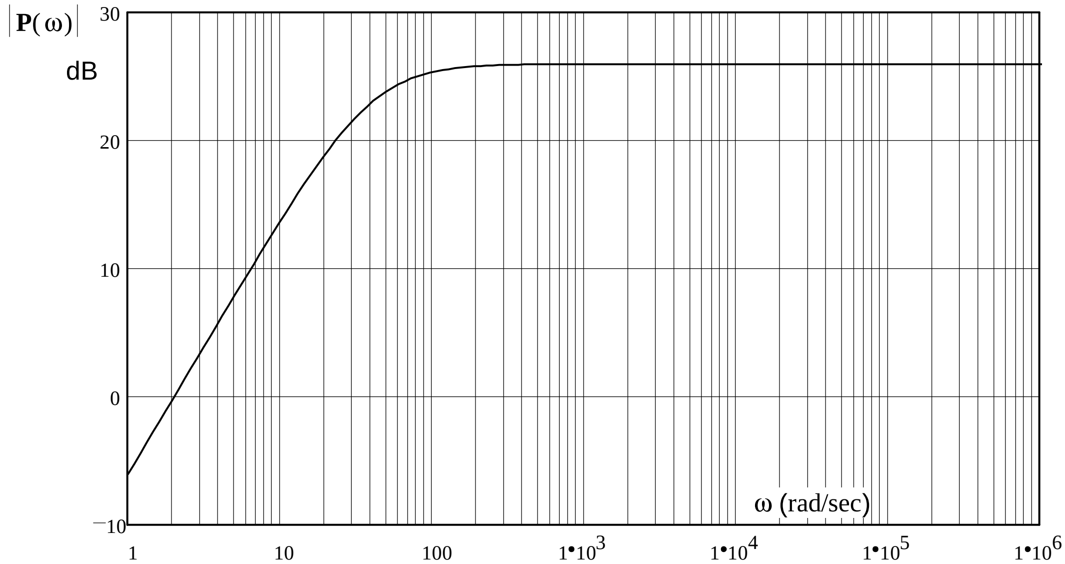


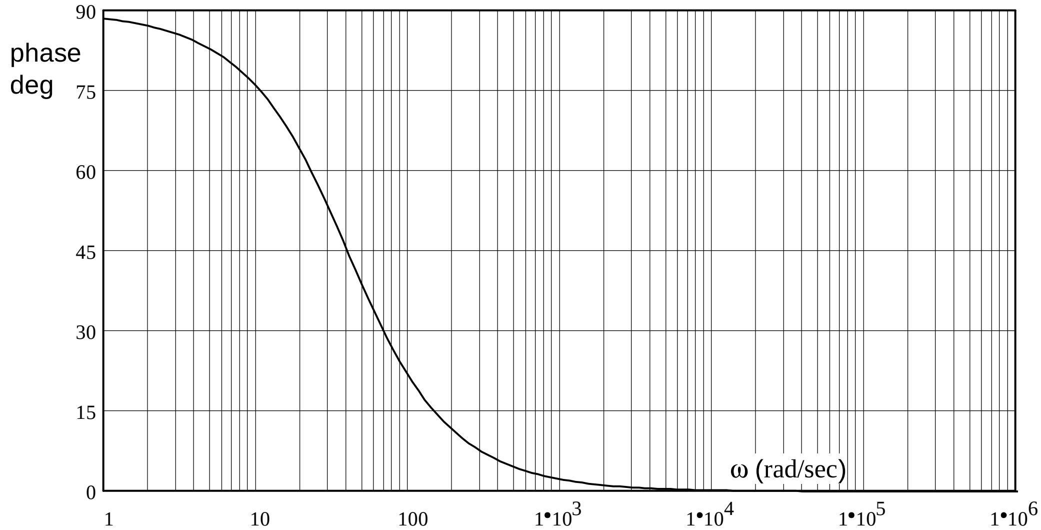



# ECE 3510 Bode Plot to Transfer Function Examples

A. Stolp
4/6/14, 4/4/17

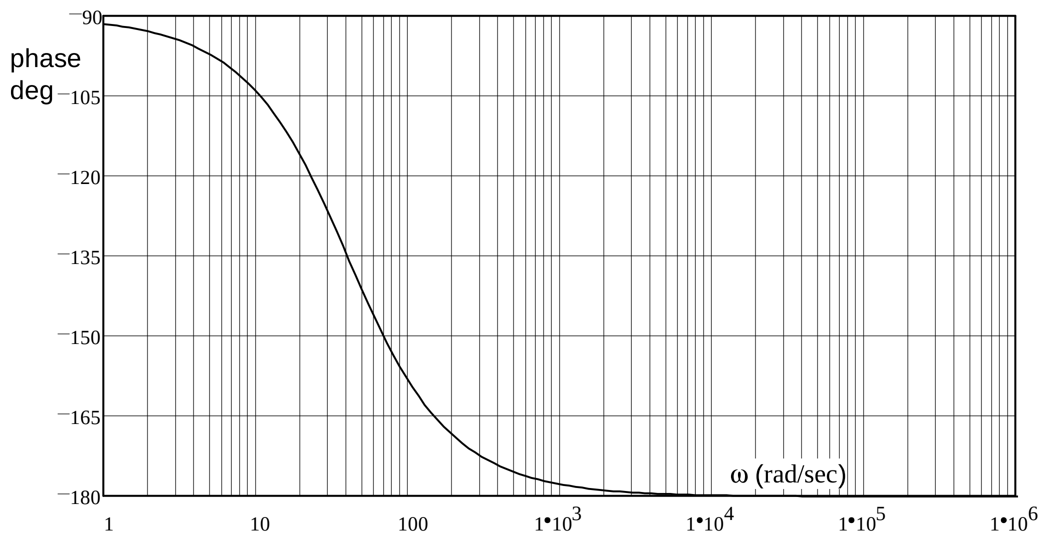
**Ex. 1** $\mathbf{P}(s) = ?$



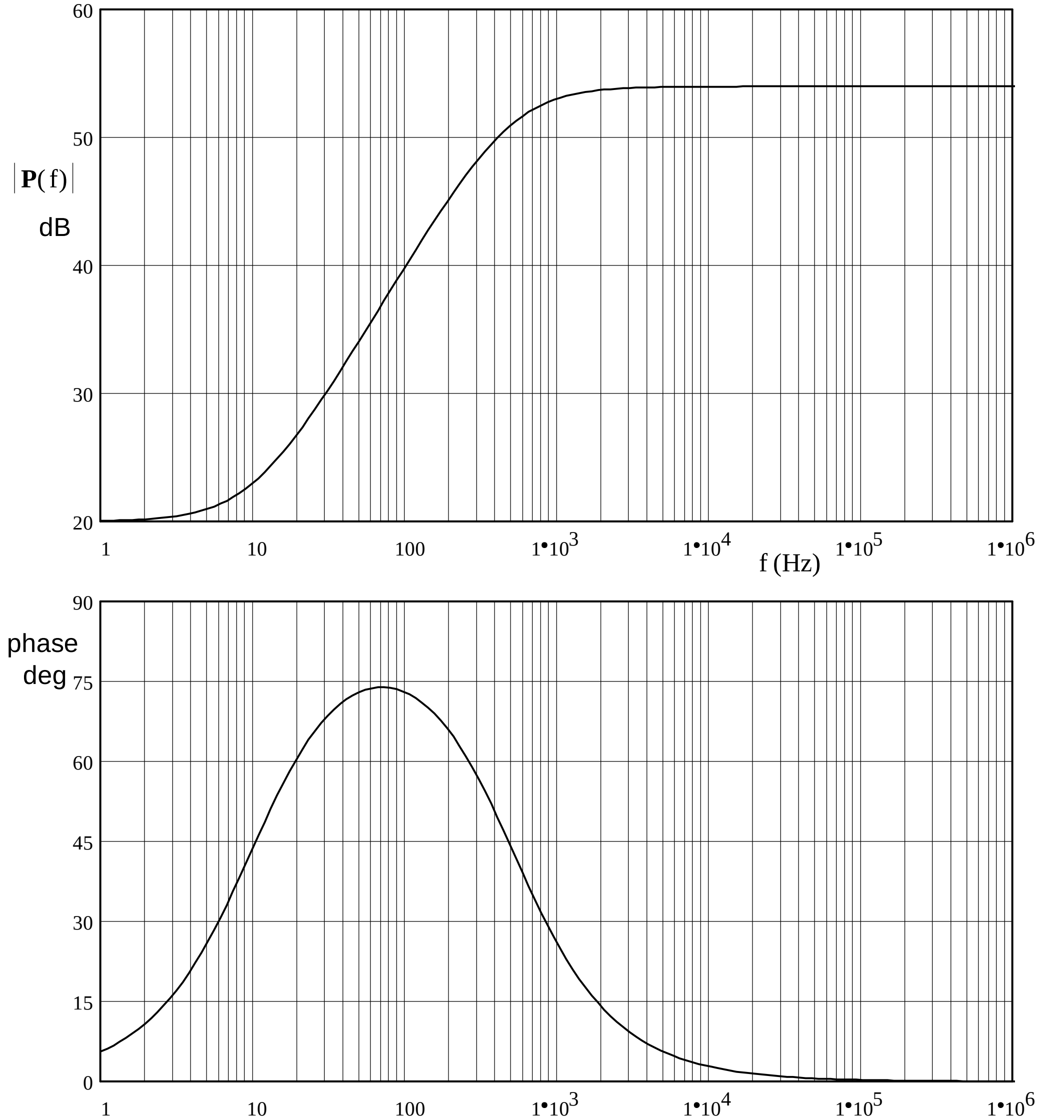

**Ex. 2** What if the phase plot was:

$\mathbf{P}(s) = ?$



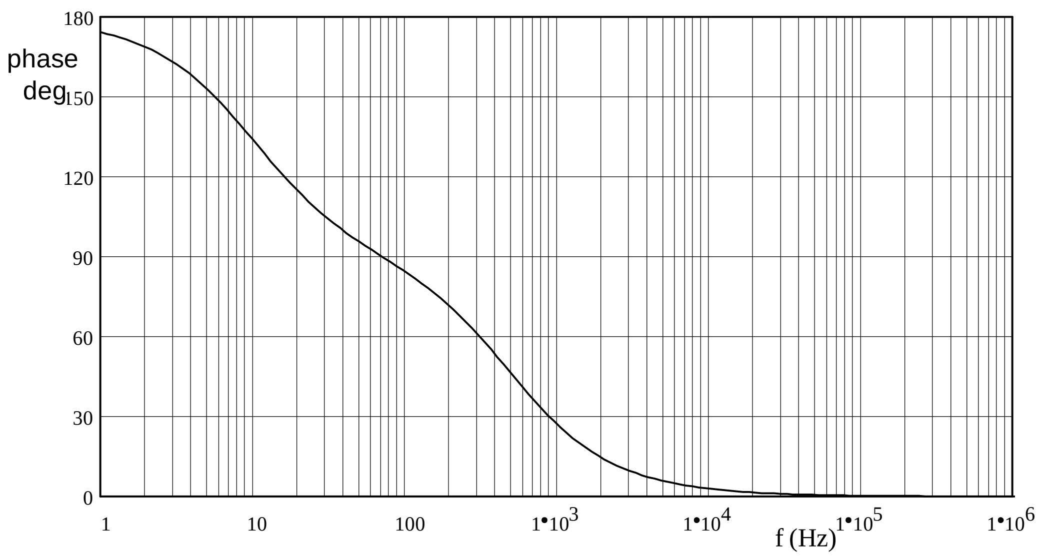
# Bode Plot to Transfer Function Examples   p.2

**Ex. 3**   $\mathbf{P}(s) = ?$

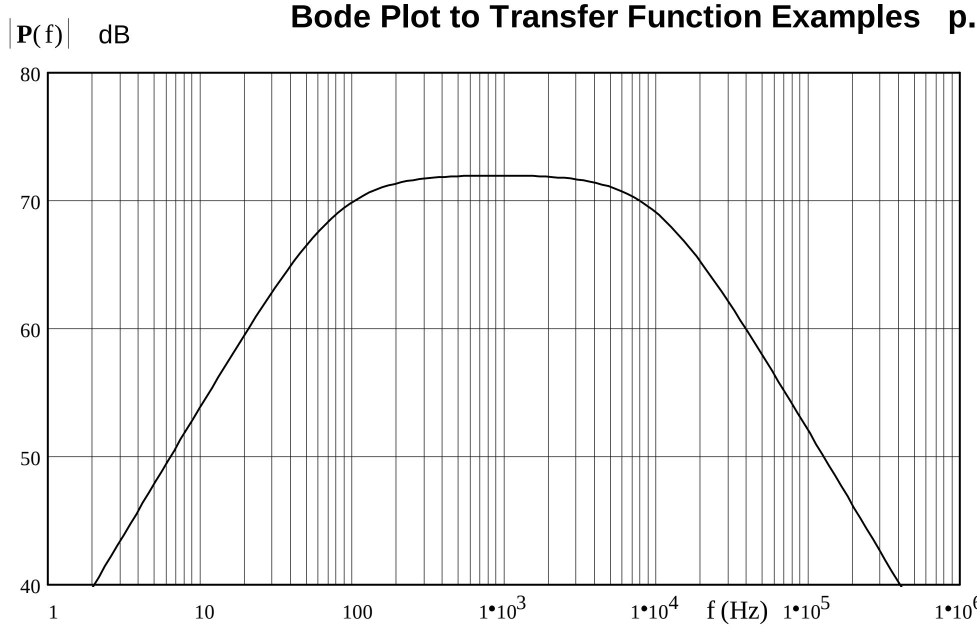


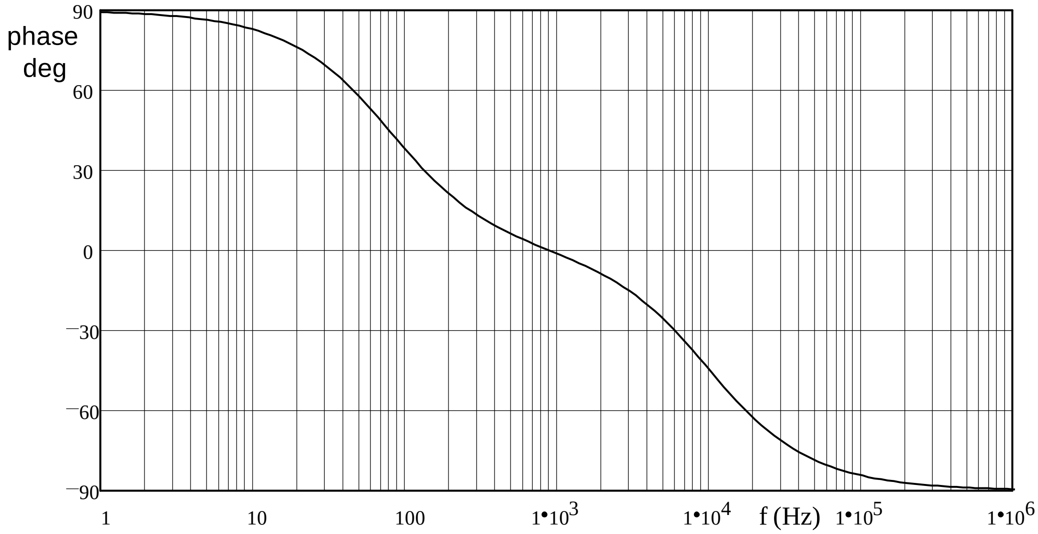

**Ex. 4**   What if the phase plot was:

$\mathbf{P}(s) = ?$



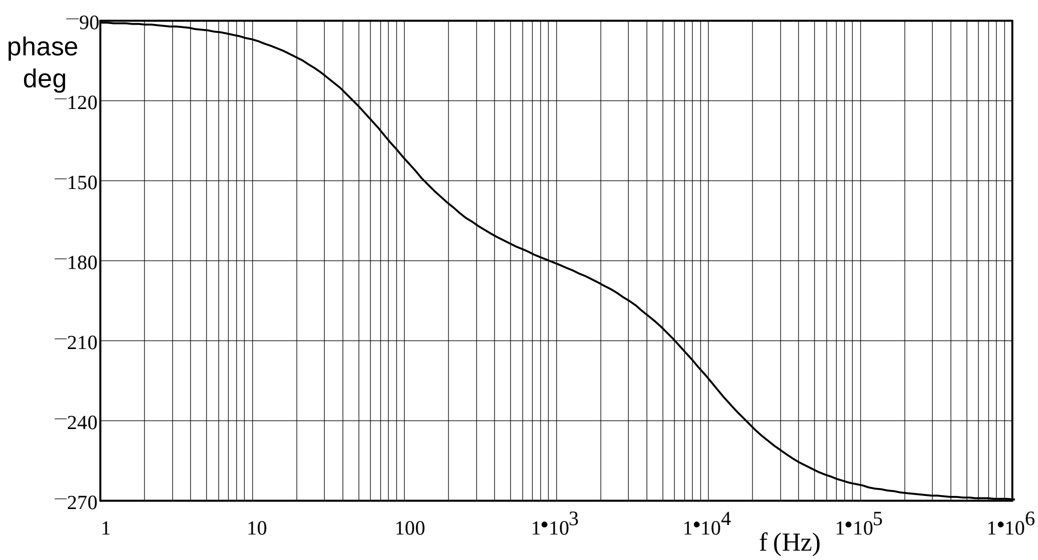
**Ex. 5** $\mathbf{P}(s) = ?$



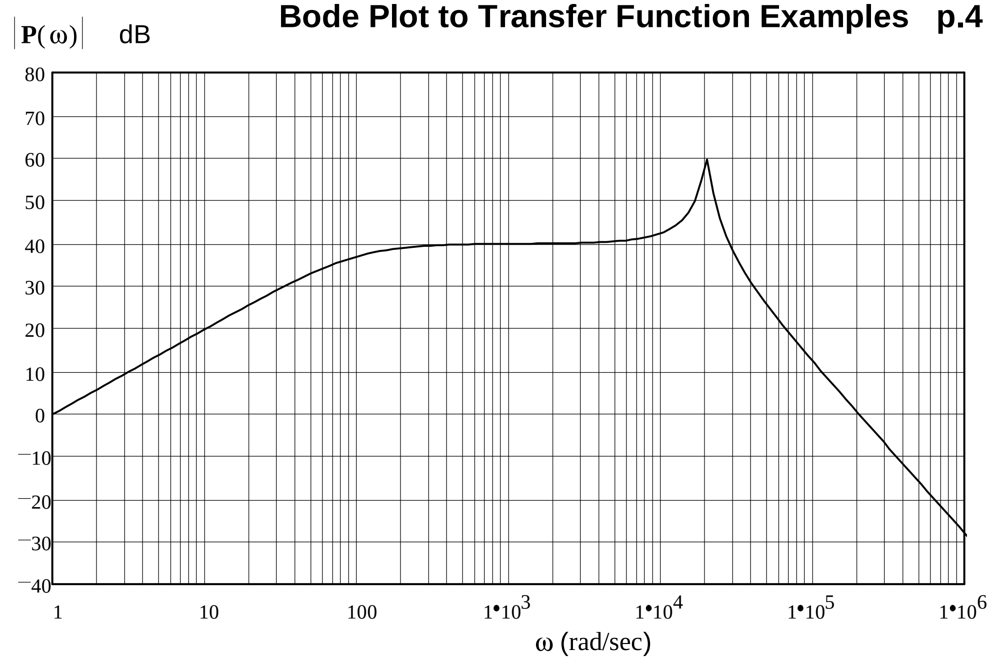



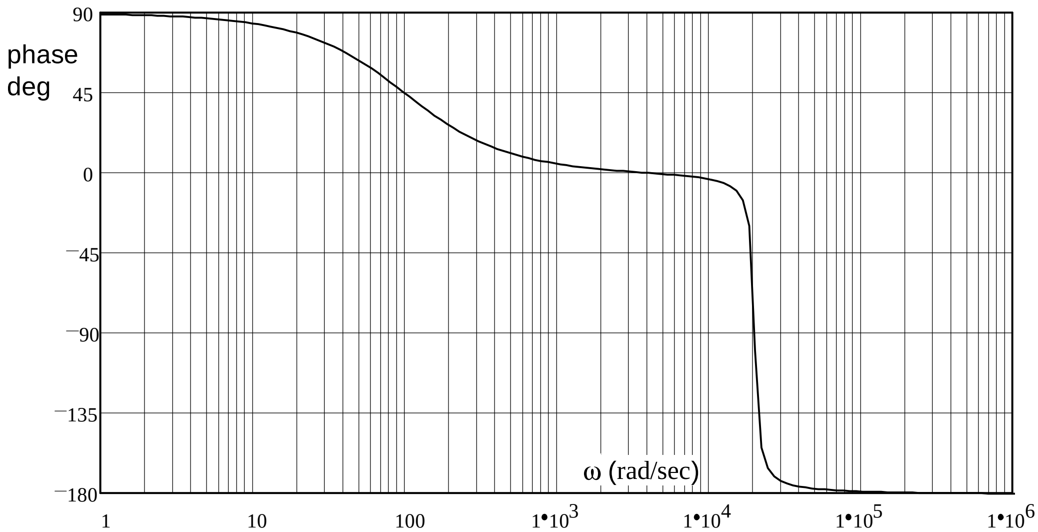

**Ex. 6** What if the phase plot was:

$\mathbf{P}(s) = ?$



**Ex. 7** $\mathbf{P}(s) = ?$

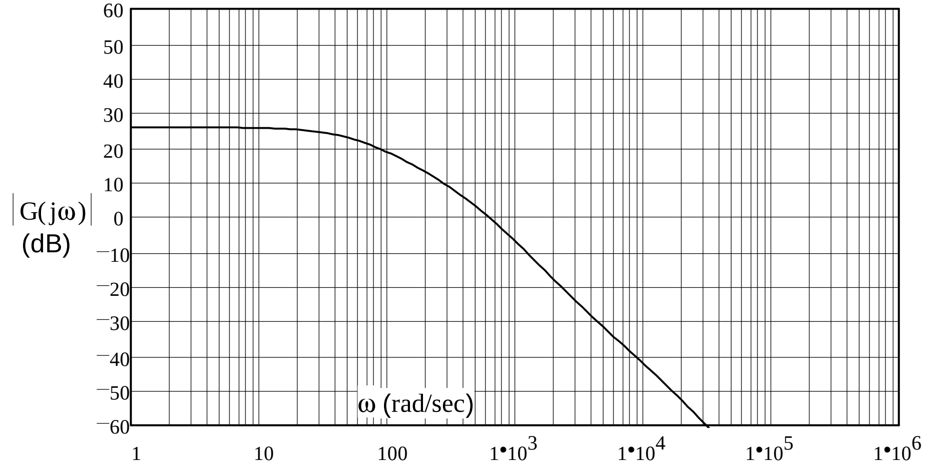


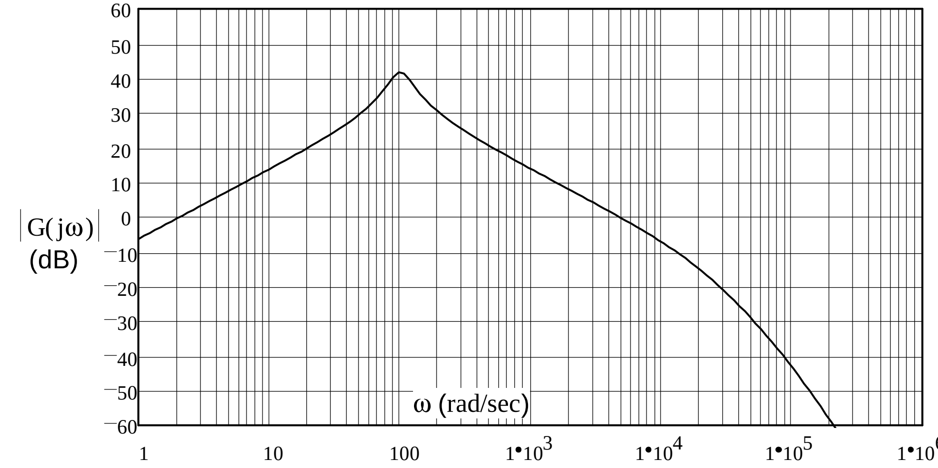



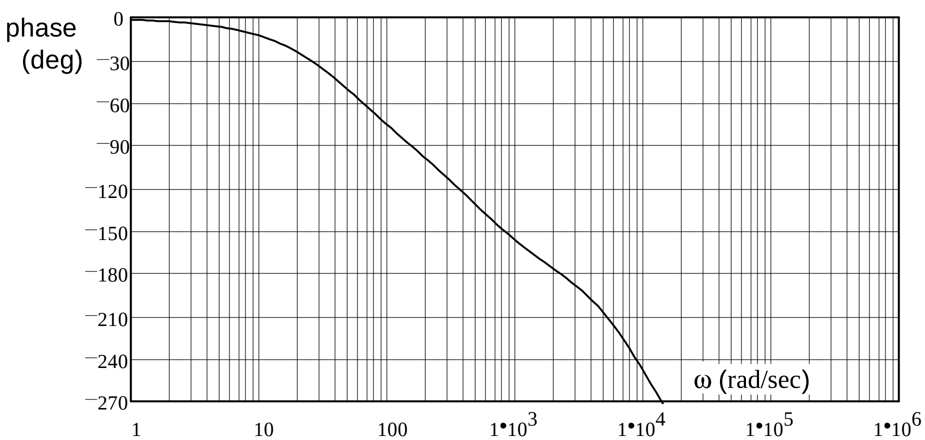

Note: A somewhat more involved method is outlined in Nise section 10.13 (p.660 in 3rd ed., 665 in 4th). That method involves estimating only one pole or zero at a time and then subtracting the effect from original to more clearly see the others. This can work much better with real experimental data. Real data always has delay effects and other non-linearities which make the process much harder.

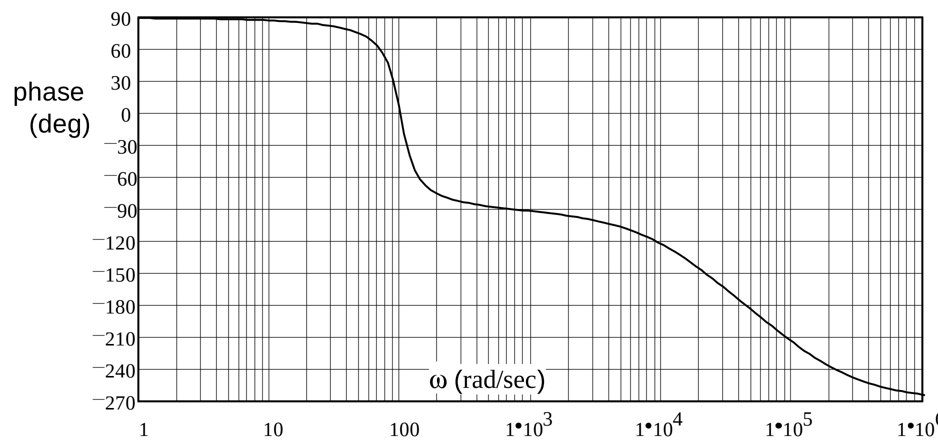
# ECE 3510  Gain, Phase, and Delay margins

A. Stolp
4/8/14
4/13/20




Gain Margin

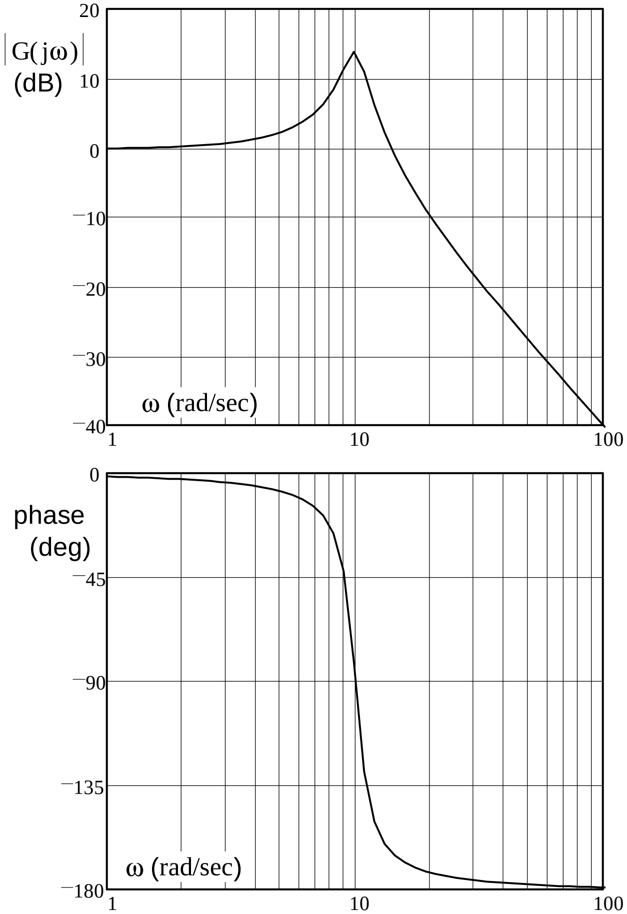
Phase Margin

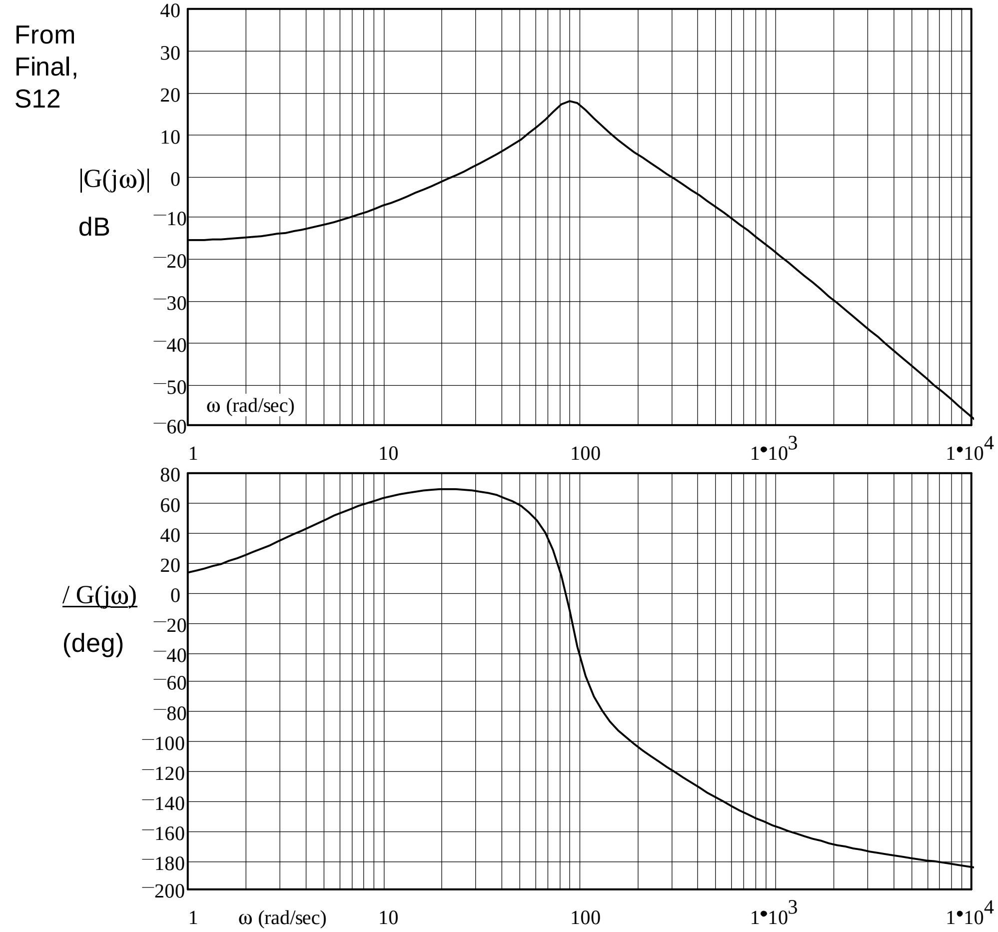
Delay Margin

Gain Margin

Phase Margin

Delay Margin




Gain Margin

Phase Margin

Delay Margin

Gain Margin

Phase Margin

Delay Margin