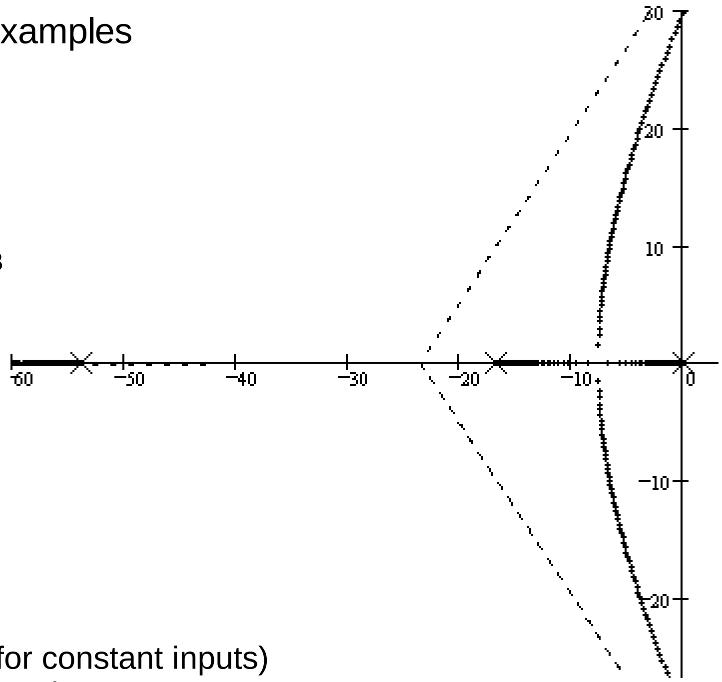
ECE 3510 Root Locus Design Examples

Recall the simple crude servo from lab 1

$$G(s) := \frac{1643}{s\cdot(s+16.64)\cdot(s+53.78)}$$

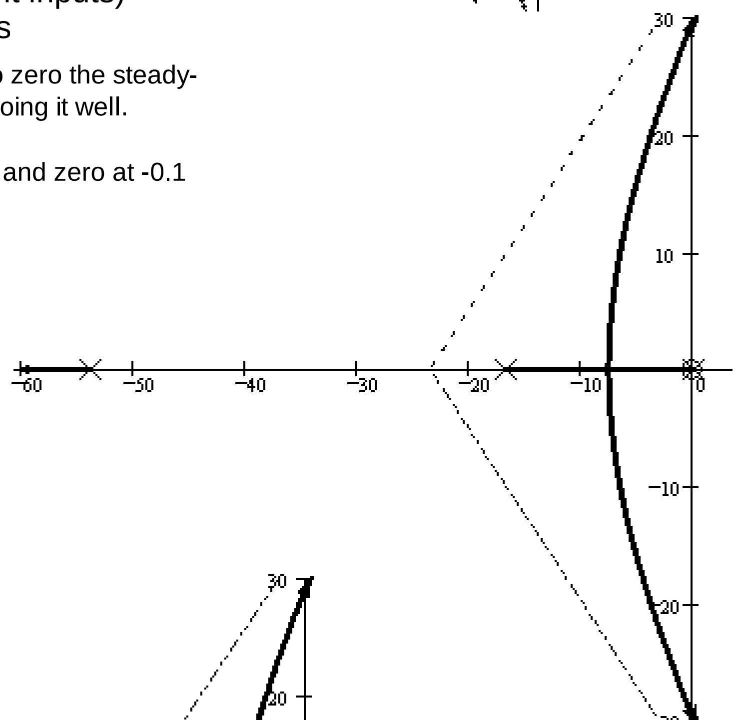
$$\sigma = \frac{-0-16.64-53.78}{3} = -23.473$$

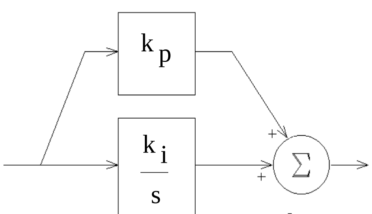


PI To eliminate steady-state error (for constant inputs) & perfect rejection of constant disturbances

Note: The DC motor has a pole at zero and should do zero the steady-state error by itself, but nonlinearities prevent it from doing it well.

$$G_c(s) := \frac{1643}{s\cdot(s+16.64)\cdot(s+53.78)}\cdot\frac{s+0.1}{s}$$ Add pole at 0 and zero at -0.1



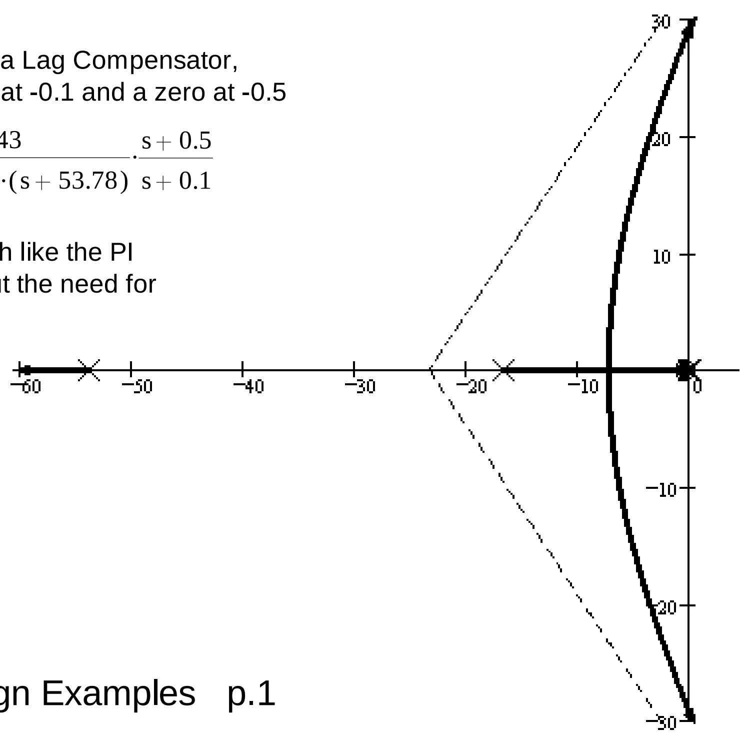


$$C(s) = k_p + \frac{k_i}{s} = k_p\cdot\frac{s+\frac{k_i}{k_p}}{s}$$

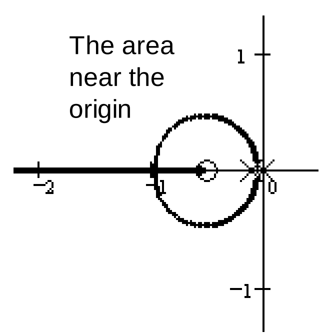
LAG An alternative is a Lag Compensator, here with a pole at -0.1 and a zero at -0.5

$$G_c(s) = \frac{1643}{s\cdot(s+16.64)\cdot(s+53.78)}\cdot\frac{s+0.5}{s+0.1}$$

This works very much like the PI controller, but without the need for active components.



The area near the origin



Root Locus Design Example   p.2

Let's keep the pole at 0 and zero at -0.1 for elimination of steady-state errors and rejection of disturbances

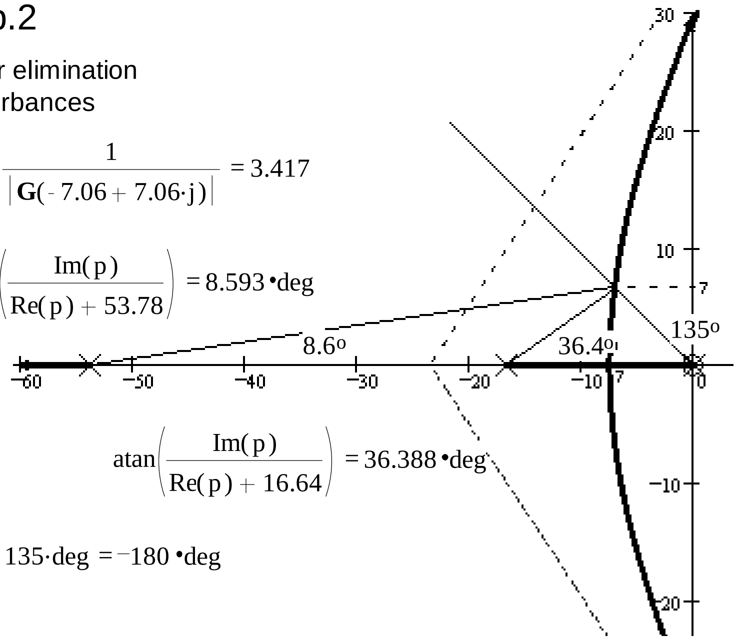
CL poles at $p := -7.06 + 7.06\cdot j$ and $-7.06 - 7.06\cdot j$

$$k = \frac{1}{\left|\mathbf{G}(-7.06 + 7.06\cdot j)\right|} = 3.417$$

At gain of 3.44

$$\operatorname{atan}\left(\frac{\operatorname{Im}(p)}{\operatorname{Re}(p) + 53.78}\right) = 8.593 \cdot \text{deg}$$

$$\operatorname{atan}\left(\frac{\operatorname{Im}(p)}{\operatorname{Re}(p) + 16.64}\right) = 36.388 \cdot \text{deg}$$



This is a point in the root locus because:

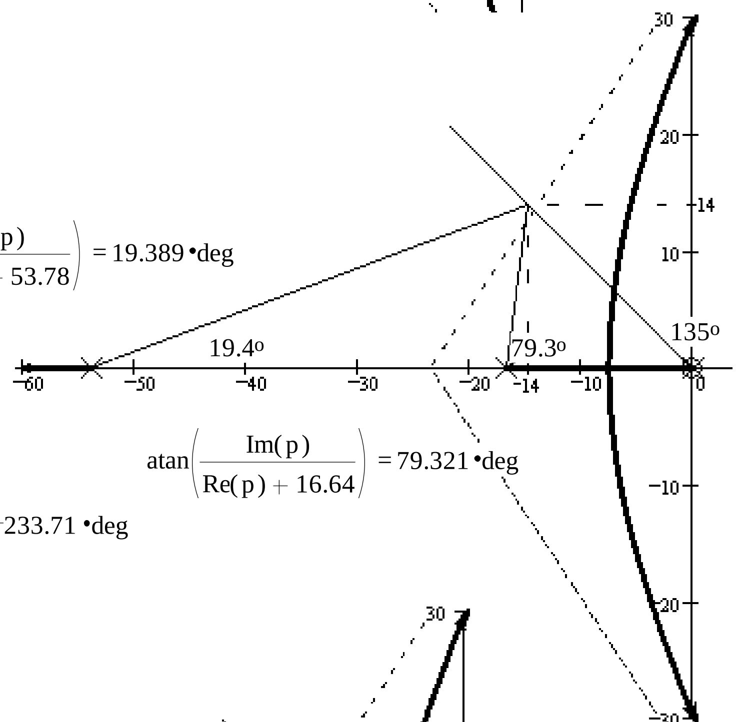
$$-8.6\cdot\text{deg} - 36.4\cdot\text{deg} - 135\cdot\text{deg} - 135\cdot\text{deg} + 135\cdot\text{deg} = -180 \cdot \text{deg}$$

PD or PID To Improve the dynamic response

Want to double the speed

Want poles to move to: $p := -14 + 14\cdot j$
$-14 - 14\cdot j$

$$\operatorname{atan}\left(\frac{\operatorname{Im}(p)}{\operatorname{Re}(p) + 53.78}\right) = 19.389 \cdot \text{deg}$$



$$\operatorname{atan}\left(\frac{\operatorname{Im}(p)}{\operatorname{Re}(p) + 16.64}\right) = 79.321 \cdot \text{deg}$$

Unfortunately, this point in NOT on the root locus

$$-\operatorname{atan}\left(\frac{\operatorname{Im}(p)}{\operatorname{Re}(p) + 53.78}\right) - \operatorname{atan}\left(\frac{\operatorname{Im}(p)}{\operatorname{Re}(p) + 16.64}\right) - 135\cdot\text{deg} = -233.71 \cdot \text{deg}$$

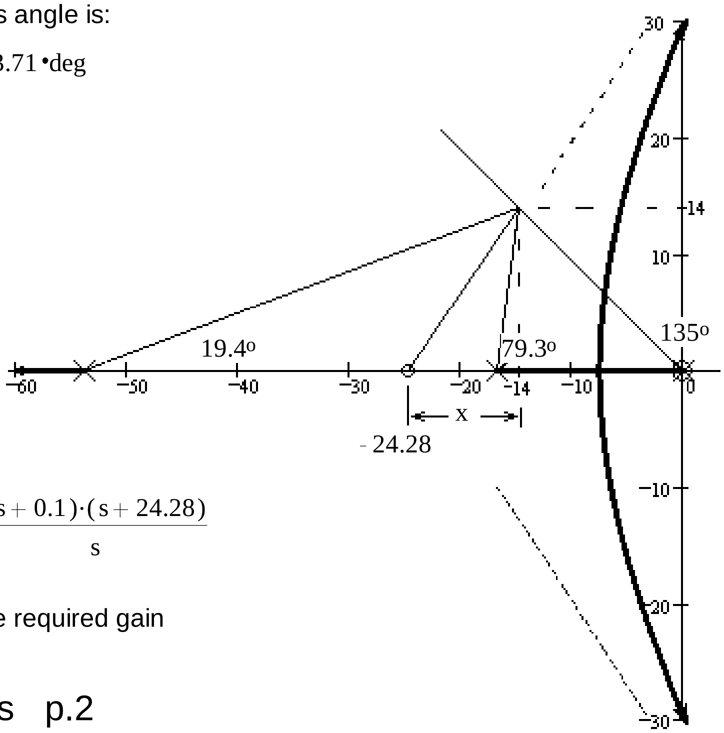
Maybe we could add a zero so that it's angle is:

$$\theta_z := 233.71\cdot\text{deg} - 180\cdot\text{deg} \qquad \theta_z = 53.71 \cdot \text{deg}$$

$$x = \operatorname{Im}(p)\cdot\frac{1}{\tan(\theta_z)} = 10.28$$

$$z := \operatorname{Re}(p) - \operatorname{Im}(p)\cdot\frac{1}{\tan(\theta_z)}$$

$$z = -24.28$$



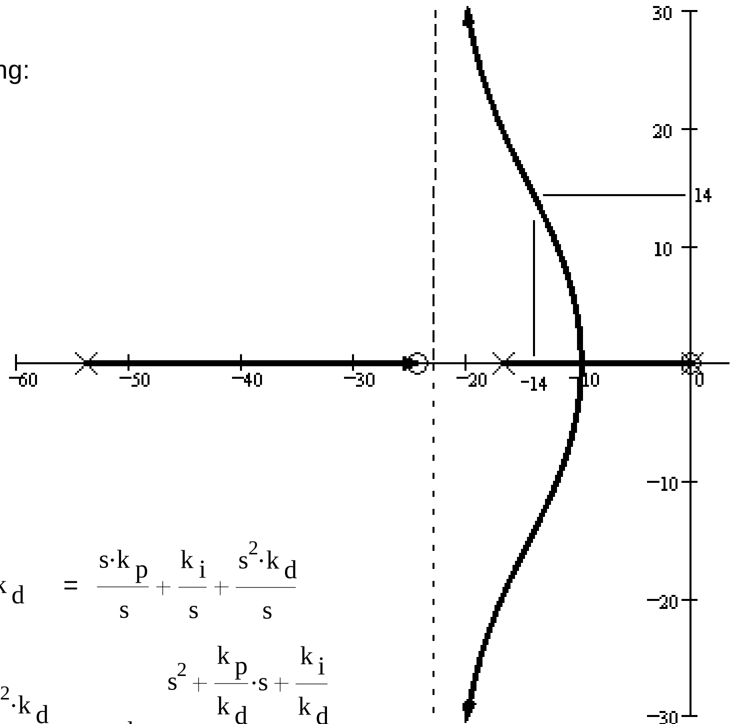
$$\mathbf{G_c}(s) := \frac{1643}{s\cdot(s + 16.64)\cdot(s + 53.78)}\cdot\frac{(s + 0.1)\cdot(s + 24.28)}{s}$$

$$k = \frac{1}{\left|\mathbf{G}(-14 + 14\cdot j)\right|} = 7.24$$ is the required gain

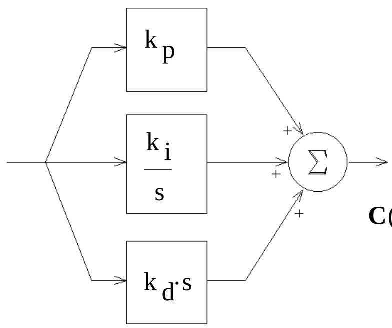
Root Locus Design Examples p.3

We have designed a our compensation with the following:

- A pole at the origin
- A zero at -0.1
- A zero at -24.28
- Gain of 0.418



Find the k_p, k_i, & k_d of a PID controller.



$$\mathbf{C}(s) = k_p + \frac{k_i}{s} + s \cdot k_d = \frac{s \cdot k_p}{s} + \frac{k_i}{s} + \frac{s^2 \cdot k_d}{s}$$

$$= \frac{s \cdot k_p + k_i + s^2 \cdot k_d}{s} = k_d \cdot \frac{s^2 + \frac{k_p}{k_d} \cdot s + \frac{k_i}{k_d}}{s}$$

gain = $k_d := 0.418$

$$(s + 0.1) \cdot (s + 24.28) = s^2 + 24.38 \cdot s + 2.43$$

$$= s^2 + \frac{k_p}{k_d} \cdot s + \frac{k_i}{k_d}$$

$$\frac{k_i}{k_d} = 2.43 \qquad k_i := k_d \cdot 2.43 \qquad k_i = 1.016$$

$$\frac{k_p}{k_d} = 24.38 \qquad k_p := k_d \cdot 24.38 \qquad k_p = 10.191$$

Notice: $\frac{k_i}{k_p} = 0.1 \simeq 0.1$

Notice that the proportional gain is actually almost 3 times higher than it was before. $3 \cdot 3.44 = 10.32$

LEAD An alternative to the differentiator is a Lead Compensator.

Instead of a single zero with: $\theta_z = 53.71 \cdot deg$

How about a zero with $\theta_z := 70 \cdot deg$ And a pole with $\theta_p := 70 \cdot deg - 53.71 \cdot deg$

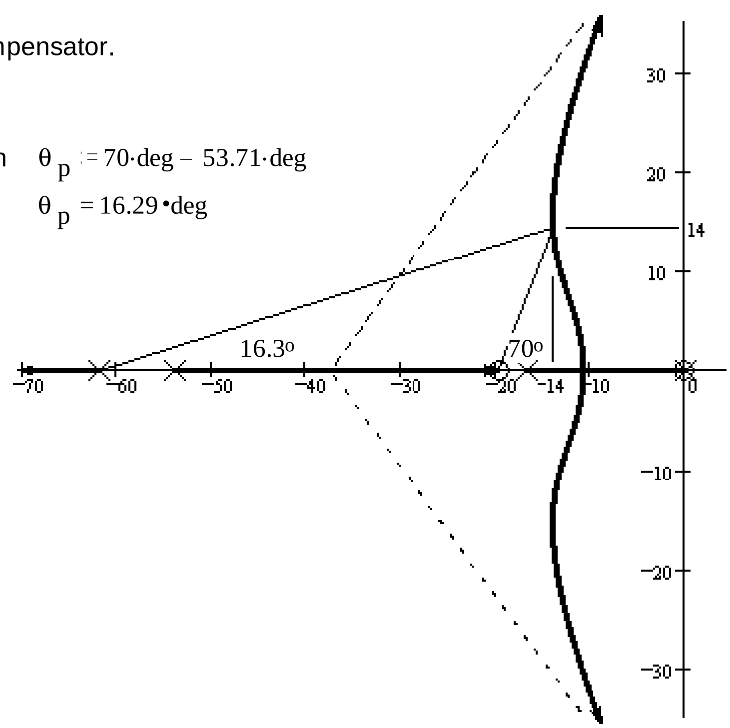
$\theta_p = 16.29 \cdot deg$

$$x = Im(p) \cdot \frac{1}{\tan(\theta_z)} = 5.096$$

$$z := Re(p) - Im(p) \cdot \frac{1}{\tan(\theta_z)} \qquad z = -19.096$$

$$xp = Im(p) \cdot \frac{1}{\tan(\theta_p)} = 47.907$$

$$p := Re(p) - Im(p) \cdot \frac{1}{\tan(\theta_p)} \qquad p = -61.907$$



This example is actually a PI-Lead controller

Problems with the differentiator

1. Tries to differentiate a step input into an impulse -- not likely.
   You'll have to consider how your differentiator will actually handle a step input and how your amplifier will saturate.

   If the differentiator and amplifiers saturate in such a way the the "area under the curve" approximates the impulse "area under the curve", then this may not be such a problem. It may not be as fast as predicted from the linear model, but it may be as fast as the system limits allow. (Pedal-to-the-metal.)

2. It's a high-pass filter and can accentuate noise.
   This is actually common to all compensators that speed up the response.

3. Requires active components and a power supply to build.
   Usually no big deal since your amplifier (source of gain) does too.

4. Is never perfect (always has higher-order poles), but then neither is anything else. Especially in mechanical systems, these poles usually are well beyond where they could cause problems.

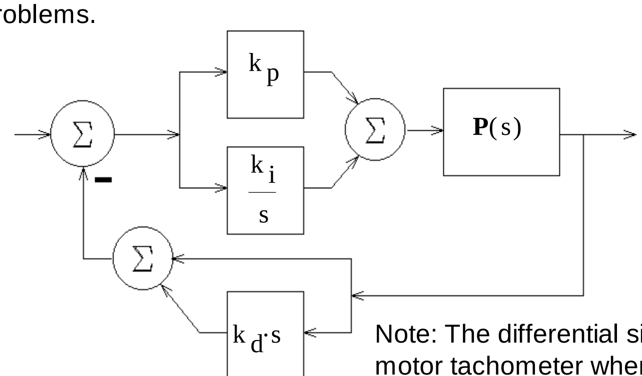
<u>Alternatives:</u>

1. Lag-Lead or PI-Lead compensation. This eliminates the differentiator, but it is still a high-pass filter that can be a noise problem and it could still saturate the amplifier if the input changes too rapidly.

   Be sure to check for saturation problems.

2. Place the differentiator in the feedback loop. The output of the plant is much less likely to be a step or to change so rapidly that it causes problems.

   Differentiation in the feedback



Note: The differential signal is often taken from a motor tachometer when the output is a position. Then you don't need a separate differentiator circuit, just a separate gain for that signal.

Find the k_p, k_i, & k_d of this controller.

$$\mathbf{F}(s)\cdot\mathbf{C}(s) \quad = \quad \underbrace{\left(k_p + \frac{k_i}{s}\right)}_{\mathbf{C}(s)}\cdot\underbrace{\left(1 + k_d \cdot s\right)}_{\mathbf{F}(s)} \quad = \quad k_p \cdot k_d \cdot \left[\frac{s + \frac{k_i}{k_p}}{s}\right]\cdot\left(\frac{1}{k_d} + s\right) \quad = \quad k_p \cdot k_d \cdot \frac{\left(s + \frac{k_i}{k_p}\right)\cdot\left(s + \frac{1}{k_d}\right)}{s}$$

For our example:

$$= 0.418\cdot\frac{(s+0.1)\cdot(s+24.28)}{s}$$

$$k_d := \frac{1}{24.38} \qquad k_d = 0.041$$

$$k_p := \frac{0.418}{k_d} \qquad k_p = 10.191$$

$$k_i := k_p \cdot 0.1 \qquad k_i = 1.019$$

In this case the open-loop zero in the feedback loop IS NOT in the closed-loop. This turns out to make the step response slower than predicted by the second-order approximation, but try a simulation, you may be able to use significantly more gain with no more overshoot. The differentiator in this position inhibits overshoot.

Ex.2, from S16 Exam 3 Consider the transfer function: $\mathbf{G}(s) := \dfrac{s+5}{(s+1)\cdot\left(s^2+4\cdot s+20\right)}$

a) Find the departure angle from a complex pole.

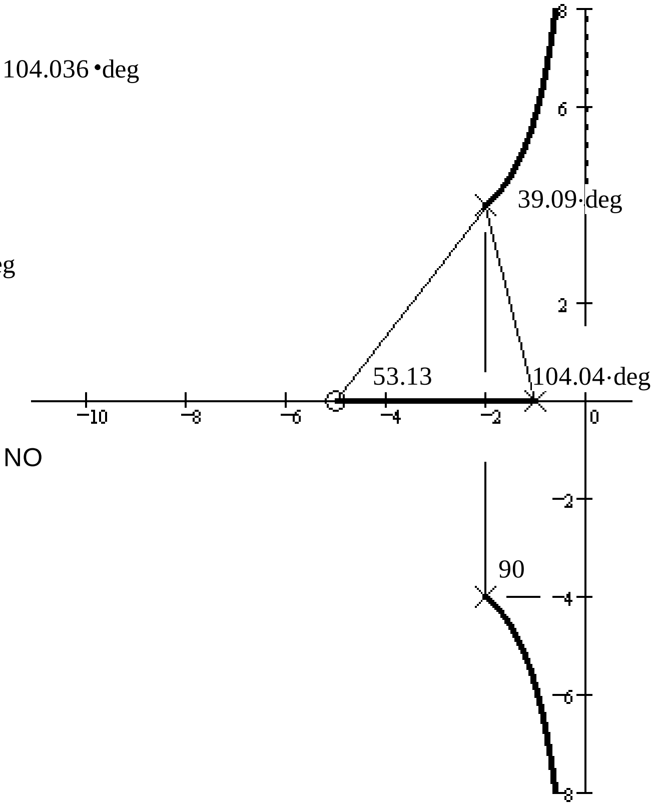
Angles:

from pole at -1 $\theta_{p1} := 180\cdot deg - \operatorname{atan}\left(\dfrac{4}{1}\right)$ $\theta_{p1} = 104.036\cdot deg$

from pole at -2-4j $\theta_{p2} := 90\cdot deg$ $\theta_{p2} = 90\cdot deg$

from zero at -5 $\theta_{z} := \operatorname{atan}\left(\dfrac{4}{3}\right)$ $\theta_{z} = 53.13\cdot deg$

$\theta = 53.13\cdot deg - 90\cdot deg - 104.036\cdot deg + 180\cdot deg = 39.094\cdot deg$



b) Draw a root locus plot. Calculate the centroid and accurately draw the departure angle.

$\sigma := \dfrac{5-1-2-2}{2}$ $\sigma = 0$

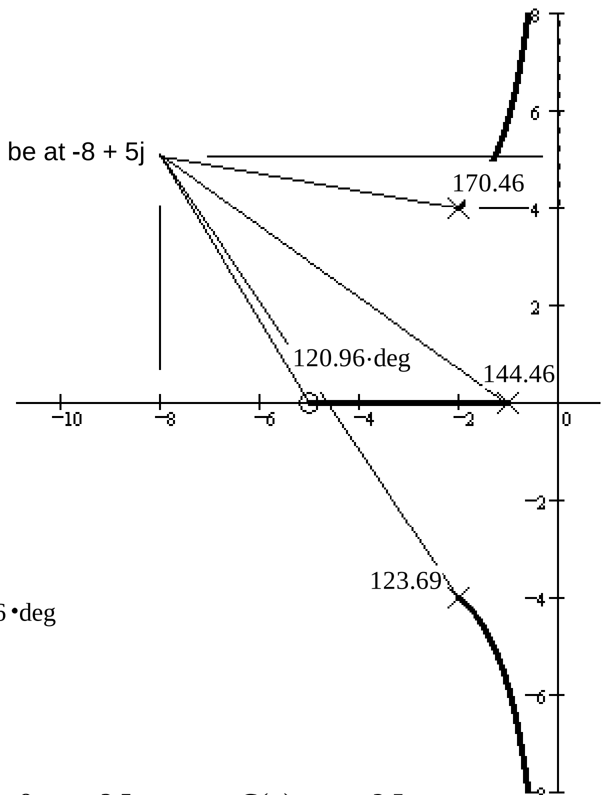
c) Is there any decent place to locate the closed-loop poles? NO

d) You would like to place your closed-loop poles to get a settling time of 1/2 sec and 0.656% overshoot. Add the simplest possible compensator to accomplish this and calculate what the compensator should be.

2% settling time: $T_s = \dfrac{4}{a}$ $a = \dfrac{4}{\left(\dfrac{1}{2}\right)} = 8$

Overshoot: $OS = e^{-\pi\cdot\frac{a}{b}}$ $\%OS = 100\%\cdot e^{-\pi\cdot\frac{a}{b}}$

$\dfrac{a}{b} = \dfrac{\ln(OS)}{-\pi} = \dfrac{\ln(0.00656)}{-\pi} = 1.6$ $b = \dfrac{8}{1.6} = 5$



Angles:

from pole at -1 $180\cdot deg - \operatorname{atan}\left(\dfrac{5}{7}\right) = 144.462\cdot deg$

from pole at -2+4j $180\cdot deg - \operatorname{atan}\left(\dfrac{1}{6}\right) = 170.538\cdot deg$

from pole at -2-4j $180\cdot deg - \operatorname{atan}\left(\dfrac{9}{6}\right) = 123.69\cdot deg$

from zero at -5 $180\cdot deg - \operatorname{atan}\left(\dfrac{5}{3}\right) = 120.964\cdot deg$

$144.462\cdot deg + 170.538\cdot deg + 123.69\cdot deg - 120.964\cdot deg = 317.726\cdot deg$

$\theta_z := 317.726\cdot deg - 180\cdot deg$ $\theta_z = 137.726\cdot deg$

$\tan(137.726\cdot deg - 90\cdot deg) = 1.1 = \dfrac{x}{5}$ $x := 5\cdot 1.1$ $8 - x = 2.5$ $\mathbf{C}(s) = s + 2.5$

$\mathbf{G}_c(s) := \dfrac{(s+5)\cdot(s+2.5)}{(s+1)\cdot\left(s^2+4\cdot s+20\right)}$ $s := -8+5\cdot j$ Check: $\arg\left[\dfrac{(s+5)\cdot(s+2.5)}{(s+1)\cdot\left(s^2+4\cdot s+20\right)}\right] = 180\cdot deg$

e) What is the gain? $k := \dfrac{1}{\left|G_c(s)\right|} = \left|\dfrac{(-8+5\cdot j+1)\cdot\left[(-8+5\cdot j)^2+4\cdot(-8+5\cdot j)+20\right]}{(-8+5\cdot j+5)\cdot(-8+5\cdot j+2.5)}\right| = 13.059$

f) What is the steady-state error for a unit-step input?

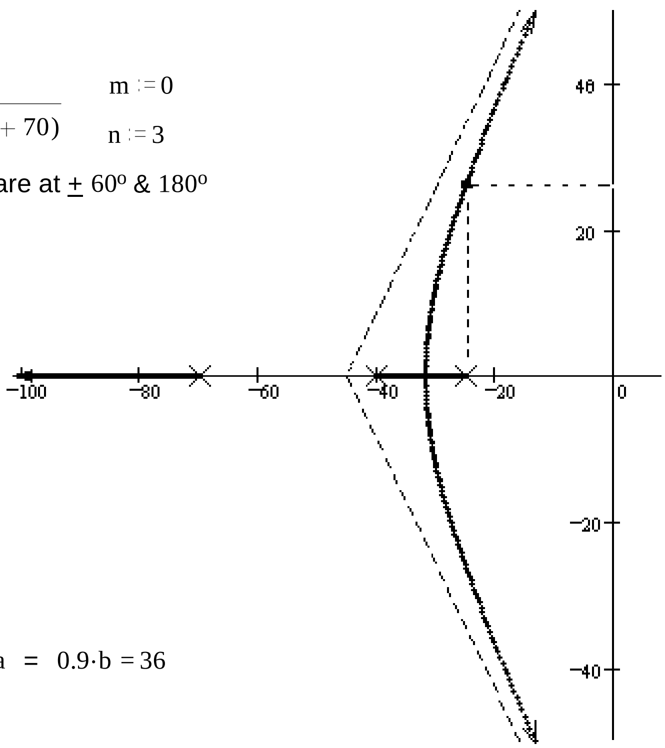
$$G_c(s) := \frac{(s+5)\cdot(s+2.5)}{(s+1)\cdot\left(s^2+4\cdot s+20\right)} \qquad G_c(0) = \frac{(0+5)\cdot(0+2.5)}{(0+1)\cdot\left(0^2+4\cdot 0+20\right)} = \frac{(5)\cdot(2.5)}{(1)\cdot(20)} = 0.625$$

$$G_c(0) = 0.625 \qquad e_{step} = \frac{1}{1+k\cdot 0.625} = 10.91\cdot\%$$

g) If this steady-state error was a little too big, what would be the very simplest way to reduce it?

turn up the gain

Ex.3, from S17 Exam 3



a) Sketch the root locus plot of, $G(s) := \dfrac{100}{(s+25)\cdot(s+40)\cdot(s+70)}$ $m := 0$ $n := 3$

$\sigma_C = \dfrac{-25+-40+-70}{n-m} = -45$ $n-m=3$ so asymptotes are at $\pm$ 60º & 180º

The gain is set at 452, so that one of the closed-loop poles is at,

$s := -24.48 + 27.2\cdot j$

Further calculations yield:

Settling time: $0.163\cdot sec$

% overshoot: $5.92\cdot\%$

Steady-state error to a unit-step input: 60.8%

b) You wish to increase the frequency of ringing to 40 rad/sec without changing the % overshoot at all. Where should the closed-loop pole be located?

$$\frac{a}{b} = \frac{24.48}{27.2} = 0.9 \qquad \text{new } b := 40 \qquad \text{new } a = 0.9\cdot b = 36$$

New location: $s := -36 + 40\cdot j$

c) Add a LEAD compensator so that you will be able to place the closed-loop pole at the location found in b).
Add the new zero at -30. Find the location of the new pole.

Angles:

from pole at -25

$$\theta_{25} := 180\cdot deg - atan\left(\frac{40}{36-25}\right) \qquad \theta_{25} = 105.376\cdot deg$$

from pole at -40

$$\theta_{40} := atan\left(\frac{40}{40-36}\right) \qquad \theta_{40} = 84.289\cdot deg$$

from pole at -70

$$\theta_{70} := atan\left(\frac{40}{70-36}\right) \qquad \theta_{70} = 49.635\cdot deg$$

from new zero at -30

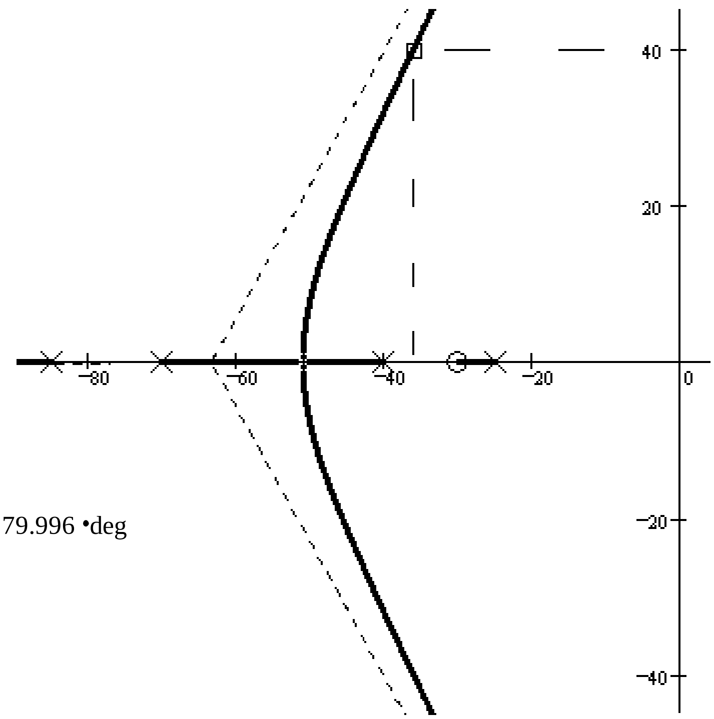
$$\theta_{30} := 180\cdot deg - atan\left(\frac{40}{36-30}\right) \qquad \theta_{30} = 98.531\cdot deg$$

$$\theta_{25} + \theta_{40} + \theta_{70} - \theta_{30} + \theta_p \quad = \quad 180 \cdot deg$$

$$\theta_p := 180 \cdot deg - \theta_{25} - \theta_{40} - \theta_{70} + \theta_{30}$$

$$\theta_p = 39.23 \cdot deg$$

$$p := 36 + \frac{40}{\tan(\theta_p)} \qquad p = 84.993 = \;\; 85$$



$$\mathbf{G_c}(s) := \frac{100 \cdot (s+30)}{(s+25) \cdot (s+40) \cdot (s+70) \cdot (s+85)}$$

Check: $\arg\left[\frac{100 \cdot (s+30)}{(s+25) \cdot (s+40) \cdot (s+70) \cdot (s+85)}\right] = -179.996 \cdot deg$

d) With the compensator in place and a closed-loop pole at the location desired in part b)

i) What is the gain? $\quad k := \frac{1}{\left|\mathbf{G_c}(s)\right|} \qquad k = 1369$

ii) What is the 2% settling time? Use the second-order approximation.

$$T_s = \frac{4}{36} = 0.111 \quad \text{sec}$$

iii) What is the steady-state error to a unit-step input?

$$\mathbf{G_c}(0) = \frac{100 \cdot (0+30)}{(0+25) \cdot (0+40) \cdot (0+70) \cdot (0+85)} = 5.042 \cdot 10^{-4} \qquad e_{step} = \frac{1}{1 + k \cdot \mathbf{G_c}(0)} = 59.161 \cdot \%$$

e) Add another compensator: $\mathbf{C_2}(s) := \frac{s+2}{s}$ and maintain the gain of part d)

i) What is this type of compensator called and what is its purpose?

PI, used to eliminate steady-state error

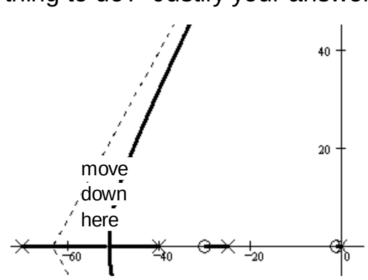
ii) Calculate what you need to to show that this compensator achieved its purpose.

$$\mathbf{G_c}(s) := \frac{100 \cdot (s+30)}{(s+25) \cdot (s+40) \cdot (s+70) \cdot (s+85)} \cdot \frac{(s+2)}{s}$$

$$\mathbf{G_c}(0) = \infty \qquad e_{step} = \frac{1}{1 + k \cdot \infty} = 0 \cdot \%$$

f) With both compensators in place, is there possibility for improvement (quicker settling time speed and/or lower ringing)? If yes, what would be the simplest thing to do? Justify your answer.

A quick sketch of the new root-locus shows that simply decreasing the gain would improve the system



ECE 3510 Root Locus Design Crib Sheet

A.Stolp
3/11/09,
3/8/10

Using 2nd-order approximation: $\dfrac{N(s)}{(s+a)^2+b^2} = \dfrac{N(s)}{s^2+2\cdot a\cdot s+a^2+b^2} = \dfrac{N(s)}{s^2+2\cdot\zeta\cdot\omega_n\cdot s+\omega_n^{\,2}}$

$\omega_n^{\,2} = a^2+b^2$ $\quad\omega_n$ = natural frequency

$\zeta\cdot\omega_n = a$

$\zeta = \dfrac{a}{\omega_n} = \dfrac{a}{\sqrt{a^2+b^2}}$ = damping factor $\qquad \zeta = \sin\left(\operatorname{atan}\left(\dfrac{a}{b}\right)\right)$

Overshoot: $\quad OS = e^{-\pi\cdot\frac{a}{b}} \qquad \%OS = 100\%\cdot e^{-\pi\cdot\frac{a}{b}} \qquad \dfrac{a}{b} = \dfrac{\ln(OS)}{-\pi}$

angle of constant damping line: $90\cdot\deg+\operatorname{atan}\left(\dfrac{a}{b}\right)$

2% settling time: $\quad T_s = \dfrac{4}{a} = \dfrac{4}{\zeta\cdot\omega_n}$

Time of first peak: $\quad T_p = \dfrac{\pi}{b}$

Static error constant (position): $\quad K_p = \lim_{s\to 0} K\cdot C(s)\cdot G(s) \qquad e_{step}(\infty) = e_{step} = \dfrac{1}{1+K_p}$ $\qquad$ Nise p378

Lag compensation improves K_p, K_v and K_a by $\dfrac{z_c}{p_c}$ $\qquad$ IE: $K_{pc} \simeq K_{puc}\cdot\dfrac{z_c}{p_c}$

Searching along a line of constant damping:

Try s values, choosing b: $\quad s = -\left(\dfrac{a}{b}\cdot b\right)+b\cdot j$ $\qquad$ Test: $\arg(G(s))\ \underline{+}180^o$ $\quad$ or $\quad -\mathrm{Re}(G(s)) >> \mathrm{Im}(G(s))$

Linear interpolation: $\quad$ new b = $b_1 - \dfrac{b_2-b_1}{\mathrm{Im}(G(s_2))-\mathrm{Im}(G(s_1))}\cdot\mathrm{Im}(G(s_1))$

Can also try "a" values with slight modification of the above.

Weird forms from Nise book:

$\sigma_d = a \qquad \%OS = 100\%\cdot e^{\frac{-\zeta\cdot\pi}{\sqrt{1-\zeta^2}}}$ $\quad$ p195

$\omega_d = b \qquad \zeta = \dfrac{-\ln(OS)}{\sqrt{\pi^2+(\ln(OS))^2}}$ $\quad$ p195 $\qquad T_p = \dfrac{\pi}{\omega_n\cdot\sqrt{1-\zeta^2}}$ $\quad$ p194

p378 $\quad$ Static error constant (ramp): (velocity) $\quad K_v = \lim_{s\to 0} s\cdot K\cdot C(s)\cdot G(s) \qquad e_{ramp} = \dfrac{1}{K_v}$

Static error constant (parabola): (acceleration) $\quad K_a = \lim_{s\to 0} s^2\cdot K\cdot C(s)\cdot G(s) \qquad e_{parabola} = \dfrac{1}{K_a}$

ECE 3510   homework # RL6   **Root Locus Design**   Due: Sat, 3/18   A.Stolp
a

1. Choice of gain. Each root-locus plot below shows a number of closed-loop pole locations labeled "a", "b", "c", etc.. Each plot has at least two poles. In answering the questions below consider all the closed-loop poles, not just the pole at the labeled location. That is, consider where the other pole(s) are when the gain places the labeled pole at the labeled location. Use a 2nd order approximation in all cases and neglect the partial-fraction coefficients of the poles

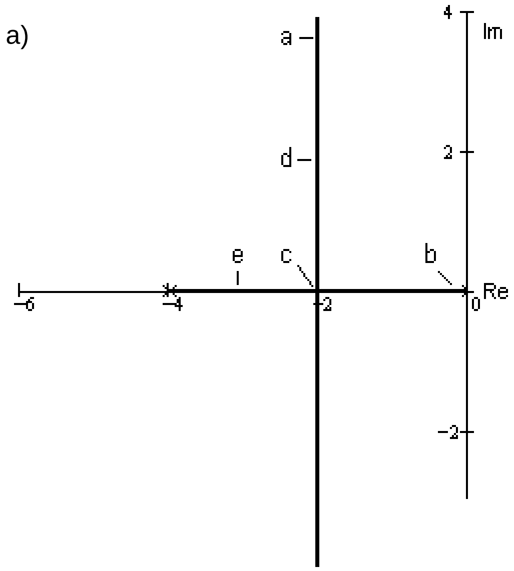
   i) List the closed-loop pole locations (labeled "a", "b", "c", etc.) in order of gain factor, smallest to largest.

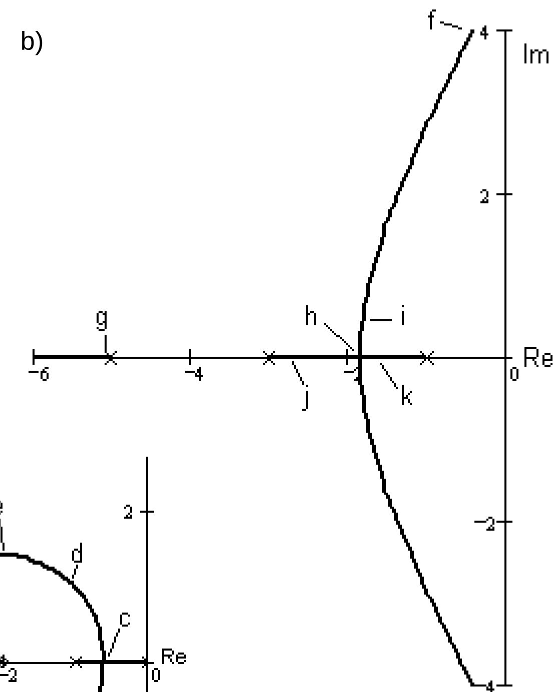
   ii) List the closed-loop pole locations in order of speed of response (measured as the time to get within 4.4% of the final step resonse). List them slowest to fastest.

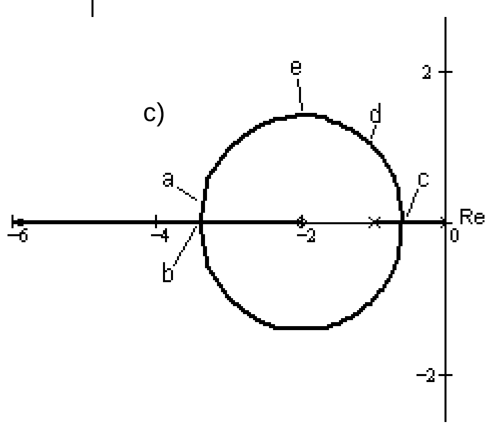
   iii) List the closed-loop pole locations which would result in a step response with absolutely no overshoot.

   iv) List the closed-loop pole locations (not listed in part b) in order of % overshoot. List them least to most.

   v) List the closed-loop pole locations in order of steady-state error to a step input. List them worst to best. (most error to least)







2. Nise 3rd & 4th: Ch.8, problem 46.

   5th ed.: Ch.8, prob 55, 6th: Ch.8, p 57.

   Read sec 4.6 in Nise book. Modify eq. 4.38 (all ed.) with: $\%OS = e^{-\pi \cdot \left|\frac{a}{b}\right|}$ (see Bodson p.51).

   Modify eq. 4.42 with: $T_s = \dfrac{4}{\zeta \cdot \omega_n} = \dfrac{4}{a}$

   a) If you find that more than one value of K will work, choose the highest K. Usually this results in the best steady-state error. In this case that should not theoretically matter because of the motor's pole at 0, but in reality, it still will.

Answers

1. a) i) b, e, c, d, a   ii) b, e, c, a, d OR b, e, a, c, d   iii) b, e, c   iv) d, a

   v) all will result in $e_{ss}(\infty) = 0$ because of open-loop pole at origin. If that were not so then list in order of gain.

   b) i) g, j, k, h, i, f   ii) f, g, j, k, h, i   iii) g, j, k, h,   iv) i, f   v) same as i)

   c) i) c, d, e, a, b   ii) c, d, e, b, a   iii) b, c   iv) a, e, d

   v) all will result in $e_{ss}(\infty) = 0$ because of open-loop pole at origin. If that were not so then list in order of gain.

2. a) 102300   b) 11.14%   c) $K < 715000$

ECE 3510 hw RL7 Root Locus Design Due: Mon, 3/20 A.Stolp b

You may sketch root locus plots and make calculations using a computer program.
Questions and problems from Nise are the same for 3rd & 4th editions unless specified otherwise.

1. Nise Ch. 9 review questions: 3, 4, 5, 9, 10, 11, & 12.

2. Nise Ch. 9 problem 1. For an explanation of the static error constants & calculation of steady-state error, see Nise, section 7.3 or Root Locus Design Crib Sheet. If you use Bodson eq 4.6, include the gain factor (multiply $P(0)C(0)$ by K).

   Use $G(s)$ and damping ratio (factor) from 3rd ed:

   $$G_{uc}(s) := \frac{1}{(s+3)\cdot(s+6)}$$ uc =uncompensated $\zeta := 0.707$

   Design a PI controller and show that it works.

3. Nise Ch. 9 problem 3 Use $G(s)$ and 10% overshoot from 3rd ed: $G_{uc}(s) := \dfrac{1}{(s+1)\cdot(s+3)\cdot(s+5)}$

   a) The static error constant is K_p on our Crib Sheet.

   b) Want to improve to $K_p = 4$ using lag controller.

   c) I suggest you use the SISO tool to show the improvement.

4. Nise Ch. 9 problem 6 Use $G(s)$ from 3rd ed: $G_{uc}(s) := \dfrac{1}{(s+1)\cdot(s+2)\cdot(s+3)\cdot(s+6)}$ use: $\zeta := 0.707$

   a) Shorten settling time to half of what it is without PD compensation.

   b) Calculate the steady state error for a step input.

   For the justification of the 2nd-order assumption, see section 8.7 in Nise. Especially, read the first numbered list and item 3 in the second list. (p. 452 in 3rd ed, p455 in 4th ed, p.416 in 6th ed.)

5. Nise Ch. 9 problem 8 Use $G(s)$ and 20% overshoot from 3rd ed: $G_{uc}(s) := \dfrac{1}{s\cdot(s+5)\cdot(s+15)}$

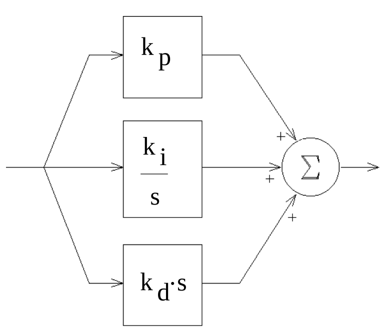
   a) Shorten settling time to 1/4 of what it is without PD compensation.

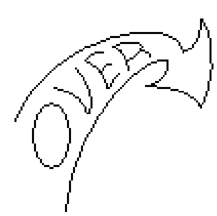
   b) Change design a lead compensator. Move the zero you found in part a) to -3 and finding the required pole.

6. You have designed a compensator with the following:

   A pole at the origin A zero at -0.5 A zero at -10 Gain of 20

   Find the k_p, k_i, & k_d of a PID controller.





Answers

1. ANSWERS TO REVIEW QUESTIONS
 1. Chapter 8: Design via gain adjustment. Chapter 9: Design via cascaded or feedback filters.
 2. A. Permits design for transient responses not on original root locus and unattainable through simple gain adjustments.
    B. Transient response and steady-state error specifications can be met separately and independently without the need for tradeoffs
 3. PI or lag compensation 4. PD or lead compensation 5. PID or lag-lead compensation
 6. A pole is placed on or near the origin to increase or nearly increase the system type, and the zero is placed near the pole in order not to change the transient response.
 7. The zero is placed closer to the imaginary axis than the pole. The total contribution of the pole and zero along with the previous poles and zeros must yield 180^o at the design point. Placing the zero closer to the imaginary axis tends to speed up a slow response.
 8. A PD controller yields a single zero, while a lead network yields a zero and a pole. The zero is closer to the imaginary axis.
 9. Further out along the same radial line drawn from the origin to the uncompensated poles
 10. The PI controller places a pole right at the origin, thus increasing the system type and driving the error to zero. A lag network places the pole only close to the origin yielding improvement but not zero error.
 11. The transient response is approximately the same as the uncompensated system, except after the original settling time has passed. A slow movement toward the new final value is noticed.
 12. 25 times; the improvement equals the ratio of the zero location to the pole location.
 13. No; the feedback compensator's zero is not a zero of the closed-loop system.
 14. A. Response of inner loops can be separately designed; B. Faster responses possible; C. Amplification may not be necessary since signal goes from high amplitude to low.

2. Uncompensated: CL pole $s_{uc} := -4.5 + 4.5 \cdot j$ $K_{uc} := 22.5$ 44.4% steady-state error

   %OS = 4.32% $T_s = 0.889$ sec

   For: $C(s) = \frac{s+0.1}{s}$ CL pole $s_c := -4.472 + 4.472 \cdot j$ $K_{uc} := 22.5$ no steady-state error

   %OS = 4.32% $T_s = 0.894$ sec Using 2nd-order approximation

3. Uncompensated: $s_{uc} := -1.4 + 1.91 \cdot j$ $K := 19.9$ steady-state error is about 43%

   Compensated, want $K_p = 4$, steady-state error of 20% Try: $C(s) = \frac{s+0.3}{s+0.1}$ That should yield a 3x improvement in K_p.

   Matlab output shows a good reduction in steady-state error.

4. Uncompensated: $s_{uc} := -1.05 + 1.05 \cdot j$ $K := 16.65$

   Want $s_c := -2.1 + 2.1 \cdot j$ Need zero at -0.604

   Possible problems with the 2nd-order assumption: Pole at -0.771 is not close enough to the zero at -0.604 to cancel it. Pole at -7.03 is not 5 times farther from $j\omega$ axis than -2.1.

   b) 0.753 75% error! That zero close to the origin is NOT OK.

5. Uncompensated: $s_{uc} := -1.809 + 3.533 \cdot j$ $K := 258$

   Want $s_c := -7.236 + 14.132 \cdot j$ Need zero at -5.422

   Compare to example 9.7 (table 9.8), similar to compensated system except gain. Gain is similar to uncompensated system.

   b) $C(s) = \frac{s+3}{s+94.43}$

6. 210, 100, 20