

# ECE 3510 Phase-Locked Loops

a

Phase-Locked Loops are a bit of a distraction right here, but we need to cover them for next lab. (6 & 7)

Need parts and breadboard for this lab.

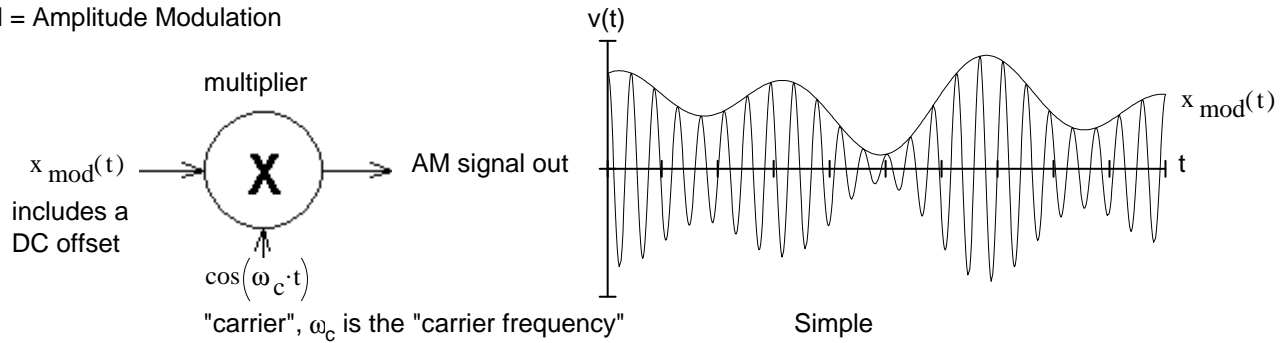
PLL IC is expensive and prone to static & handling damage.

See lab 6 handout

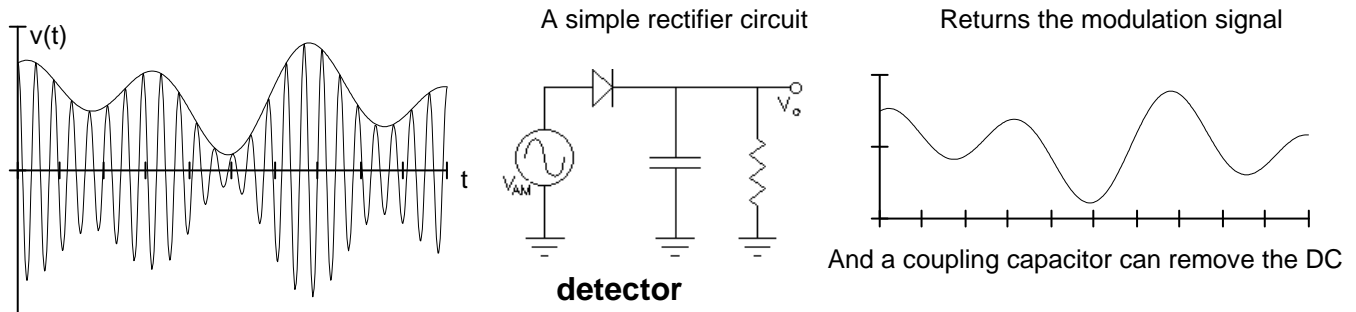
PAY ATTENTION to warnings in the lab.

## Modulation

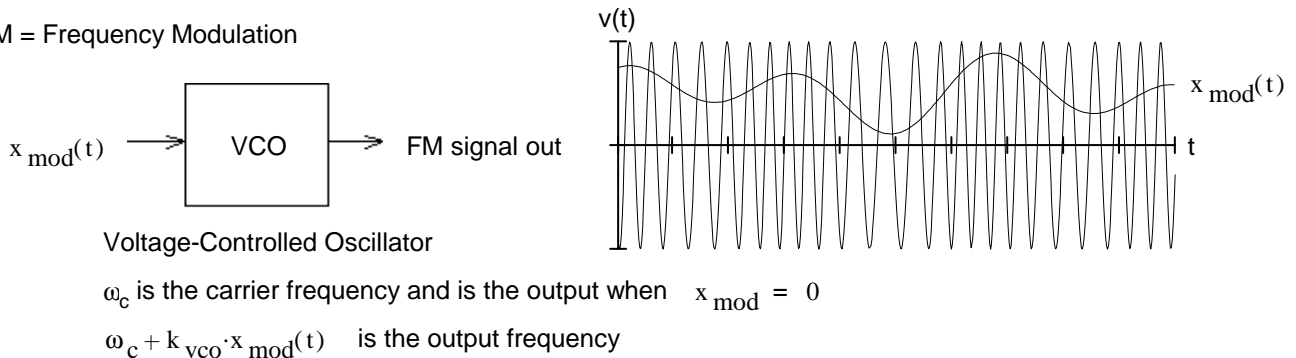
AM = Amplitude Modulation



Demodulation



FM = Frequency Modulation



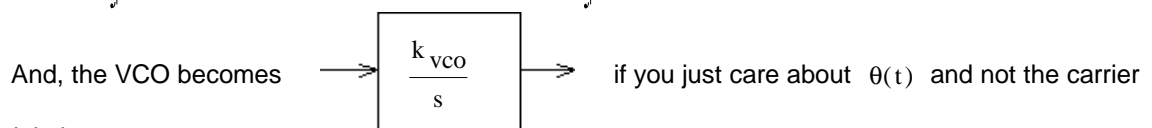
So if:  $\cos(\omega_c \cdot t)$  is the carrier, I guess  $\cos[(\omega_c + k_{vco} \cdot x_{mod}(t)) \cdot t]$  must be the output... **WRONG!**

actually:  $\cos(\theta(t))$

where:  $\theta(t) = \int \omega dt$  is the REAL relationship between  $\theta$  and  $\omega$   
 $= \omega t$  for the unmodulated (steady-state sinusoid) case

so if you want to modulate the frequency:

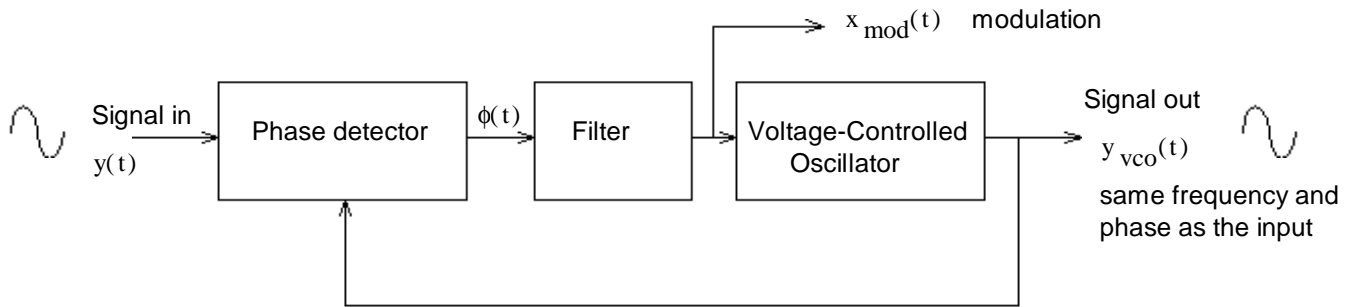
$$\theta(t) = \int (\omega_c + k_{vco} \cdot x_{mod}(t)) dt = \omega_c \cdot t + \int k_{vco} \cdot x_{mod}(t) dt$$



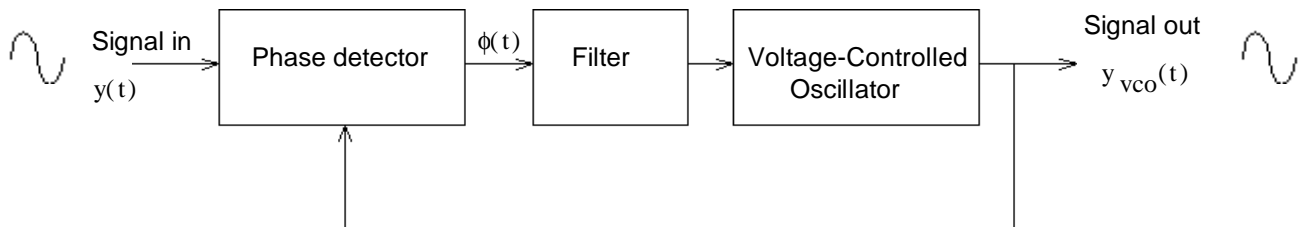
PM = Phase Modulation

One way to demodulate FM is with a Phase-Locked-Loop.

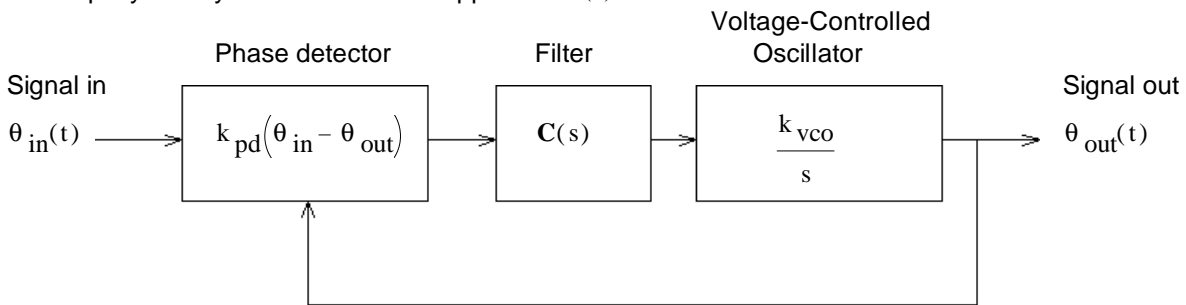
## Phase-Locked Loops p2



To analyze the Phase-Locked-Loop (PLL).



The same loop if you only care about what happens to  $\theta(t)$

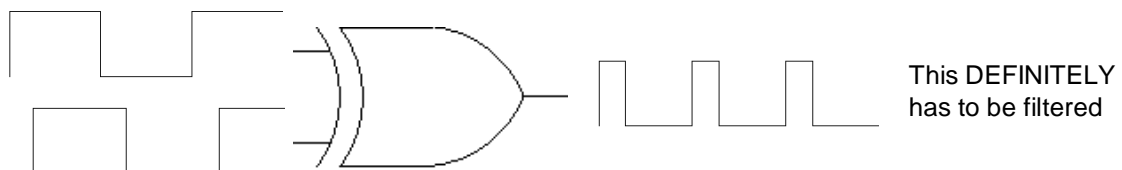


$$G(s) = k_{pd} \cdot \frac{k_{vco}}{s} \cdot C(s) = \frac{k_{pll}}{s} \cdot C(s) \quad \text{where } k_{pll} = k_{pd} \cdot k_{vco}$$

$$\text{Closed-loop: } H(s) = \frac{\frac{k_{pll}}{s} \cdot C(s)}{1 + \frac{k_{pll}}{s} \cdot C(s)} = \frac{k_{pll} \cdot C(s)}{s + k_{pll} \cdot C(s)}$$

At first glance, that filter,  $C(s)$ , doesn't look necessary, but many phase detectors don't put out a nice DC.

Our phase detector in the lab is a good example:



For filter,  $C(s)$ , design, see Bodson, section 4.5.4 and PLL labs.

Your challenge in the lab will be to get a good demodulation and a stable system.

PLLs can also be used for frequency synthesis and motor speed control, etc..

A regular root locus plot is very useful if you want to see how the positions of the closed-loop poles of a feedback system are affected by simple proportional gain in the system. But... what if you want to see how these poles are affected by some other variable in the system, like the filter time-constant in the Phase-Locked-Loop lab? Could we use some of the same concepts to see the affects of some other variable? The answer is yes, we just have to hold the gain constant and rearrange things a bit. An "unconventional root-locus plot" is like a regular root locus plot except that the gain is held constant and the plot shows how the closed-loop poles move as the result of changing some variable *other* than gain.

**To create an unconventional root locus plot:**

1. Determine the gain factor if it can be adjusted, and make it part of the open-loop transfer function,  $G(s)$ . Hold it constant at some number.
2. Determine the denominator of the closed-loop transfer function,  $H(s)$ . Let's call it  $D_H(s)$ .
3. Rearrange  $D_H(s)$  into this form:  $D'(s) + x \cdot N'(s)$  where  $x$  is the variable for which you want to draw the root locus. Notice that  $x$  occupies exactly the same position the gain would normally occupy. Normal:  $D G(s) + k \cdot N G(s)$   
Note: If you cannot rearrange  $D_H(s)$  into this form, then you cannot use this method to create an root locus plot for the variable  $x$ . Now:  $D'(s) + x \cdot N'(s)$
4. Now simply draw a root locus as though  $D'(s)$  was the open-loop denominator and  $N'(s)$  was the open-loop numerator.

**Ex.1** Sketch the unconventional root-locus plot for the open-loop transfer function below. The root-locus should be plotted for an increasing  $a$ . The gain will be held constant at 3

$$G(s) = \frac{(s + 2 \cdot a)}{(s + 5) \cdot (s + a)}$$

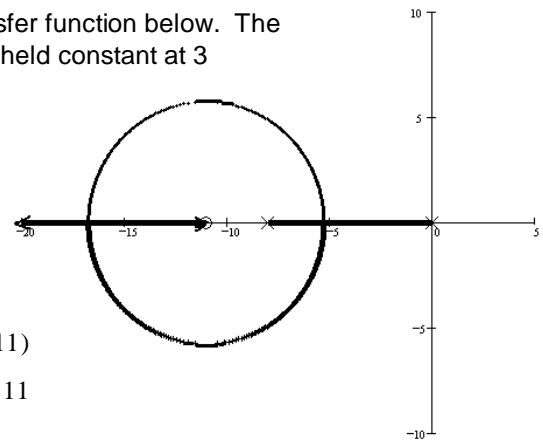
The denominator of the closed-loop transfer function:

$$(s + 5) \cdot (s + a) + 3 \cdot (s + 2 \cdot a) = s^2 + s \cdot a + 8 \cdot s + 11 \cdot a$$

$$= s^2 + 8 \cdot s + s \cdot a + 11 \cdot a = s \cdot (s + 8) + a \cdot (s + 11)$$

$$D'(s) = s \cdot (s + 8) \quad N'(s) = (s + 11)$$

poles at 0 and -8      zero at -11



**Ex.2** Sketch the unconventional root-locus plot for the open-loop transfer function below. The root-locus should be plotted for an increasing  $g$ .

$$G(s) = \frac{k \cdot s \cdot (g \cdot s + 1)}{(s + 4 \cdot g) \cdot (s + 5)} \quad k = 2 \quad \text{and is constant}$$

The denominator of the closed-loop transfer function:

$$(s + 4 \cdot g) \cdot (s + 5) + 2 \cdot s \cdot (g \cdot s + 1)$$

$$s^2 + 5 \cdot s + 4 \cdot g \cdot s + 20 \cdot g + 2 \cdot g \cdot s^2 + 2 \cdot s$$

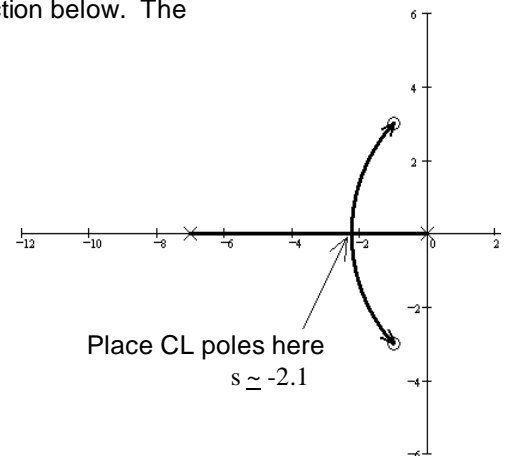
$$s^2 + 7 \cdot s + 2 \cdot g \cdot s^2 + 4 \cdot g \cdot s + 20 \cdot g$$

$$s \cdot (s + 7) + g \cdot (2 \cdot s^2 + 4 \cdot s + 20)$$

$$D'(s) \quad g \cdot N'(s)$$

$$s \cdot (s + 7) + g \cdot 2 \cdot (s^2 + 2 \cdot s + 10)$$

$$a := -1 \quad b = \sqrt{10 - 1^2} = 3$$



The arrow points to a desirable place for the closed-loop poles for minimal ringing and the shortest settling time.

To find the value

of  $g$  needed:  $0 = s \cdot (s + 7) + 2 \cdot g \cdot (s^2 + 2 \cdot s + 10)$

solve for  $g = \frac{-(s^2 + 7 \cdot s)}{(2 \cdot s^2 + 4 \cdot s + 20)}$

if  $s := -2.1$   $g = \frac{-(s^2 + 7 \cdot s)}{(2 \cdot s^2 + 4 \cdot s + 20)} = 0.504$

**Ex.3** From E3, S12 Sketch the unconventional root-locus plot for the open-loop transfer function below. The root-locus should be plotted for an increasing  $m$ .

$$G(s) = \frac{k \cdot (s + 30)}{(m \cdot s + s - 10) \cdot (s + 4)} \quad k = 2 \text{ and is fixed}$$

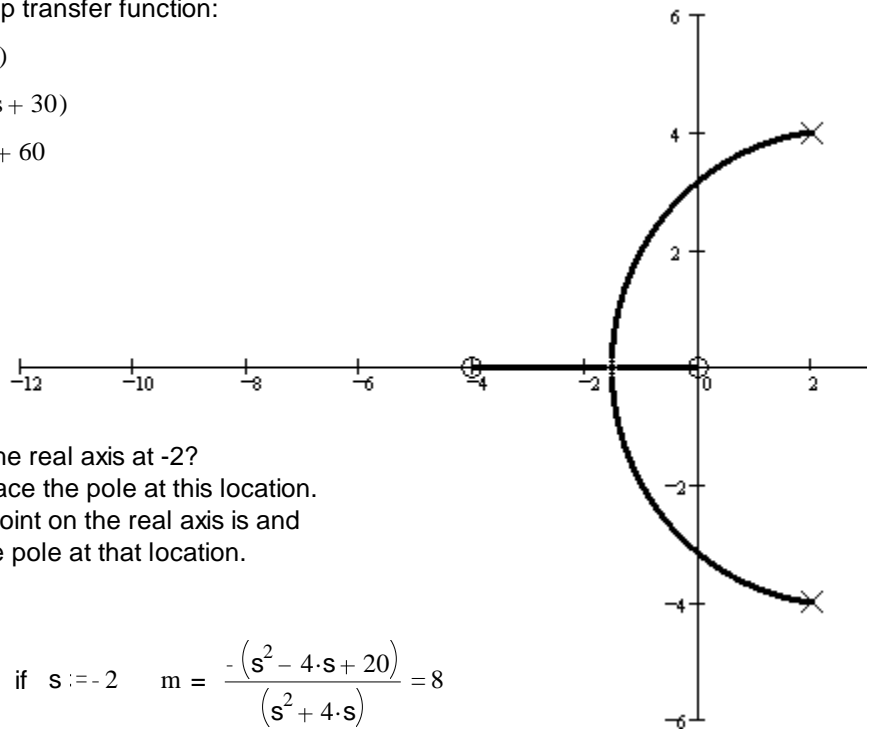
The denominator of the closed-loop transfer function:

$$\begin{aligned} &(m \cdot s + s - 10) \cdot (s + 4) + 2 \cdot (s + 30) \\ &m \cdot s^2 + 4 \cdot m \cdot s + s^2 - 6 \cdot s - 40 + 2 \cdot (s + 30) \\ &m \cdot s^2 + 4 \cdot m \cdot s + s^2 - 6 \cdot s - 40 + 2 \cdot s + 60 \\ &m \cdot s^2 + 4 \cdot m \cdot s + s^2 - 4 \cdot s + 20 \end{aligned}$$

$$\begin{aligned} &D'(s) \quad m \cdot N'(s) \\ &(s^2 - 4 \cdot s + 20) + m \cdot (s^2 + 4 \cdot s) \end{aligned}$$

$$a := 2$$

$$b := \sqrt{20 - 2^2} \quad b = 4$$



b) Can you place a closed-loop pole on the real axis at -2?

If yes, find the value of  $m$  needed to place the pole at this location.

If no, indicate what you think the best point on the real axis is and find the value of  $m$  needed to place the pole at that location.

$$0 = (s^2 - 4 \cdot s + 20) + m \cdot (s^2 + 4 \cdot s)$$

$$\text{solve for } m = \frac{-(s^2 - 4 \cdot s + 20)}{(s^2 + 4 \cdot s)} \quad \text{if } s := -2 \quad m = \frac{-(s^2 - 4 \cdot s + 20)}{(s^2 + 4 \cdot s)} = 8$$

**Ex.4** From E3, S13 Sketch the unconventional root-locus plot for the open-loop transfer function below. The root-locus should be plotted for an increasing  $x$ .

$$G(s) = \frac{k \cdot (5 \cdot (s + 2 \cdot x) - 6)}{s \cdot (x \cdot s + 2 \cdot (s + 2 \cdot x))} \quad k = 2 \text{ and is fixed}$$

The denominator of the closed-loop transfer function:

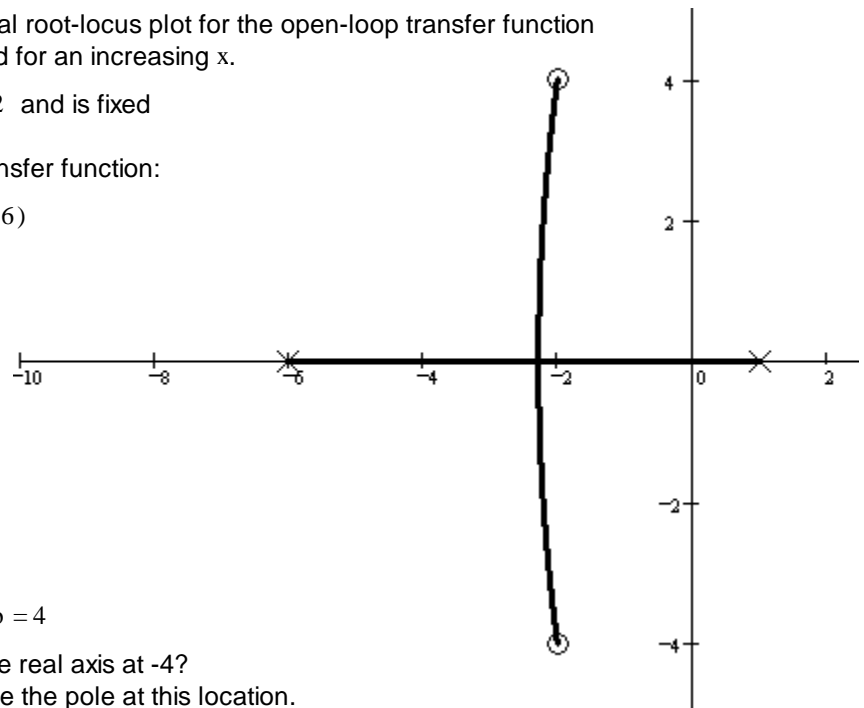
$$\begin{aligned} &s \cdot (x \cdot s + 2 \cdot (s + 2 \cdot x)) + 2 \cdot (5 \cdot (s + 2 \cdot x) - 6) \\ &x \cdot s^2 + 2 \cdot s^2 + 4 \cdot x \cdot s + 10 \cdot (s + 2 \cdot x) - 12 \\ &x \cdot s^2 + 2 \cdot s^2 + 4 \cdot x \cdot s + 10 \cdot s + 20 \cdot x - 12 \\ &2 \cdot s^2 + 10 \cdot s - 12 + x \cdot s^2 + 4 \cdot x \cdot s + 20 \cdot x \\ &2 \cdot (s^2 + 5 \cdot s - 6) + x \cdot (s^2 + 4 \cdot s + 20) \end{aligned}$$

$$D'(s) \quad x \cdot N'(s)$$

$$2 \cdot (s + 6) \cdot (s - 1) + x \cdot (s^2 + 4 \cdot s + 20)$$

$$a := -2$$

$$b := \sqrt{20 - 2^2} \quad b = 4$$



b) Can you place a closed-loop pole on the real axis at -4?

If yes, find the value of  $x$  needed to place the pole at this location.

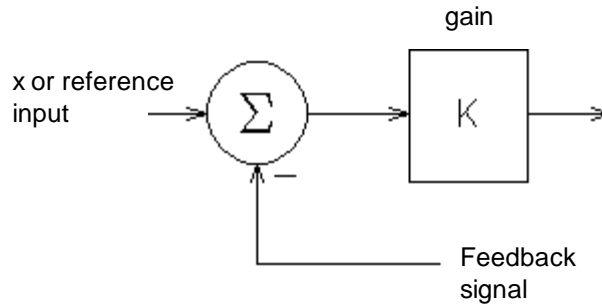
If no, indicate what you think the best point on the real axis is and find the value of  $x$  needed to place the pole at that location.

$$0 = 2 \cdot (s + 6) \cdot (s - 1) + x \cdot (s^2 + 4 \cdot s + 20)$$

$$\text{solve for } x = \frac{-(2 \cdot (s + 6) \cdot (s - 1))}{(s^2 + 4 \cdot s + 20)} \quad \text{if } s := -4 \quad x = \frac{-(2 \cdot (s + 6) \cdot (s - 1))}{(s^2 + 4 \cdot s + 20)} = 1$$

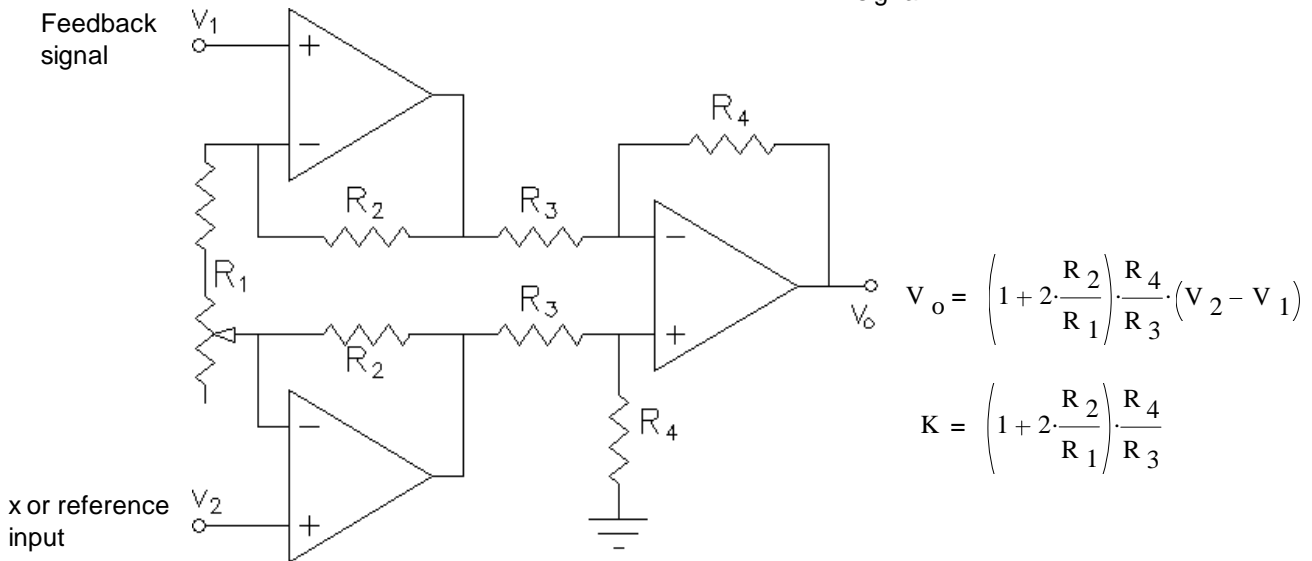
**Implementation (Physical Realization) of Feedback System Components and Compensators**

One way to implement this:

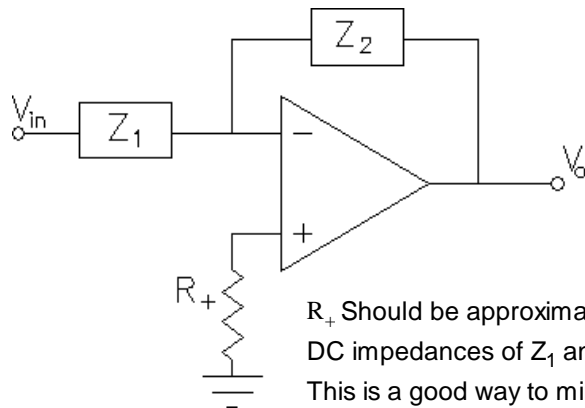


A. Stolp  
2/24/09,  
rev,

Is the **instrumentation amplifier**:



To build **active compensators**, use this basic circuit and then consult Table 9.10 (p555) in the Nise textbook.



$R_+$  Should be approximately the parallel combination of the DC impedances of  $Z_1$  and  $Z_2$ .  
This is a good way to minimize the effects of the Op-amp's bias currents.

**Beware!** This is an inverting circuit. You will have to follow it with another inverter.






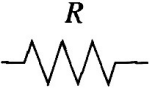
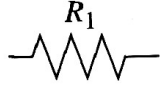
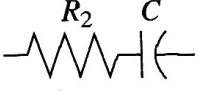
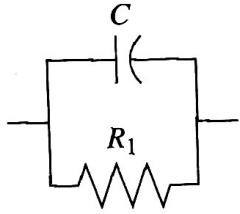

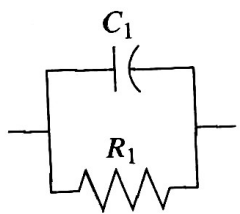
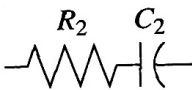
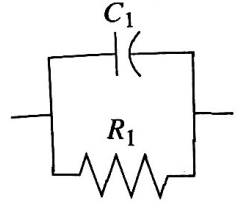
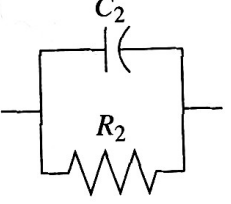
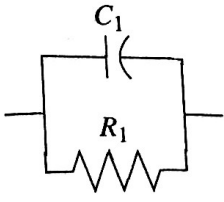
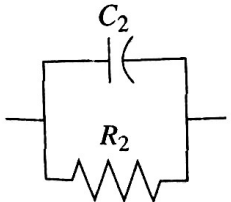
Or... you could just swap the inputs to the instrumentation amplifier, if you are using one.

The resistors used in Op-amp circuits should be 100Ω to 1MΩ, and preferably 1kΩ to 100kΩ.

These Op-amp circuits require + and - power supplies.

To build **passive compensators**, consult Table 9.11 (p558) in the Nise textbook.

Table 9.10 Active realization of controllers and compensators, using an operational amplifier

Function	$Z_1(s)$	$Z_2(s)$	$G_c(s) = -\frac{Z_2(s)}{Z_1(s)}$
Gain			$-\frac{R_2}{R_1}$
Integration			$-\frac{1}{RCs}$
Differentiation			$-RCs$
PI controller			$-\frac{R_2}{R_1} \left( s + \frac{1}{R_2 C} \right)$
PD controller			$-R_2 C \left( s + \frac{1}{R_1 C} \right)$
PID controller			$-\left[ \left( \frac{R_2}{R_1} + \frac{C_1}{C_2} \right) + R_2 C_1 s + \frac{R_1 C_2}{s} \right]$
Lag compensation			$-\frac{C_1}{C_2} \frac{\left( s + \frac{1}{R_1 C_1} \right)}{\left( s + \frac{1}{R_2 C_2} \right)}$ where $R_2 C_2 > R_1 C_1$
Lead compensation			$-\frac{C_1}{C_2} \frac{\left( s + \frac{1}{R_1 C_1} \right)}{\left( s + \frac{1}{R_2 C_2} \right)}$ where $R_1 C_1 > R_2 C_2$

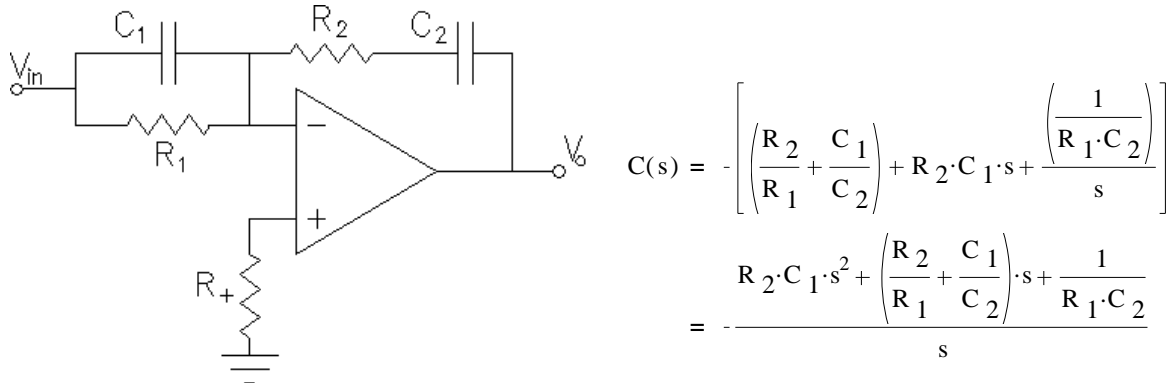
## PID Design Example Implementation p.1

$$C(s) = 0.418 \cdot \frac{(s+0.1) \cdot (s+24.28)}{s} = 0.418 \cdot \frac{s^2 + 24.38s + 2.48}{s} = \frac{0.418 \cdot s^2 + 10.19s + 1.037}{s}$$

$$= \frac{k_d \cdot s^2 + k_p \cdot s + k_i}{s}$$

$k_d := 0.418 \cdot \text{sec}$   
 $k_p := 10.19$   
 $k_i := \frac{1.037}{\text{sec}}$

Using the PID design from table 9.10 (p.555) in Nise:  
 This could be implemented with:



If we use an instrumentation amplifier with a gain of, say 3, and invert the two inputs to "fix" the inversion above, then:

$$R_2 \cdot C_1 = \frac{k_d}{3} = 0.139 \cdot \text{sec} \quad \left( \frac{R_2}{R_1} + \frac{C_1}{C_2} \right) = \frac{10.19}{3} = 3.397 \quad \frac{1}{R_1 \cdot C_2} = \frac{k_i}{3} = 0.346 \cdot \text{sec}^{-1}$$

There are 4 component values to select and only 3 coefficients to match, so arbitrarily select 1 component.

Try  $C_1 := 0.1 \cdot \mu\text{F}$        $R_2 := \frac{0.139 \cdot \text{sec}}{C_1}$        $R_2 = 1.39 \cdot \text{M}\Omega$       too high

Try  $C_1 := 10 \cdot \mu\text{F}$        $R_2 := \frac{0.139 \cdot \text{sec}}{C_1}$        $R_2 = 13.9 \cdot \text{k}\Omega$       Use  $R_2 := 14 \cdot \text{k}\Omega$

Now  $\left( \frac{R_2}{R_1} + \frac{C_1}{C_2} \right) = 3.397$       And  $\frac{1}{R_1 \cdot C_2} = \frac{0.346}{\text{sec}}$       So,  $C_2 = \frac{\text{sec}}{0.346 \cdot R_1}$

Combining:  $= \frac{14 \cdot \text{k}\Omega}{R_1} + \frac{10 \cdot \mu\text{F} \cdot (0.346 \cdot R_1)}{\text{sec}} - 3.397 = 0$

Solve:  $R_1 := \frac{3.397 + \sqrt{3.397^2 - 4 \cdot \frac{C_1 \cdot 0.346 \cdot R_2}{\text{sec}}}}{2 \cdot \frac{C_1 \cdot 0.346}{\text{sec}}}$        $R_1 = 977.653 \cdot \text{k}\Omega$       Use  $R_1 := 1 \cdot \text{M}\Omega$   
 Let  $R_+ = 1\text{M}\Omega$

$C_2 := \frac{\text{sec}}{0.346 \cdot R_1}$        $C_2 = 2.89 \cdot \mu\text{F}$       Use  $C_2 := 3 \cdot \mu\text{F}$

Test:  $R_2 \cdot C_1 = 0.14 \cdot \text{sec}$        $\left( \frac{R_2}{R_1} + \frac{C_1}{C_2} \right) = 3.347$        $\frac{1}{R_1 \cdot C_2} = 0.333 \cdot \text{sec}^{-1}$       Close enough

## PID Design Example Implementation p.2

Instrumentation amp gain:  $K_{inst} := 3$        $k_d := 0.418 \cdot \text{sec}$        $k_p := 10.19$        $k_i := \frac{1.037}{\text{sec}}$

$$R_2 \cdot C_1 = k'_d := \frac{k_d}{3} \qquad \left( \frac{R_2}{R_1} + \frac{C_1}{C_2} \right) = k'_p := \frac{10.19}{3} \qquad \frac{1}{R_1 \cdot C_2} = k'_i := \frac{k_i}{3}$$

$k'_d = 0.139 \cdot \text{sec}$        $k'_p = 3.397$        $k'_i = 0.346 \cdot \text{sec}^{-1}$

For standard capacitor values from  $C_{10} = 0.01 \cdot \mu\text{F}$  to  $C_{32} = 82 \cdot \mu\text{F}$

Combining equations above

$$\left( \frac{R_2}{R_1} + \frac{C_1}{C_2} \right) = k'_p = \left[ \frac{R_2}{R_1} + \frac{C_1}{\left( \frac{1}{k'_i \cdot R_1} \right)} \right] \quad \text{OR} \quad R_2 + C_1 \cdot k'_i \cdot R_1^2 - k'_p \cdot R_1 = 0$$

Rearrange:  $C_1 \cdot k'_i \cdot R_1^2 - k'_p \cdot R_1 + R_2 = 0$

And solve:  $R_{1_i} := \frac{k'_p + \sqrt{k'_p^2 - 4 \cdot (C_{1_i} \cdot k'_i \cdot R_{2_i})}}{2 \cdot (C_{1_i} \cdot k'_i)}$

Finally:  $C_{2_i} := \frac{1}{k'_i \cdot R_{1_i}}$

Possible solutions	$C_{1_i}$ $\mu\text{F}$	$R_{2_i}$ $\text{k}\Omega$	$R_{1_i}$ $\text{k}\Omega$	$C_{2_i}$ $\mu\text{F}$
	0.12	1161.11	81544	0.035
	0.15	928.89	65235	0.044
	0.18	774.07	54362	0.053
	0.22	633.33	44478	0.065
	0.33	422.22	29652	0.098
	0.39	357.26	25090	0.115
	0.47	296.45	20820	0.139
	0.56	248.81	17474	0.166
	0.68	204.9	14390	0.201
	0.82	169.92	11933	0.242
	1	139.33	9785	0.296
	1.2	116.11	8154	0.355
	1.5	92.89	6523	0.443
	1.8	77.41	5436	0.532
	2.2	63.33	4448	0.65
	3.3	42.22	2965	0.976
	3.9	35.73	2509	1.153
	4.7	29.65	2082	1.39
	5.6	24.88	1747	1.656
	6.8	20.49	1439	2.01
	8.2	16.99	1193	2.424
	10	13.93	979	2.956
	12	11.61	815	3.548
	15	9.29	652	4.435
	18	7.74	544	5.322
	22	6.33	445	6.504
	33	4.22	297	9.756
	39	3.57	251	11.53
	47	2.96	208	13.895
	56	2.49	175	16.556
	68	2.05	144	20.104
	82	1.7	119	24.243



1. A compensator:  $C(s) = \frac{s + 2 \cdot a}{s + a}$  and a plant:  $P(s) = \frac{k_p}{s + 6}$

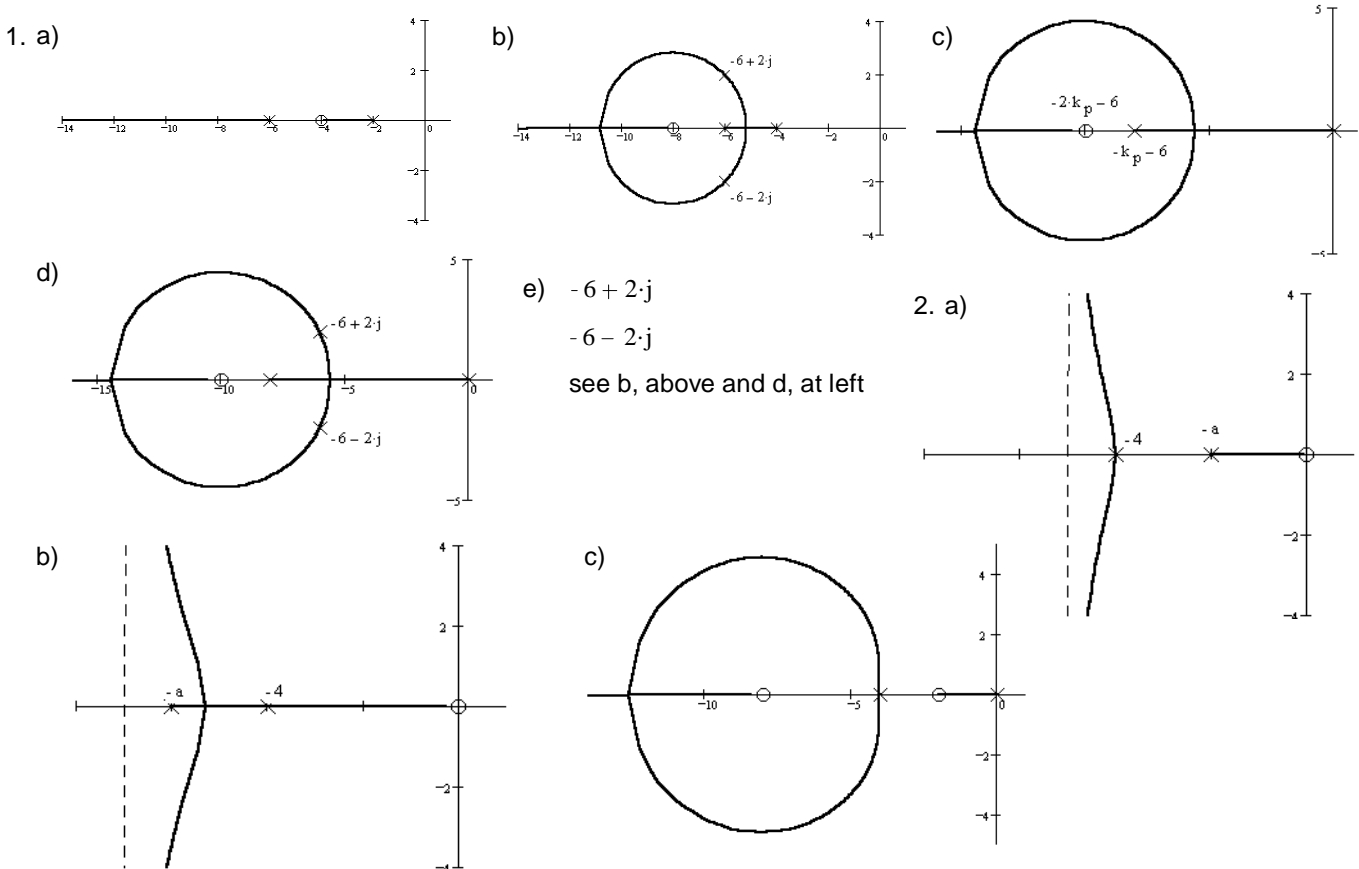
are combined to form an open-loop transfer function:  $G(s) = \frac{k_p \cdot (s + 2 \cdot a)}{(s + 6)(s + a)}$

- a) Sketch a conventional root-locus plot taking  $k_p$  as the gain and  $a = 2$ .
- b) Sketch a conventional root-locus plot taking  $k_p$  as the gain and  $a = 4$ .
- c) Sketch an unconventional root-locus plot taking  $a$  as the "gain".  $k_p$  is not specified.
- d) Sketch an unconventional root-locus plot taking  $a$  as the "gain" and  $k_p = 2$ .
- e) What are the closed-loop poles if  $a = 4$  and  $k_p = 2$ ? Show that these poles fit on the root locus drawn in part b) as well as the root locus drawn in part d).

2. A compensator:  $C(s) = \frac{a}{s + a}$  and a plant:  $P(s) = \frac{k_p \cdot s}{(s + 4)^2}$  are combined to form an open-loop transfer function.

- a) Sketch a conventional root-locus plot taking  $k_p$  as the gain and some  $a < 4$ .
- b) Sketch a conventional root-locus plot taking  $k_p$  as the gain and some  $a > 4$ .
- c) Sketch an unconventional root-locus plot taking  $a$  as the "gain" and  $k_p = 2$ .

**Answers**



Implementation (Physical Realization) of Feedback System Components and Compensators

- Design an instrumentation amplifier with a variable gain from approximately 1 to 40. This could be used as the summing (difference) block and the gain block of a feedback loop.

Your answer should use different values than my answer below.

- Design an active circuit which could follow the circuit you just designed and implement the PID compensator of Example 9.5 in the Nise textbook.

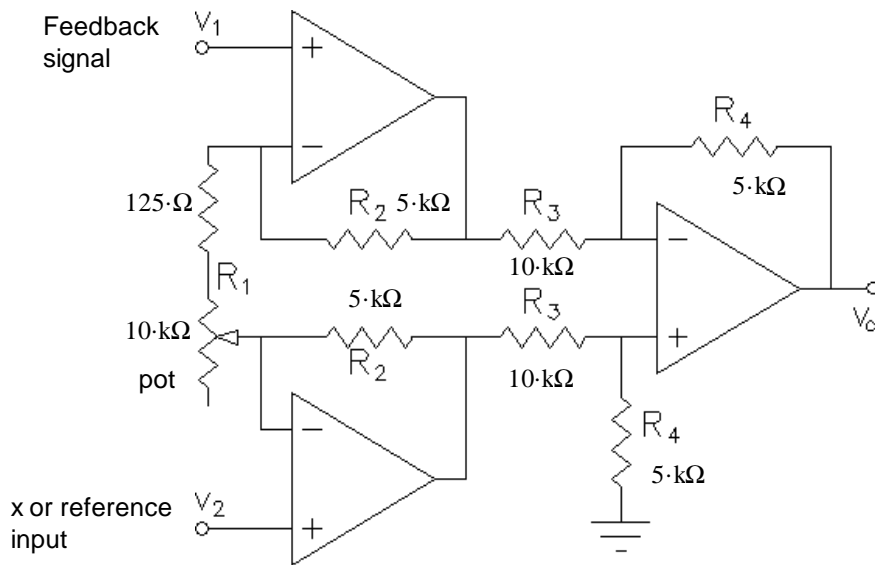
Note: your capacitor values may come out overly large due to the artificially small poles and zeroes.

Your answer should use different values than my answer below.

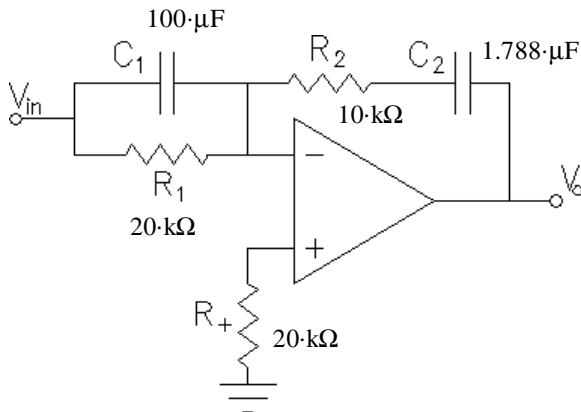
**Answers**

Your answers should use different values than my answers.

1.



2.



And either swap the inputs to the Instrumentation amplifier or follow this with a unity gain inverting amplifier.