Complex Numbers

ECE 3510

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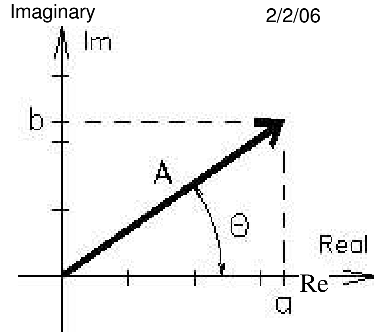
10/18/00
2/2/06



$j = \sqrt{-1}$ the imaginary number

Rectangular Form $\mathbf{A} = a + b\cdot j$

$\mathrm{Re}(\mathbf{A}) = a \qquad \mathrm{Im}(\mathbf{A}) = b$

Polar Form $\mathbf{A} = A\cdot e^{j\cdot\theta}$

$\mathrm{Re}(\mathbf{A}) = A\cdot\cos(\theta) \qquad \mathrm{Im}(\mathbf{A}) = A\cdot\sin(\theta)$

Conversions $A = |\mathbf{A}| = \sqrt{a^2 + b^2} \qquad \theta = \arg(\mathbf{A}) = \mathrm{atan}\left(\frac{b}{a}\right)$

$a = A\cdot\cos(\theta) \qquad b = A\cdot\sin(\theta)$

$\mathbf{A} = A\cdot e^{j\cdot\theta} = A\cdot\cos(\theta) + A\cdot\sin(\theta)\cdot j \qquad \mathbf{A} = a + b\cdot j = \left(\sqrt{a^2+b^2}\right)\cdot e^{j\cdot\mathrm{atan}\left(\frac{b}{a}\right)}$

Special Cases $j := \sqrt{-1} = e^{j\cdot 90\cdot deg} \qquad \frac{1}{j} = -j = e^{-j\cdot 90\cdot deg} \qquad e^{j\cdot 0\cdot deg} = 1 \qquad e^{-j\cdot 180\cdot deg} = e^{-j\cdot 180\cdot deg} = -1$

$j\cdot e^{j\cdot\theta} = e^{j\cdot(\theta + 90\cdot deg)}$

Define a 2nd number: rect: $\mathbf{D} = c + d\cdot j$ polar: $\mathbf{D} = D\cdot e^{j\cdot\phi}$

Equality $\mathbf{A} = \mathbf{D}$ if and only if $a = c$ and $b = d$ OR $A = D$ and $\theta = \phi$

Addition and Subtraction

$\mathbf{A} + \mathbf{D} = (a + b\cdot j) + (c + d\cdot j) = (a + c) + (b + d)\cdot j$

$\mathbf{A} - \mathbf{D} = (a + b\cdot j) - (c + d\cdot j) = (a - c) + (b - d)\cdot j$

Convert polars to rectangular form first

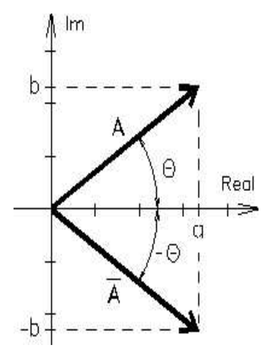
Multiplication and Division $\mathbf{A}\cdot\mathbf{D} = (a + b\cdot j)\cdot(c + d\cdot j) = (a\cdot c - b\cdot d) + (b\cdot c + a\cdot d)\cdot j$

Rectangular: $\frac{\mathbf{A}}{\mathbf{D}} = \frac{a + b\cdot j}{c + d\cdot j} = \frac{a + b\cdot j}{c + d\cdot j}\cdot\frac{c - d\cdot j}{c - d\cdot j} = \frac{a\cdot c + b\cdot d}{c^2 + d^2} + \frac{b\cdot c - a\cdot d}{c^2 + d^2}\cdot j$

Polar: $\mathbf{A}\cdot\mathbf{D} = A\cdot e^{j\cdot\theta}\cdot D\cdot e^{j\cdot\phi} = A\cdot D\cdot e^{j\cdot(\theta + \phi)}$

$\frac{\mathbf{A}}{\mathbf{D}} = \frac{A\cdot e^{j\cdot\theta}}{D\cdot e^{j\cdot\phi}} = \frac{A}{D}\cdot e^{j\cdot(\theta - \phi)}$

Powers $\mathbf{A}^n = A^n\cdot e^{j\cdot n\cdot\theta} = A^n\cdot\cos(n\cdot\theta) + A^n\cdot\sin(n\cdot\theta)\cdot j$ Convert rectangulars first, usually



Conjugates

complex number	Conjugate
$\mathbf{A} = a + b\cdot j$	$\overline{\mathbf{A}} = a - b\cdot j$
$\mathbf{A} = A\cdot e^{j\cdot\theta}$	$\overline{\mathbf{A}} = A\cdot e^{-j\cdot\theta}$
$\mathbf{F} = \frac{3 + 5\cdot j}{(2 - 6\cdot j)\cdot e^{j\cdot 40\cdot deg}}$	$\overline{\mathbf{F}} = \frac{3 - 5\cdot j}{(2 + 6\cdot j)\cdot e^{-j\cdot 40\cdot deg}}$

$\overline{\overline{\mathbf{A}}} = \mathbf{A}$

Euler's equation $e^{j\cdot\alpha} = \cos(\alpha) + j\cdot\sin(\alpha)$ OR: $\cos(\alpha) = \frac{e^{j\cdot\alpha} + e^{-j\cdot\alpha}}{2} \qquad \sin(\alpha) = \frac{e^{j\cdot\alpha} - e^{-j\cdot\alpha}}{2\cdot j}$

$e^{j\cdot(\omega\cdot t + \theta)} = \cos(\omega\cdot t + \theta) + j\cdot\sin(\omega\cdot t + \theta)$

$\mathrm{Re}\left[e^{j\cdot(\omega\cdot t + \theta)}\right] = \cos(\omega\cdot t + \theta)$

If we freeze this at time t=0, then we can represent $\cos(\omega\cdot t + \theta)$ by $e^{j\cdot\theta}$

Calculus Remember, when we write $e^{j\cdot\theta}$, we really mean $e^{j\cdot(\omega\cdot t + \theta)}$

$$\frac{d}{dt}\mathbf{A} = \frac{d}{dt}\left(A\cdot e^{j\cdot\theta}\right) = j\cdot\omega\cdot A\cdot e^{j\cdot\theta} = \omega\cdot A\cdot e^{j\cdot(\theta + 90\cdot deg)}$$

$$\int \mathbf{A}\, dt = \int A\cdot e^{j\cdot\theta}\, dt = \frac{1}{j\cdot\omega}\cdot A\cdot e^{j\cdot\theta} = \frac{1}{\omega}\cdot A\cdot e^{j\cdot(\theta - 90\cdot deg)}$$

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Sinusoidal Response Notes

For BIBO Systems

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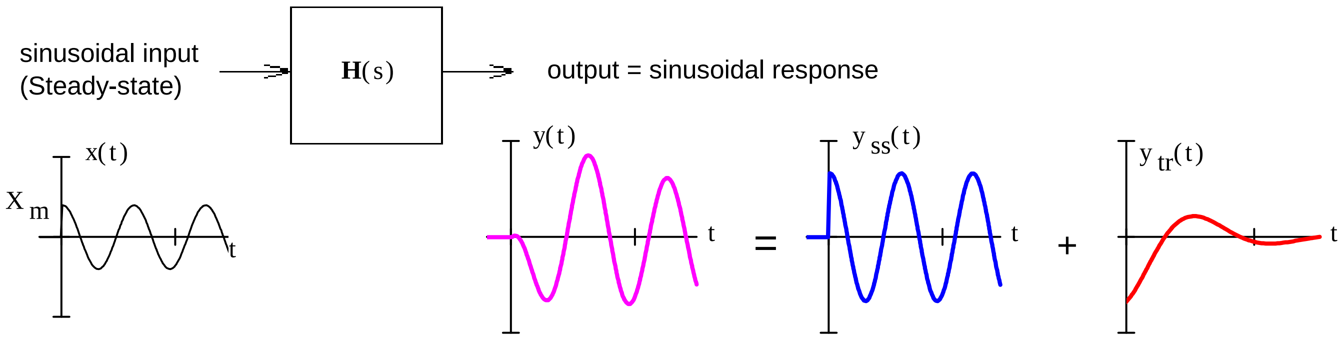
The sinusoidal response of a system is the output when the input is a sinusoidal (which starts at time = 0).

System Sinusoidal Response



Complete step response = steady-state response + transient response

$$\mathbf{Y}(s) = \mathbf{X}(s)\cdot\mathbf{H}(s) \quad = \quad X_m\cdot\mathbf{H}(j\omega)\cdot u(t) \quad + \quad \mathbf{Y_{tr}}(s)$$

$\mathbf{H}(j\omega)$ = phasor-type transfer function

Sinusoidal Input

$$\cos(\omega t)\cdot u(t) \quad \Longleftrightarrow \quad \frac{s}{s^2+\omega^2} \qquad\qquad \sin(\omega t)\cdot u(t) \quad \Longleftrightarrow \quad \frac{\omega}{s^2+\omega^2}$$

General sinusoidal input: $\left[X_{mc}\cdot\cos(\omega t) + X_{ms}\cdot(\sin(\omega t)\cdot u(t))\right]\cdot u(t)$ $\qquad \mathbf{X}(s) = \dfrac{X_{mc}\cdot s + X_{ms}\cdot\omega}{s^2+\omega^2}$

OR $X_m\cdot\cos(\omega t+\theta)\cdot u(t) = \left[X_{mc}\cdot\cos(\omega t) + X_{ms}\cdot(\sin(\omega t)\cdot u(t))\right]\cdot u(t)$

$X_{mc} = X_m\cdot\cos(\theta) \qquad X_{ms} = -X_m\cdot\sin(\theta)$ $\quad$ note that the sine carries the opposite sign as you might expect.

Steady-State Response & $\mathbf{H}(j\omega)$

$$\mathbf{Y}(s) = \mathbf{X}(s)\cdot\mathbf{H}(s) = \frac{X_{mc}\cdot s + X_{ms}\cdot\omega}{s^2+\omega^2}\cdot\mathbf{H}(s)$$ = Complete sinusoidal response

$$= \frac{A\cdot s + B\cdot\omega}{s^2+\omega^2} + \frac{C}{(\ )} + \frac{D}{(\ )} + \frac{E}{(\ )} + \ldots$$

partial fraction expansion:

$$\mathbf{Y}(s) = \frac{X_{mc}\cdot s + X_{ms}\cdot\omega}{s^2+\omega^2}\cdot\mathbf{H}(s) = \frac{A\cdot s + B\cdot\omega}{s^2+\omega^2} + \left[\frac{C}{(\ )} + \frac{D}{(\ )} + \frac{E}{(\ )}\right]\cdot s$$

steady-state response + transient response

$\mathbf{Y_{ss}}(s) \quad + \quad \mathbf{Y_{tr}}(s)$

multiply both sides by: $\left(s^2+\omega^2\right)$

$$\left(X_{mc}\cdot s + X_{ms}\cdot\omega\right)\cdot\mathbf{H}(s) = A\cdot s + B\cdot\omega + \left[\frac{C}{(\ )} + \frac{D}{(\ )} + \frac{E}{(\ )}\right]\cdot\left(s^2+\omega^2\right)$$

set $s := j\omega$

$$\left(X_{mc}\cdot j\omega + X_{ms}\cdot\omega\right)\cdot\mathbf{H}(j\omega) = A\cdot j\omega + B\cdot\omega + \left[\frac{C}{(\ )} + \frac{D}{(\ )} + \frac{E}{(\ )}\right]\cdot 0$$

divide both sides by $j\omega$ $\quad \left(X_{mc} - X_{ms}\cdot j\right)\cdot\mathbf{H}(j\omega)$

$$\mathbf{X}(\omega)\cdot\mathbf{H}(j\omega) = A - B\cdot j = \mathbf{Y_{ss}}(\omega)$$ = steady-state response in phasor form

(real is cosine, imaginary is **-**sine)

$\mathbf{X}(\omega)$ = the input expressed in phasor form NOT $\mathbf{X}(s)$ with $s := \omega$ or $s := j\omega$ (that would be ∞)

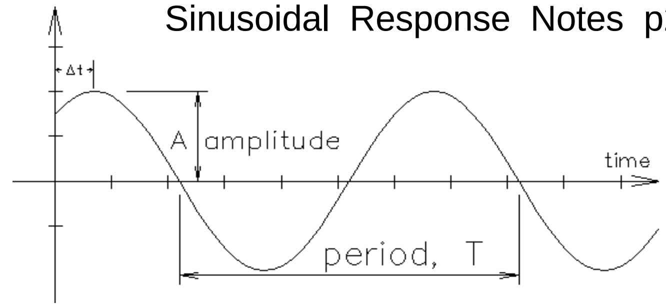
$\mathbf{H}(j\omega)$ = the steady-state sinusoidal transfer function

= phasor-type transfer function

The **transient part** would be found by finishing the partial-fraction expansion.

Steady-State Response by Phasors

Time-domain sinusoidals



T = Period

f = frequency, cycles / second $\quad f = \frac{1}{T} = \frac{\omega}{2\cdot\pi}$

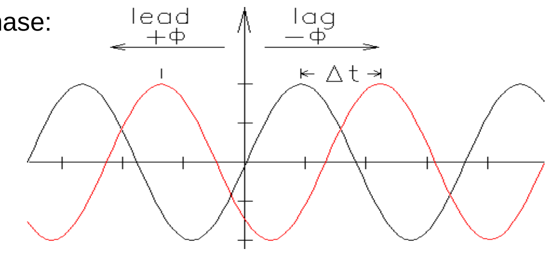
ω = radian frequency, radians/sec $\quad \omega = 2\cdot\pi\cdot f$

A = amplitude

Phase: $\phi = -\frac{\Delta t}{T}\cdot 360\cdot deg$ or: $\phi = -\frac{\Delta t}{T}\cdot 2\cdot\pi\cdot rad$

$y(t) = A\cdot\cos(\omega\cdot t + \phi)$

Phase:



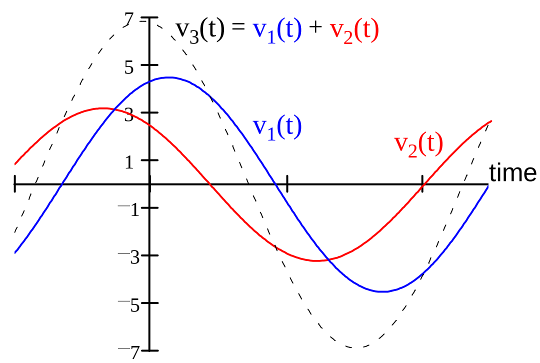
Expression of signals as phasors

<u>Phasor</u>

voltage: $v(t) = V_p\cdot\cos(\omega\cdot t + \phi)$ $\qquad \mathbf{V}(\omega) = V_p\cdot e^{j\cdot\phi}$

current: $i(t) = I_p\cdot\cos(\omega\cdot t + \phi)$ $\qquad \mathbf{I}(\omega) = I_p\cdot e^{j\cdot\phi}$

Ex1 What if a signal is the sum of two sinusoids of the same frequency.



—— $v_1(t) = 3.2\cdot V\cdot\cos(\omega\cdot t + 40\cdot deg)$

—— $v_2(t) = 4.5\cdot V\cdot\sin(\omega\cdot t + 75\cdot deg)$

- - - - $v_3(t) = v_1(t) + v_2(t)$

I'm going to drop the (w) notation from the phasor notation, it gets cumbersome, but remember that phasors are in the frequency domain.

Express the time-domain voltages as phasors:

$\mathbf{V_1} := 3.2\cdot V\cdot e^{j\cdot 40\cdot deg}$ = 3.2V /40°

Convert to rectangular: $3.2\cdot V\cdot\cos(40\cdot deg) = 2.451\cdot V$ $\qquad 3.2\cdot V\cdot\sin(40\cdot deg) = 2.057\cdot V$ $\qquad \mathbf{V_1} = 2.451 + 2.057j \cdot V$

Phasors are based on cosines, so express $v_2(t)$ as a cosine. Remember: $\sin(\omega t) = \cos(\omega\cdot t - 90\cdot deg)$

So: $v_2(t) = 4.5\cdot V\cdot\cos(\omega\cdot t + 75\cdot deg - 90\cdot deg) = 4.5\cdot V\cdot\cos(\omega\cdot t - 15\cdot deg)$

$\mathbf{V_2}$ = 4.5V /-15° or: $\mathbf{V_2} := 4.5\cdot V\cdot e^{-j\cdot 15\cdot deg}$

$4.5\cdot V\cdot\cos(-15\cdot deg) = 4.347\cdot V$ $\qquad 4.5\cdot V\cdot\sin(-15\cdot deg) = -1.165\cdot V$ $\qquad \mathbf{V_2} = 4.347 - 1.165j \cdot V$ \

$\mathbf{V_1} = 2.451 + 2.057j \cdot V$ } add /

Add real parts: $4.347 + 2.451 = 6.798$ $\qquad \mathbf{V_3} := \mathbf{V_1} + \mathbf{V_2}$

Add imaginary parts: $-1.165 + 2.057 = 0.892$ $\qquad \mathbf{V_3} = 6.798 + 0.892j \cdot V$ sum

Change $\mathbf{V_3}$ back to polar coordinates:

$\sqrt{6.798^2 + 0.892^2} = 6.856$ $\qquad \text{atan}\left(\frac{0.892}{6.798}\right) = 7.475\cdot deg$

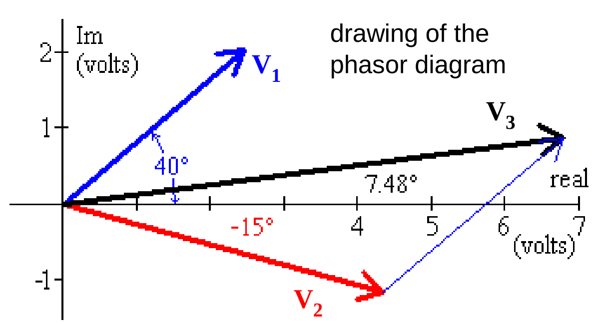
OR, in Mathcad notation (which you'll see a lot more):

$|\mathbf{V_3}| = 6.856\cdot V$ $\qquad \arg(\mathbf{V_3}) = 7.48\cdot deg$

$\mathbf{V_3}(\omega)$ = 6.856V <u>/ 7.48°</u> or: $\mathbf{V_3}(\omega) = 6.856\cdot V\cdot e^{-j\cdot 7.48\cdot deg}$

$\mathbf{V_3}$ may also be converted back to the time domain:

$v_3(t) = v_1(t) + v_2(t) = 6.856\cdot\cos(\omega\cdot t + 7.48\cdot deg)\cdot V$



Impedances

Resistor

$v_R = i_R \cdot R$

$\mathbf{V_R}(s) = R \cdot \mathbf{I}(s)$

$\mathbf{Z_R} = R$

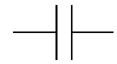
Inductor

$v_L(t) = L \cdot \frac{d}{dt} i_L(t)$

$\mathbf{V_L}(s) = s \cdot L \cdot \mathbf{I_L}(s)$

$\mathbf{Z_L} = L \cdot s$

Capacitor



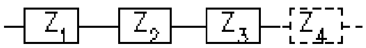
$i_C(t) = C \cdot \frac{d}{dt} v_C(t)$

$\mathbf{I_C}(s) = C \cdot s \cdot \mathbf{V_C}(s)$

$\mathbf{V_C}(\omega) = \frac{1}{C \cdot s} \cdot \mathbf{I}(s)$

$\mathbf{Z_C} = \frac{1}{C \cdot s}$

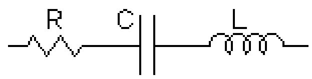
series:



$\mathbf{Z_{eq}} = \mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} + \ldots$

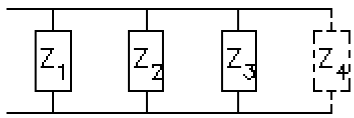
Voltage divider: $\mathbf{V_{Zn}} = \mathbf{V_{total}} \cdot \frac{\mathbf{Z_n}}{\mathbf{Z_1} + \mathbf{Z_2} + \mathbf{Z_3} + \ldots}$

Example:



$\mathbf{Z_{eq}} = R + \frac{1}{C \cdot s} + L \cdot s$

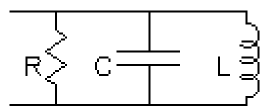
parallel:



$\mathbf{Z_{eq}} = \frac{1}{\frac{1}{\mathbf{Z_1}} + \frac{1}{\mathbf{Z_2}} + \frac{1}{\mathbf{Z_3}} + \ldots}$

Current divider: $\mathbf{I_{Zn}} = \mathbf{I_{total}} \cdot \frac{\frac{1}{\mathbf{Z_n}}}{\frac{1}{\mathbf{Z_1}} + \frac{1}{\mathbf{Z_2}} + \frac{1}{\mathbf{Z_3}} + \ldots}$

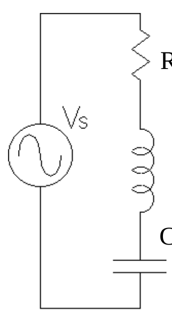
Example:



$\mathbf{Z_{eq}} = \frac{1}{\frac{1}{R} + \frac{1}{\frac{1}{C \cdot s}} + \frac{1}{L \cdot s}} = \frac{1}{\frac{1}{R} + C \cdot s + \frac{1}{L \cdot s}}$

For Steady-State Response, replace s with jω & 1/s with -jω .

Ex2 Find V_R, V_L, and V_C in polar phasor form. $f := 2 \cdot kHz$ Steady-state



$\mathbf{V_S}(\omega) = 6 \cdot V \cdot e^{j0}$

$f := 2 \cdot kHz$

$\omega := 2 \cdot \pi \cdot f \qquad \omega = 12566 \cdot \frac{rad}{sec}$

$R := 500 \cdot \Omega$

$L := 80 \cdot mH \qquad \mathbf{Z_L} := j \cdot \omega \cdot L \qquad \mathbf{Z_L} = 1.005j \cdot k\Omega$

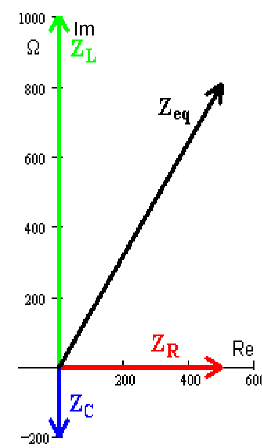
$= 1005\Omega \ /\ 90^o$

$C := 0.4 \cdot \mu F \qquad \mathbf{Z_C} := \frac{1}{j \cdot \omega \cdot C} \qquad \mathbf{Z_C} = -0.199j \cdot k\Omega$

$= -199\Omega \ /\ 90^o = 199\Omega \ /\ {-90^o}$

$\mathbf{Z_{eq}} := R + j \cdot \omega \cdot L + \frac{1}{j \cdot \omega \cdot C} \qquad \mathbf{Z_{eq}} = 500 + 806.366j \ \cdot \Omega$

$\sqrt{500^2 + 806^2} = 948.491 \qquad \text{atan}\left(\frac{806}{500}\right) = 58.187 \cdot deg \qquad Z_{eq} = 948.5\Omega \ /\ 58.2^o$



b) find the current: $\mathbf{I} := \frac{6 \cdot V \cdot e^{j \cdot 0}}{\mathbf{Z_{eq}}}$ magnitude: $\frac{6 \cdot V}{948.5 \cdot \Omega} = 6.326 \cdot mA$ angle: $0 \cdot deg - 58.2 \cdot deg = -58.2 \cdot deg$

$\mathbf{I} = 6.326mA \ /\ {-58.2^o}$

c) Draw a phasor diagram of V_S, V_R, V_L, and V_C

$\mathbf{V_R} := \mathbf{I} \cdot R$

magnitude $\quad 6.326 \cdot mA \cdot 500 \cdot \Omega = 3.163 \cdot V$

angle $\quad -58.2 \cdot deg + 0 \cdot deg = -58.2 \cdot deg$

$V_R = 3.163V \ /\ {-58.2^o}$

$\mathbf{V_L} := \mathbf{I} \cdot \mathbf{Z_L} \qquad 6.326 \cdot mA \cdot 1005 \cdot \Omega = 6.358 \cdot V$

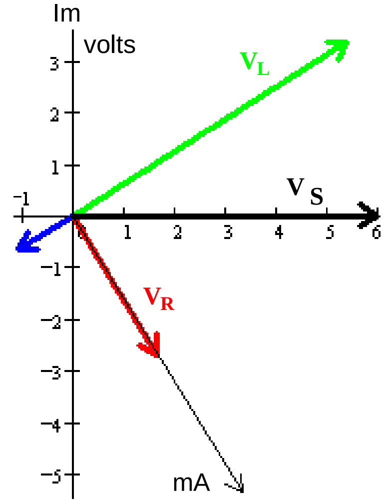
$-58.2 \cdot deg + 90 \cdot deg = 31.8 \cdot deg$

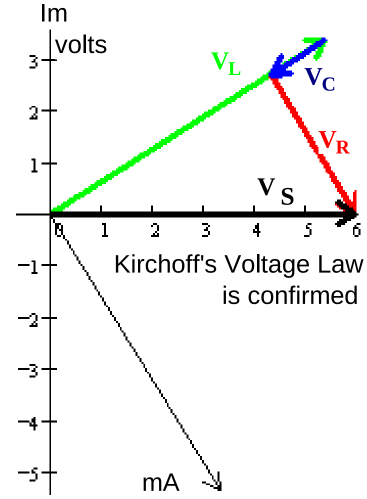
$V_L = 6.358V \ /\ 31.8^o$

$\mathbf{V_C} := \mathbf{I} \cdot \mathbf{Z_C} \qquad 6.326 \cdot mA \cdot (199) \cdot \Omega = 1.259 \cdot V$

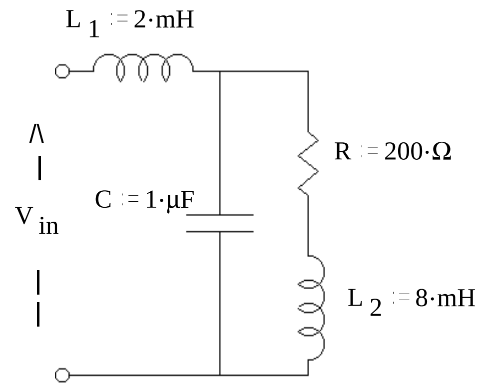
$-58.2 \cdot deg + (-90) \cdot deg = -148.2 \cdot deg$

$V_C = 1.259V \ /\ {-148.2^o}$





Ex3 a) Find the steady-state V_C and $v_C(t)$
given $v_{in}(t)$ is a 12-V cosine wave at: $f := 2.5\cdot kHz$
with a 20º leading phase angle. $\mathbf{V_{in}} := 12\cdot V\cdot e^{j\cdot 20\cdot deg}$



Transfer function for $\mathbf{V_C}$ as the output: $\mathbf{H}(s) = \dfrac{\mathbf{V_C}(s)}{\mathbf{V_{in}}(s)}$

$$\mathbf{H}(s) = \frac{\dfrac{1}{\dfrac{1}{R+L_2\cdot s}+C\cdot s}}{L_1\cdot s+\dfrac{1}{\dfrac{1}{R+L_2\cdot s}+C\cdot s}} \qquad \frac{\left(\dfrac{1}{R+L_2\cdot s}+C\cdot s\right)}{\left(\dfrac{1}{R+L_2\cdot s}+C\cdot s\right)}$$

$$= \frac{1}{L_1\cdot s\cdot\left(\dfrac{1}{R+L_2\cdot s}+C\cdot s\right)+1} = \frac{1}{L_1\cdot(j\cdot\omega)\cdot\left[\dfrac{1}{R+L_2\cdot(j\cdot\omega)}+C\cdot(j\cdot\omega)\right]+1}$$

$f = 2.5\cdot kHz$
$\omega := 2\cdot\pi\cdot f \qquad \omega = 15708\cdot\dfrac{rad}{sec}$

$$\mathbf{H}(j\omega) = \frac{1}{0.002\cdot(j\cdot 15708)\cdot\left[\dfrac{1}{200+0.008\cdot(j\cdot 15708)}+10^{-6}\cdot(j\cdot 15708)\right]+1}$$

$$= \frac{1}{31.416\cdot j\cdot\left[3.585\cdot 10^{-3}-2.252\cdot 10^{-3}\cdot j+(j\cdot 0.015708)\right]+1} = \frac{1}{1-0.423+0.113\cdot j} = \frac{1}{0.588\cdot e^{j\cdot 11.039\cdot deg}}$$

$$= 1.7\cdot e^{-j\cdot 11.039\cdot deg}$$

$$\mathbf{V_C} = \mathbf{V_{in}}(\omega)\cdot\mathbf{H}(j\omega) = 12\cdot V\cdot e^{j\cdot 20\cdot deg}\cdot 1.7\cdot e^{-j\cdot 11.039\cdot deg} = 12\cdot V\cdot 1.7\ \underline{/\ 20 - 11.039^o} = 20.4V\ \underline{/\ 8.96^o}$$

$$v_c(t) = 20.4\cdot V\cdot\cos\left(15708\cdot\frac{rad}{sec}\cdot t+8.96\cdot deg\right)$$

a) Find the steady-state I_{L2} and $i_{L2}(t)$.

Transfer function for $\mathbf{I_{L2}}$ as the output: $\mathbf{H}(s) = \dfrac{\mathbf{I_{L2}}(s)}{\mathbf{V_{in}}(s)} = \dfrac{\left(\dfrac{\mathbf{V_C}(s)}{R+L_2\cdot s}\right)}{\mathbf{V_{in}}(s)} = \dfrac{\mathbf{V_C}(s)}{\mathbf{V_{in}}(s)}\cdot\dfrac{1}{R+L_2\cdot s}$

$$\mathbf{H}(s) = \frac{1}{L_1\cdot s\cdot\left(\dfrac{1}{R+L_2\cdot s}+C\cdot s\right)+1}\cdot\frac{1}{(R+L_2\cdot s)} = \frac{1}{L_1\cdot s+L_1\cdot s\cdot C\cdot s\cdot(R+L_2\cdot s)+(R+L_2\cdot s)}$$

$$\mathbf{H}(j\omega) = \frac{1}{L_1\cdot(j\cdot\omega)+L_1\cdot C\cdot(j\cdot\omega)^2\cdot\left[R+L_2\cdot(j\cdot\omega)\right]+\left[R+L_2\cdot(j\cdot\omega)\right]} = 5.249\cdot 10^{-3}-4.926\cdot 10^{-3}j \quad \cdot\frac{1}{\Omega} = \frac{7.198}{k\Omega}\cdot e^{-j\cdot 43.181\cdot deg}$$

$$\mathbf{I_{L2}} = \mathbf{V_{in}}(\omega)\cdot\mathbf{H}(j\omega) = 12\cdot V\cdot e^{j\cdot 20\cdot deg}\cdot\left(\frac{7.198}{k\Omega}\cdot e^{-j\cdot 43.181\cdot deg}\right) = 12\cdot V\cdot\frac{7.198}{k\Omega}\ \underline{/\ 20 - 43.181^o} = 86.38mA\ \underline{/\ -23.18^o}$$

$$i_{L2}(t) = 86.38\cdot mA\cdot\cos\left(15708\cdot\frac{rad}{sec}\cdot t-23.18\cdot deg\right)$$

Magnitude and Phase of transfer functions

With steady-state sinusoidal inputs which start at $t = 0$

Ex4 a) Find the magnitude and phase of the following transfer function at this frequency: $f := 5 \cdot Hz$ $\quad \omega := 2 \cdot \pi \cdot f$

$$\omega = 31.42 \cdot \frac{rad}{sec}$$

$$\mathbf{H}(s) = \frac{s^2 + \frac{20}{sec} \cdot s + \frac{1000}{sec^2}}{s^2 + \frac{8}{sec} \cdot s + \frac{800}{sec^2}}$$

$$s := j \cdot \omega = j \cdot 31.42 \cdot \frac{rad}{sec} \qquad \mathbf{H}(j \cdot \omega) = \frac{(j\cdot\omega)^2 + \frac{20}{sec}\cdot(j\cdot\omega) + \frac{1000}{sec^2}}{(j\cdot\omega)^2 + \frac{8}{sec}\cdot(j\cdot\omega) + \frac{800}{sec^2}} = \frac{(j\cdot 31.42)^2 + 20\cdot(j\cdot 31.42) + 1000}{(j\cdot 31.42)^2 + 8\cdot(j\cdot 31.42) + 800}$$ without units

$$= \frac{13.04 + 628.319 \cdot j}{-187.2164 + 251.36 \cdot j} = 1.583 - 1.231j$$

$$|\mathbf{H}(j\cdot\omega)| = M = \sqrt{(1.583)^2 + (1.231)^2} = 2.005 \qquad \underline{/\,\mathbf{H}(j\omega)} = \operatorname{atan}\left(\frac{-1.231}{1.583}\right) = -37.87 \cdot deg \qquad \mathbf{H}(j\cdot\omega) = 2.005 \ \underline{/-37.87^{o}}$$

b) Find the steady-state sinusoidal output if the input is: $x(t) = 4 \cdot \cos(2 \cdot \pi \cdot 5 \cdot Hz \cdot t)$ $\quad \mathbf{X}(\omega) = 4 + 0j$

and then $\mathbf{Y}(\omega) = \mathbf{X}(\omega) \cdot \mathbf{H}(j \cdot \omega)$ $\quad 4 \cdot (1.583 - 1.231 \cdot j) = 6.332 - 4.924j$

Note that you can use the rectangular form of $\mathbf{H}(j\cdot\omega)$

$$y_{ss}(t) = \left(6.332 \cdot \cos\left(31.42 \cdot \frac{rad}{sec} \cdot t\right) + 4.924 \cdot \sin\left(31.42 \cdot \frac{rad}{sec} \cdot t\right)\right) \cdot u(t)$$

note that the sine carries the opposite sign as the imaginary part of phasor.

$$\sqrt{6.332^2 + 4.924^2} = 8.021 \qquad \operatorname{atan}\left(\frac{-4.924}{6.332}\right) = -37.87 \cdot deg \qquad y_{ss}(t) := 8.021 \cdot \cos(31.42 \cdot t - 37.87 \cdot deg) \cdot u(t)$$

c) Find the transient response for the same input. $x(t) = 4 \cdot \cos(2 \cdot \pi \cdot 5 \cdot Hz \cdot t)$ $\quad \mathbf{X}(s) = 4 \cdot \frac{s}{s^2 + 31.42^2}$

$$\mathbf{Y}_{tr}(s) = \frac{4 \cdot s}{s^2 + 31.42^2} \cdot \frac{s^2 + 20 \cdot s + 1000}{s^2 + 8 \cdot s + 800} \qquad a := -4 \qquad b := \sqrt{800 - 4^2} \qquad b = 28$$

$$= A \cdot \frac{s + 4}{s^2 + 8 \cdot s + 800} + B \cdot \frac{28}{s^2 + 8 \cdot s + 800} + C \cdot \frac{s}{s^2 + 31.42^2} + D \cdot \frac{31.42}{s^2 + 31.42^2}$$

multiply both sides by $\left(s^2 + 8 \cdot s + 800\right)$ and let $s := -4 + 28 \cdot j$

$$\frac{4 \cdot (-4 + 28 \cdot j) \cdot \left[(-4 + 28 \cdot j)^2 + 20 \cdot (-4 + 28 \cdot j) + 1000\right]}{(-4 + 28 \cdot j)^2 + 31.42^2} = -115.969 - 65.365j = A((-4 + 28 \cdot j) - 4) + B \cdot 28$$

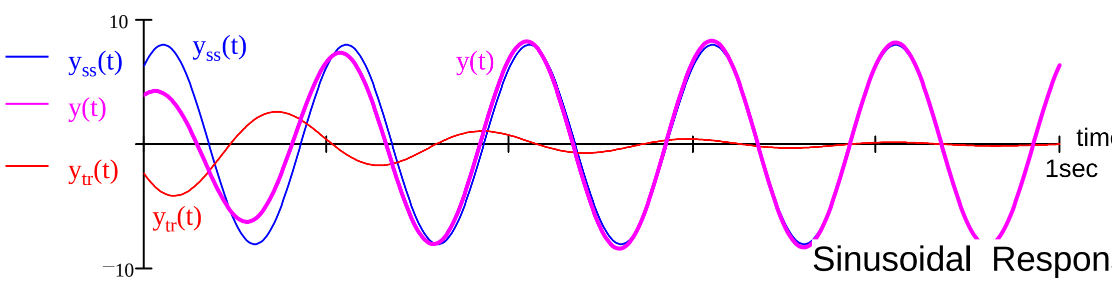
$$\frac{-115.969 - 65.365 \cdot j}{28} = -4.142 - 2.334j = A \cdot j + B$$

$$A := -2.334 \qquad B := -4.142$$

$$y_{tr}(t) := \left(-2.334 \cdot e^{-4 \cdot t} \cdot \cos(28 \cdot t) - 4.142 \cdot e^{-4 \cdot t} \cdot \sin(28 \cdot t)\right) \cdot u(t)$$

d) Find the total response for the same input.

$$y(t) := \left(-2.334 \cdot e^{-4 \cdot t} \cdot \cos(28 \cdot t) - 4.142 \cdot e^{-4 \cdot t} \cdot \sin(28 \cdot t) + 8.021 \cdot \cos(31.42 \cdot t - 37.87 \cdot deg)\right) \cdot u(t)$$



Ex4 a) Find the magnitude and phase of the following transfer function at this frequency: $\omega := 2\cdot\frac{rad}{sec}$

$$\mathbf{H}(s) = \frac{2\cdot s^2 + 5\cdot s + 20}{s^2 + 2\cdot s + 10} = \frac{2\cdot s^2 + \frac{5}{sec}\cdot s + \frac{20}{sec^2}}{s^2 + \frac{2}{sec}\cdot s + \frac{10}{sec^2}}\cdot\frac{V}{V}$$

Expressed with proper units.
In this case the transfer function is for a circuit where both the input and output are voltages.

$s := j\cdot\omega$

$$\mathbf{H}(j\cdot\omega) = \frac{2\cdot(j\cdot\omega)^2 + 5\cdot(j\cdot\omega) + 20}{(j\cdot\omega)^2 + \frac{2}{sec}\cdot(j\cdot\omega) + \frac{10}{sec^2}} = \frac{\left(20 - 2\cdot\omega^2\right) + (5\cdot\omega)\cdot j}{\left(10 - \omega^2\right) + (2\cdot\omega)\cdot j} = \frac{\left(20 - 2\cdot 2^2\right) + (5\cdot 2)\cdot j}{\left(10 - 2^2\right) + (2\cdot 2)\cdot j} = \frac{12 + 10\cdot j}{6 + 4\cdot j}$$

without units

$$|\mathbf{H}(j\cdot\omega)| = M = \frac{\sqrt{12^2 + 10^2}}{\sqrt{6^2 + 4^2}} = 2.166 \qquad \underline{/\,\mathbf{H}(j\omega)} = \operatorname{atan}\left(\frac{10}{12}\right) - \operatorname{atan}\left(\frac{4}{6}\right) = 6.12\cdot deg \qquad \mathbf{H}(j\cdot\omega) = 2.166\ \underline{/6.12^o}$$

b) Find the steady-state response if the input is: $v_{in}(t) = 3.2\cdot\cos(2\cdot t + 15\cdot deg)$ $\quad \mathbf{V_{in}} := 3.2\cdot V\cdot e^{j\cdot 15\cdot deg} = 3.2V\ \underline{/15^o}$

$$\mathbf{V_{out\_ss}}(\omega) = \mathbf{V_{in}}(\omega)\cdot\mathbf{H}(j\cdot\omega) = \left(3.2\cdot V\cdot e^{j\cdot 15\cdot deg}\right)\cdot\left(2.166\cdot e^{j\cdot 6.12\cdot deg}\right) = 3.2\cdot V\cdot 2.166\cdot e^{j\cdot(15\cdot deg + 6.12\cdot deg)}$$

$$= 6.931\cdot V\cdot e^{j\cdot 21.12\cdot deg} = 6.465 + 2.497j\ \cdot V$$

$$v_{out\_ss}(t) = 6.931\cdot V\cdot\cos(2\cdot t + 21.12\cdot deg)\cdot u(t) = (6.465\cdot\cos(2\cdot t) - 2.497\cdot\sin(2\cdot t))\cdot u(t)$$

Note that the sine carries the opposite sign as the imaginary part of phasor.

c) Find the transient response for the same input.

Phasor: $\mathbf{V_{in}} = 3.091 + 0.828j\ \cdot V$ $\qquad \mathbf{V_{in}}(s) = 3.091\cdot\frac{s}{s^2 + 2^2} - 0.828\cdot\frac{2}{s^2 + 2^2} = \frac{3.091\cdot s - 0.828\cdot 2}{s^2 + 2^2}$

Note the sign change for the sine.

$$\mathbf{V_{out\_tr}}(s) = \frac{(3.091\cdot s - 0.828\cdot 2)}{s^2 + 2^2}\cdot\frac{\left(2\cdot s^2 + 5\cdot s + 20\right)}{s^2 + 2\cdot s + 10} = \frac{6.182\cdot s^3 + 12.143\cdot s^2 + 53.54\cdot s - 33.12}{\left(s^2 + 4\right)\cdot\left(s^2 + 2\cdot s + 10\right)}$$

$$= A\cdot\frac{s + 1}{s^2 + 2\cdot s + 10} + B\cdot\frac{3}{s^2 + 2\cdot s + 10} + C\cdot\frac{s}{s^2 + 4} + D\cdot\frac{2}{s^2 + 4}$$

multiply both sides by $\left(s^2 + 2\cdot s + 10\right)$ and let $s := -1 + 3\cdot j$

$$\frac{6.182\cdot s^3 + 12.143\cdot s^2 + 53.54\cdot s - 33.12}{\left(s^2 + 4\right)} = A\cdot((-1 + 3\cdot j) + 1) + B\cdot 3$$

$$\frac{6.182\cdot(-1 + 3\cdot j)^3 + 12.143\cdot(-1 + 3\cdot j)^2 + 53.54\cdot(-1 + 3\cdot j) - 33.12}{\left[(-1 + 3\cdot j)^2 + 4\right]} = \frac{-23.072 - 23.514\cdot j}{-4 - 6\cdot j} = 4.488 - 0.853j = A\cdot 3\cdot j + B\cdot 3$$

$$\frac{4.488 - 0.853\cdot j}{3} = 1.496 - 0.284j \qquad A := -0.284 \qquad B := 1.496$$

$$v_{out\_tr}(t) = \left(-0.281\cdot V\cdot e^{-1\cdot t}\cdot\cos(3\cdot t) + 1.496\cdot V\cdot e^{-1\cdot t}\cdot\sin(3\cdot t)\right)\cdot u(t)$$

d) Find the total response for the same input.

$$v_{out}(t) = \left(-0.281\cdot V\cdot e^{-1\cdot t}\cdot\cos(3\cdot t) + 1.496\cdot V\cdot e^{-1\cdot t}\cdot\sin(3\cdot t) + 6.931\cdot V\cdot\cos(2\cdot t + 21.12\cdot deg)\right)\cdot u(t)$$

ECE 3510 Effect of initial conditions

See Bodson text, section 3.5

Best explained by example, IF: $\mathbf{H}(s) = \dfrac{b_2 \cdot s^2 + b_1 \cdot s + b_0}{s^2 + a_1 \cdot s + a_0} \cdot \mathbf{X}(s)$

We would normally say: $\mathbf{Y}(s) = \dfrac{b_2 \cdot s^2 + b_1 \cdot s + b_0}{s^2 + a_1 \cdot s + a_0} \cdot \mathbf{X}(s)$

But that ignores initial conditions. So let's deconstruct and then reconstruct with initial conditions included.

$$\mathbf{Y}(s) \cdot \left(s^2 + a_1 \cdot s + a_0\right) = \left(b_2 \cdot s^2 + b_1 \cdot s + b_0\right) \cdot \mathbf{X}(s)$$

$$s^2 \cdot \mathbf{Y}(s) + a_1 \cdot s \cdot \mathbf{Y}(s) + a_0 \cdot \mathbf{Y}(s) = b_2 \cdot s^2 \cdot \mathbf{X}(s) + b_1 \cdot s \cdot \mathbf{X}(s) + b_0 \cdot \mathbf{X}(s)$$

$$\frac{d^2}{dt^2} y(t) + a_1 \cdot \frac{d}{dt} y(t) + a_0 \cdot y(t) = b_2 \cdot \frac{d^2}{dt^2} x(t) + b_1 \cdot \frac{d}{dt} x(t) + b_0 \cdot x(t)$$

Laplace Properties

Operation	f(t)	F(s)
Time differentiation	$\frac{d}{dt} f(t)$	$s \cdot \mathbf{F}(s) - f(0-)$
	$\frac{d^2}{dt^2} f(t)$	$s^2 \cdot \mathbf{F}(s) - s \cdot f(0-) - \frac{d}{dt} f(0-)$

initial slope

$$\frac{d^2}{dt^2} y(t) + a_1 \cdot \frac{d}{dt} y(t) + a_0 \cdot y(t) = b_2 \cdot \frac{d^2}{dt^2} x(t) + b_1 \cdot \frac{d}{dt} x(t) + b_0 \cdot x(t)$$

$$\left(s^2 \cdot \mathbf{Y}(s) - s \cdot y(0-) - \frac{d}{dt} y(0-)\right) + a_1 \cdot \left(s \cdot \mathbf{Y}(s) - y(0-)\right) + a_0 \cdot \mathbf{Y}(s) =$$

$$b_2 \cdot \left(s^2 \cdot \mathbf{X}(s) - s \cdot x(0-) - \frac{d}{dt} x(0-)\right) + b_1 \cdot \left(s \cdot \mathbf{X}(s) - x(0-)\right) + b_0 \cdot \mathbf{X}(s)$$

$$\left(s^2 \cdot \mathbf{Y}(s) + a_1 \cdot s \cdot \mathbf{Y}(s) + a_0 \cdot \mathbf{Y}(s)\right) - s \cdot y(0-) - \frac{d}{dt} y(0-) - a_1 \cdot y(0-) =$$

$$\left(b_2 \cdot s^2 \cdot \mathbf{X}(s) + b_1 \cdot s \cdot \mathbf{X}(s) + b_0 \cdot \mathbf{X}(s)\right) - b_2 \cdot s \cdot x(0-) - b_2 \cdot \frac{d}{dt} x(0-) - b_1 \cdot x(0-)$$

$$\mathbf{Y}(s) \cdot \left(s^2 + a_1 \cdot s + a_0\right) = \left(b_2 \cdot s^2 + b_1 \cdot s + b_0\right) \cdot \mathbf{X}(s) + \left(s \cdot y(0-) + \frac{d}{dt} y(0-) + a_1 \cdot y(0-) - b_2 \cdot s \cdot x(0-) - b_2 \cdot \frac{d}{dt} x(0-) - b_1 \cdot x(0-)\right)$$

Response to input — Response to initial conditions

$$\mathbf{Y}(s) = \underbrace{\frac{b_2 \cdot s^2 + b_1 \cdot s + b_0}{s^2 + a_1 \cdot s + a_0}}_{\mathbf{H}(s)} \cdot \mathbf{X}(s) + \frac{s \cdot y(0-) + \frac{d}{dt} y(0-) + a_1 \cdot y(0-) - b_2 \cdot s \cdot x(0-) - b_2 \cdot \frac{d}{dt} x(0-) - b_1 \cdot x(0-)}{s^2 + a_1 \cdot s + a_0}$$

Forced response — Natural response

Zero-state response — Zero-input response

Initial conditions

$$\mathbf{Y}(s) = \overbrace{\frac{b_2 \cdot s^2 + b_1 \cdot s + b_0}{s^2 + a_1 \cdot s + a_0}}^{\mathbf{H}(s)} \cdot \mathbf{X}(s) \quad + \quad \frac{s \cdot y(0-) + \frac{d}{dt} y(0-) + a_1 \cdot y(0-) - b_2 \cdot s \cdot x(0-) - b_2 \cdot \frac{d}{dt} x(0-) - b_1 \cdot x(0-)}{s^2 + a_1 \cdot s + a_0}$$

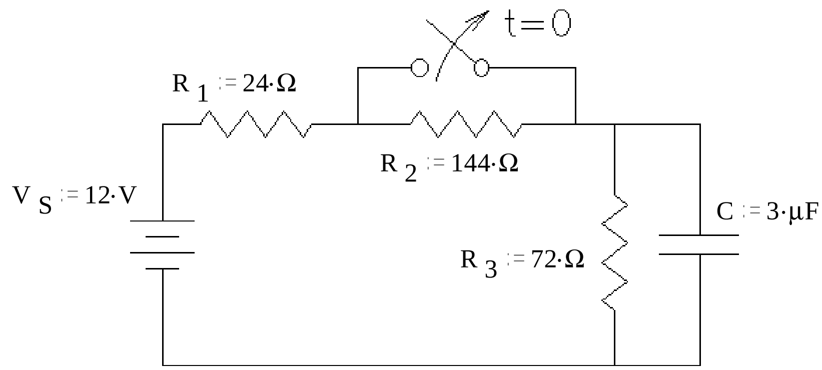
Observations

1. The total response is the sum of two independent components.
2. These values together fully describe the *state* of the 2nd-order system at time $t = 0^-$ (the initial state): $y(0-)$ $\quad \frac{d}{dt} y(0-)$ $\quad x(0-)$ $\quad \frac{d}{dt} x(0-)$
3. Similar denominator for both parts = Share poles = Similar responses
4. Response to Initial conditions always go to zero if system is BIBO.
5. Pole-zero cancellations in right-half plane can cause major problems with internal states of the system.

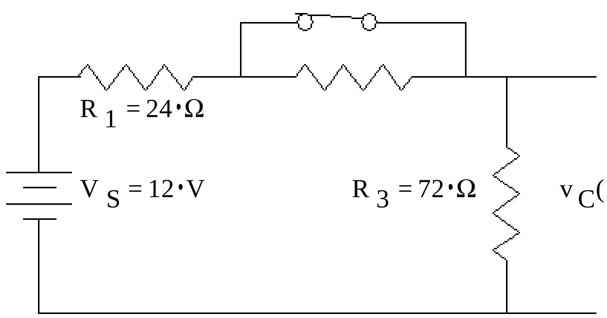
A simple first-order example

The switch has been closed for a long time and is opened at time $t=0$.

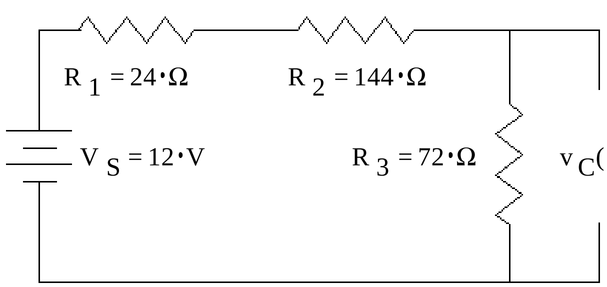
Find the complete expression for $v_C(t)$.



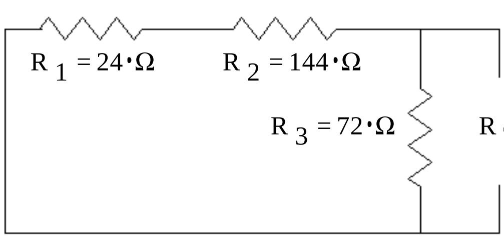
The way you've solved it before:



$$v_C(0) = V_S \cdot \frac{R_3}{R_1 + R_3} = 9 \cdot V$$



$$v_C(\infty) = V_S \cdot \frac{R_3}{R_1 + R_2 + R_3} = 3.6 \cdot V$$



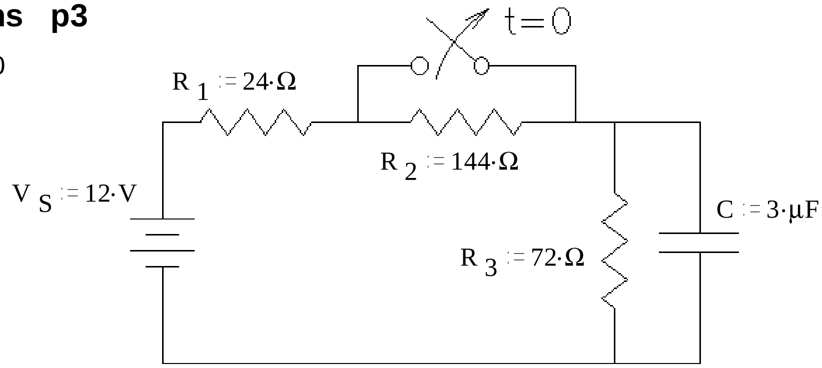
$$R_{Th} := \frac{1}{\left(\frac{1}{R_3} + \frac{1}{R_1 + R_2}\right)} \qquad R_{Th} = 50.4 \cdot \Omega$$

$$\tau := R_{Th} \cdot C \qquad \tau = 151.2 \cdot \mu s$$

$$v_C(t) = v_C(\infty) + \left(v_C(0) - v_C(\infty)\right) \cdot e^{-\frac{t}{\tau}} \quad = 3.6 \cdot V + (9 \cdot V - 3.6 \cdot V) \cdot e^{-\frac{t}{151 \cdot \mu s}} \quad = 3.6 \cdot V + 5.4 \cdot V \cdot e^{-\frac{t}{151 \cdot \mu s}}$$

ECE 3510 Effect of initial conditions p3

The way we would do the same thing in 3510



$$\mathbf{H}(s) = \frac{\mathbf{V_C}(s)}{\mathbf{V_S}(s)} = \frac{\dfrac{1}{\dfrac{1}{R_3}+C\cdot s}}{R_1+R_2+\dfrac{1}{\dfrac{1}{R_3}+C\cdot s}}\cdot\frac{\left(\dfrac{1}{R_3}+C\cdot s\right)}{\left(\dfrac{1}{R_3}+C\cdot s\right)} = \frac{1}{\dfrac{R_1+R_2}{R_3}+\left(R_1+R_2\right)\cdot C\cdot s+1}\cdot\frac{\left[\dfrac{1}{\left(R_1+R_2\right)\cdot C}\right]}{\left[\dfrac{1}{\left(R_1+R_2\right)\cdot C}\right]}$$

$$= \frac{1}{\dfrac{R_1+R_2}{R_3}+\left(R_1+R_2\right)\cdot C\cdot s+1}\cdot\frac{\left[\dfrac{1}{\left(R_1+R_2\right)\cdot C}\right]}{\left[\dfrac{1}{\left(R_1+R_2\right)\cdot C}\right]} = \frac{\dfrac{1}{\left(R_1+R_2\right)\cdot C}}{s+\left[\dfrac{1}{R_3\cdot C}+\dfrac{1}{\left(R_1+R_2\right)\cdot C}\right]}$$

First-order version of $\mathbf{Y}(s)$ with initial conditions

$$\mathbf{Y}(s) = \overbrace{\frac{b_1\cdot s+b_0}{s+a_0}}^{\mathbf{H}(s)}\cdot\mathbf{X}(s) + \overbrace{\frac{y(0-)-b_1\cdot x(0-)}{s+a_0}}^{\text{Initial conditions}}$$

$b_1 = 0$ $\qquad b_0 := \dfrac{1}{\left(R_1+R_2\right)\cdot C}$ $\qquad b_0 = 1984\cdot sec^{-1}$

$a_0 := \dfrac{1}{R_3\cdot C}+\dfrac{1}{\left(R_1+R_2\right)\cdot C}$ $\qquad a_0 = 6614\cdot sec^{-1}$

$$\mathbf{X}(s) = \frac{12\cdot V}{s}$$

$$\mathbf{Y}(s) = \frac{b_0}{s+a_0}\cdot\frac{12\cdot V}{s} + \frac{y(0-)}{s+a_0}$$

$y(0-) = 9\cdot V$ from above

$$\frac{b_0}{s+a_0}\cdot\frac{12\cdot V}{s} = \frac{A}{s}+\frac{B}{s+a_0}$$

$$A = \mathbf{H}(0)\cdot 12\cdot V = \frac{b_0}{a_0}\cdot 12\cdot V = 3.6\cdot V$$

$$B = b_0\cdot\frac{12\cdot V}{s}\Bigg|_{s:=-a_0} = \frac{b_0}{-a_0}\cdot 12\cdot V = -3.6\cdot V$$

$$= \frac{3.6\cdot V}{s} + \frac{-3.6\cdot V}{s+a_0} + \frac{y(0-)}{s+a_0}$$

$$y(t) = v_C(t) = \left(3.6\cdot V - 3.6\cdot V\cdot e^{-a_0\cdot t} + 9\cdot V\cdot e^{-a_0\cdot t}\right)\cdot u(t)$$

Same as above $\quad v_C(t) = 3.6\cdot V + (9\cdot V - 3.6\cdot V)\cdot e^{-\frac{t}{151\cdot\mu s}}$ $\qquad$ where $\tau = \dfrac{1}{a_0} = \dfrac{sec}{6614} = 151.2\cdot\mu s$

ECE 3510 Effect of initial conditions p3

ECE 3510 State Space

A completely different method where all math is done in the time-domain using linear algebra. See Bodson, section 3.6.

$x(t)$ = The state vector (n x 1 matrix)

n = order of the system

$\frac{d}{dt}x(t)$ = Time derivative of the state vector (n x 1 matrix)

$u(t)$ = The input vector (n_u x 1 matrix)

n_u = number of inputs

$y(t)$ = The state vector (n_y x 1 matrix)

n_y = order of the system

A = The system matrix (n x n matrix)

B = The input matrix (n x n_u matrix)

C = The output matrix (n_y x n matrix)

D = The feed-forward matrix (n_y x n_u matrix)

State Equation: $\frac{d}{dt}x(t) = A\cdot x(t) + B\cdot u(t)$

A third-order, 2-input, 2-output system

$$\begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix} = \begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{pmatrix} \cdot \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix} + \begin{pmatrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{pmatrix} \cdot \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$$

Output Equation: $y(t) = C\cdot x(t) + D\cdot u(t)$

$$\begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix} = \begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{pmatrix} \cdot \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix} + \begin{pmatrix} \blacksquare & \blacksquare \\ \blacksquare & \blacksquare \end{pmatrix} \cdot \begin{pmatrix} \blacksquare \\ \blacksquare \end{pmatrix}$$

State Equation: $\frac{d}{dt}x(t) = A\cdot x(t) + B\cdot u(t)$

A third-order, single-input, single-output system

$$\begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix} = \begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \\ \blacksquare & \blacksquare & \blacksquare \end{pmatrix} \cdot \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix} + \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix} \cdot u(t)$$

Output Equation: $y(t) = C\cdot x(t) + D\cdot u(t)$

$$y(t) = \begin{pmatrix} \blacksquare & \blacksquare & \blacksquare \end{pmatrix} \cdot \begin{pmatrix} \blacksquare \\ \blacksquare \\ \blacksquare \end{pmatrix} + D\cdot u(t)$$

Advantages of the state-space method

- Easily handles multiple inputs, multiple outputs and initial conditions
- Can be used with nonlinear systems
- Can be used with time-varying systems
- Reveals unstable systems that have stable transfer functions (pole-zero cancellations). You can determine:
  - Controllability: State variables can all be affected by the input
  - Observability: State variables are all "observable" from the output
- Basis of Optimal control methods

Advantages and disadvantages of the classical frequency-domain method used in this class

- Simpler to understand and model interconnected systems.
- Rapidly provide stability and transient response information.
- Easy to see the effects of varying system parameters to get a good design.
- Limited to linear, time-invariant systems or systems that can be approximated as such.