

Homework should be turned in to the 3510 homework locker by 5:00pm on the due date.

Good LaPlace Transform tables: Nielson p.595 (7th ed.,p.547), Lathi p.372, Bodson p.5, Nise, p.40, Class handout

Good LaPlace Property tables: Nielson p.601 (7th ed.,p.553), Lathi p.389, Bodson p.8, Nise p.41, Class handout

For problems 1 & 2:

Don't just write down what the table shows.

You must show some work of your own.

You may use simpler table entries together with properties of the Laplace transform.

1. Find the Laplace transform of the following functions: See instructions above

- a) $u(t)$
- b) $\sin(\omega \cdot t) \cdot u(t)$
- c) $t \cdot u(t)$

2. Find the Laplace transform of the following functions: See instructions above

- a) $e^{-at} \cdot \sin(\omega \cdot t) \cdot u(t)$
- b) $e^{-at} \cdot \cos(\omega \cdot t) \cdot u(t)$

3. Look at the figures on the **next page**. Each set of real (horizontal) and imaginary (vertical) axes show the poles of a signal transform on the s-plane.

a) Find the best matching time-domain signal or answer in "Answers for problem 3" section (following page).

Answers may be used more than once or not at all, but make a little check mark next to each on that you do use.

Don't overlook answers A and B, which are written only (no figure).

The axes all have the same scaling. All time scales on the ANSWERS page are the same.

Your answers should make sense relative to one another.

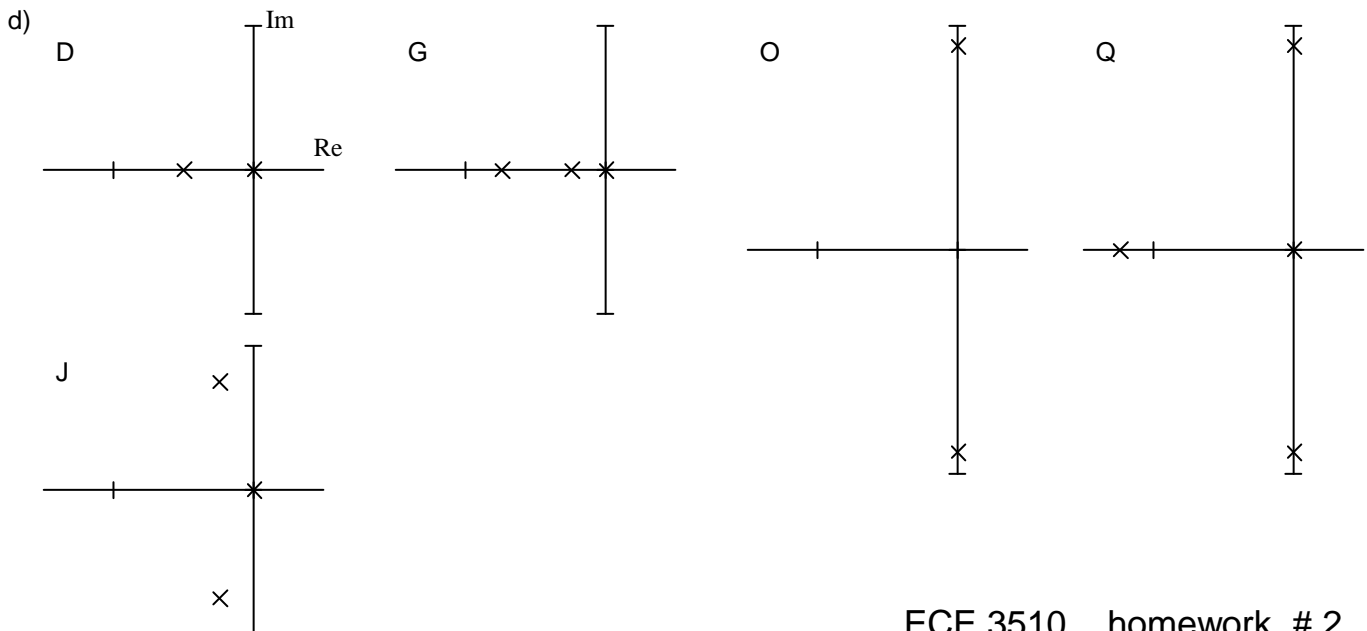
dbl = double pole at that location

Answers

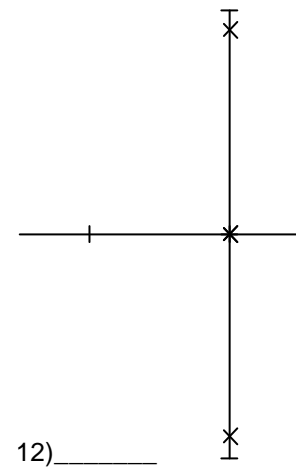
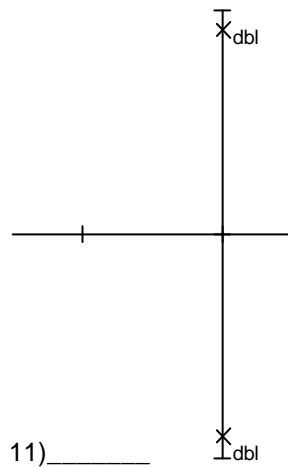
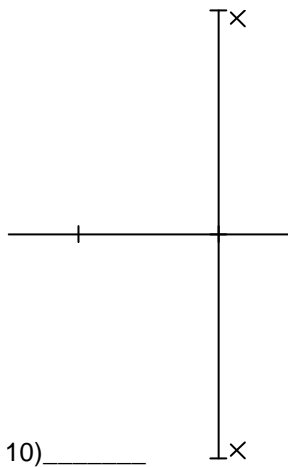
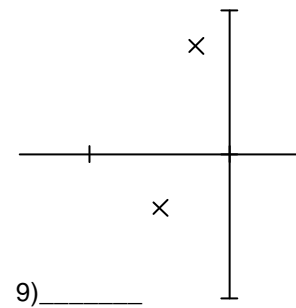
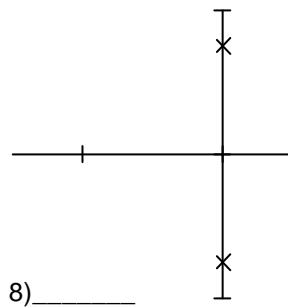
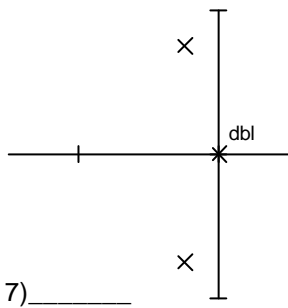
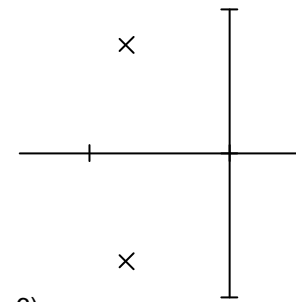
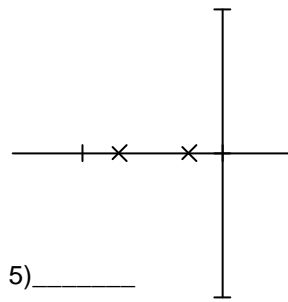
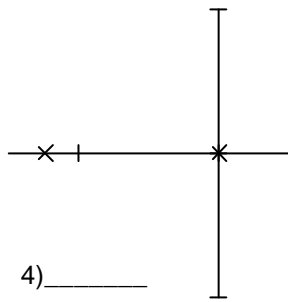
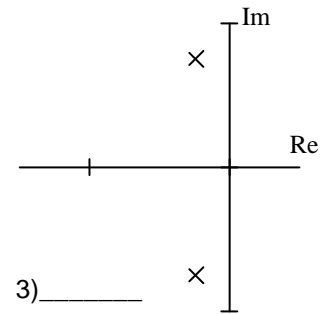
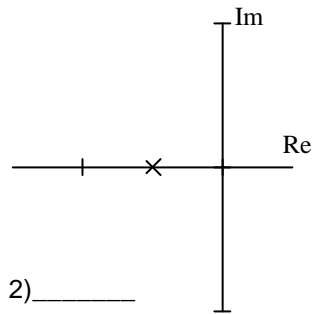
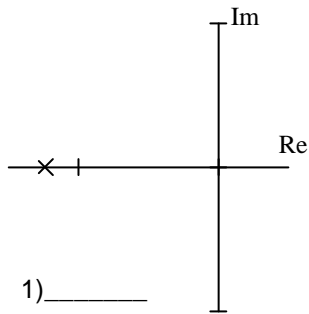
1. a) $\frac{1}{s}$ b) $\frac{\omega}{s^2 + \omega^2}$ c) $\frac{1}{s^2}$ 2. a) $\frac{\omega}{(s+a)^2 + \omega^2}$ b) $\frac{(s+a)}{(s+a)^2 + \omega^2}$

3. a) 1) E 2) F 3) K 4) C 5) H 6) L 7) M 8) N 9) A 10) R 11) S 12) P

b) 7 10 11 c) 7 8 10 11 12



3. See instructions on previous page



b) List those numbers above that represent signals that are UNBOUNDED.

c) List those numbers above that represent signals that DO NOT converge.

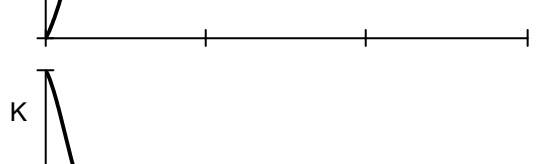
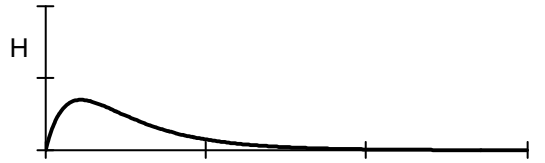
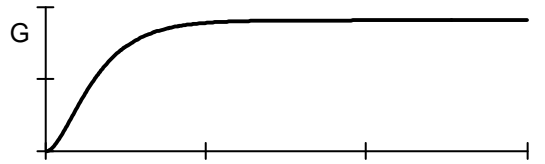
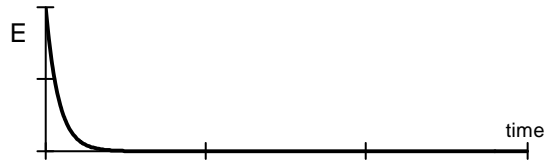
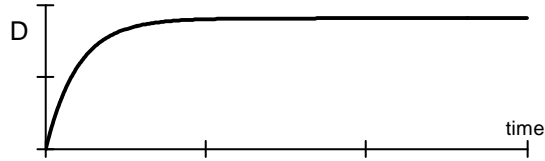
d) Several of the answers on the next page were not used.

For each of the answers that were **not** used, draw the poles of that time-domain signal on a set of real and imaginary axes (an s-plane). Scale your axes just like the ones above.

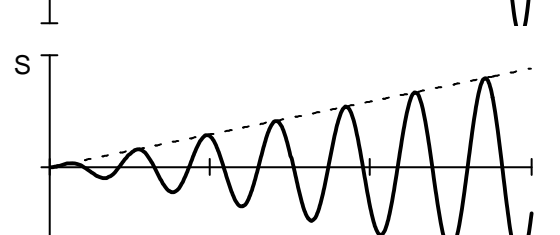
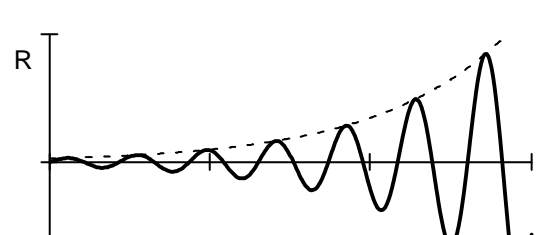
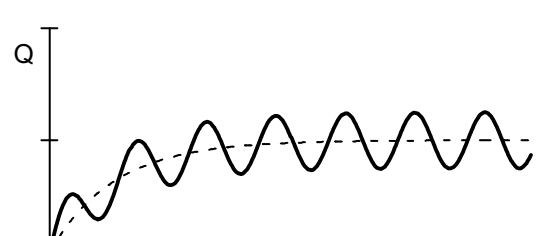
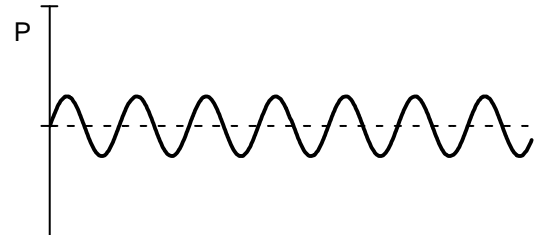
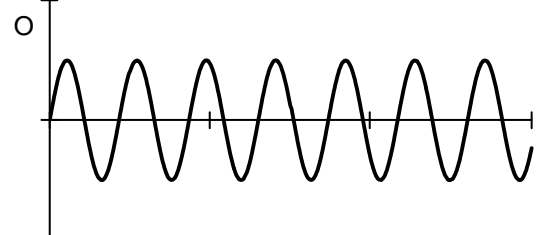
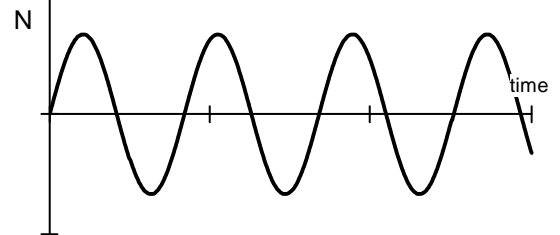
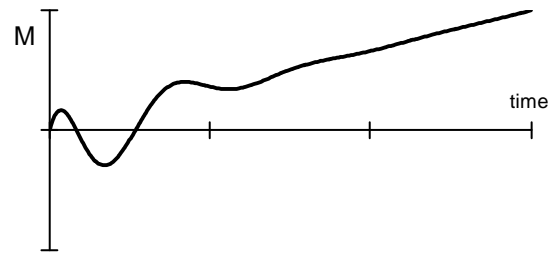
e) Keep the "Answers for problem 3" page. I may refer to it again in future homework problems.

Answers for problem 3 All horizontal axes are time.

- A No real time-domain answer could match these pole(s)
- B None of these time-domain answers match these poles



KEEP THIS PAGE FOR FUTURE USE



1. Find the inverse Laplace transform of each of the following functions:

Use partial fraction expansion and the tables.

$$a) F(s) = \frac{1}{s^2 + 5s + 6}$$

$$b) F(s) = \frac{s-1}{s \cdot (s+2)}$$

ECE 3510 homework # 3b

Due: Tue, 1/22/19

2. Find the inverse Laplace transform of each of the following functions:

Use the mixed method and the tables.

$$a) F(s) = \frac{3s+6}{(s^2+1) \cdot (s^2+4)}$$

$$b) F(s) = \frac{1}{(s+2) \cdot (s+1)^2}$$

$$c) F(s) = \frac{2s}{s^2 + 2s + \frac{5}{4}}$$

$$d) F(s) = \frac{8s+4}{s^2 \cdot (s+1)^2}$$

$$e) F(s) = \frac{\frac{1}{2} \cdot s^3 + s^2 + s + \frac{5}{2}}{s^2 \cdot (s^2 + 2s + 5)}$$

$$3. F(s) = \frac{s-1}{s^3 \cdot (s^2 + 2s + 5)^2}$$

Show the form of $f(t)$ without actually finding it.
Indicate which of the coefficients may not be 0

4. Problem 2.3a - f in textbook (p.20)

As part of your work to reach a solution, draw the pole diagram for each.

Answers (time functions below valid for $t \geq 0$ only)

$$1. a) (e^{-2t} - e^{-3t}) \cdot u(t)$$

$$b) \left(\frac{3}{2} \cdot e^{-2t} - \frac{1}{2} \right) \cdot u(t)$$

$$2. a) (\cos(t) + 2 \cdot \sin(t) - \cos(2t) - \sin(2t)) \cdot u(t)$$

$$b) (e^{-2t} + t \cdot e^{-t} - e^{-t}) \cdot u(t)$$

$$c) \left(2 \cdot e^{-t} \cdot \cos\left(\frac{1}{2} \cdot t\right) - 4 \cdot e^{-t} \cdot \sin\left(\frac{1}{2} \cdot t\right) \right) \cdot u(t)$$

$$d) (4 \cdot t - 4 \cdot t \cdot e^{-t}) \cdot u(t)$$

$$e) \left(\frac{1}{2} \cdot t + \frac{1}{2} \cdot e^{-t} \cdot \cos(2t) \right) \cdot u(t)$$

$$3. (A + B \cdot t + C \cdot t^2 + D \cdot e^{at} \cdot \cos(bt) + E \cdot e^{at} \cdot \sin(bt) + F \cdot t \cdot e^{at} \cdot \cos(bt) + G \cdot t \cdot e^{at} \cdot \sin(bt)) \cdot u(t)$$

C may not be 0 & Either F or G may be 0, but **NOT BOTH**

Alternate solution:

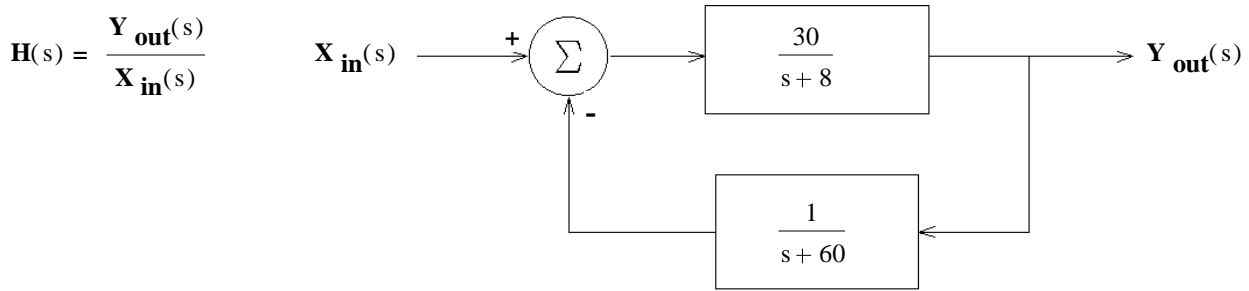
$$\left(A + B \cdot t + C \cdot t^2 + \sqrt{D^2 + E^2} \cdot e^{at} \cdot \cos(bt + \theta) + \sqrt{F^2 + G^2} \cdot t \cdot e^{at} \cdot \cos(bt + \phi) \right) \cdot u(t)$$

Can't be 0: C & $\sqrt{F^2 + G^2}$

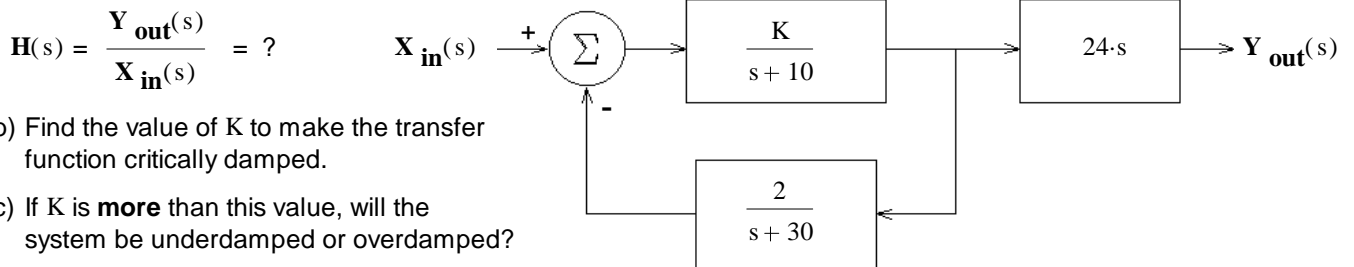
4.	<u>Bounded</u>	<u>Converges</u>	<u>f(∞)</u>
a)	yes	yes	0
b)	yes	yes	$-\frac{1}{2}$
c)	no		
d)	yes	yes	5
e)	yes	no	
f)	no		

Properly simplify all your expressions for $H(s)$. By this I mean that the numerator and denominator should both be simple polynomials or factored polynomials. There should be no $1/s^n$ terms in either the numerator or denominator. Also, there should be no coefficient on the highest-order term in the denominator

1. For the feedback system shown below, find the transfer function of the whole system, with feedback.

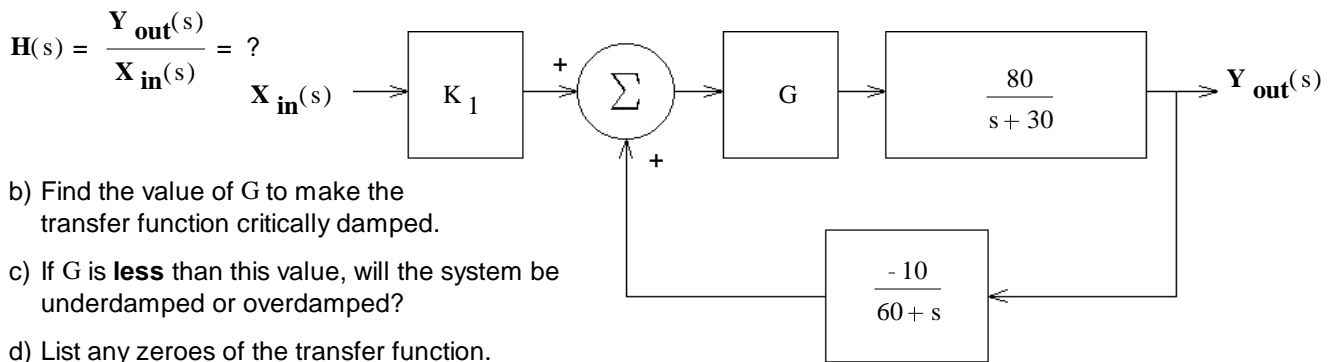


2. a) For the feedback system shown below, find the transfer function of the whole system, with feedback.



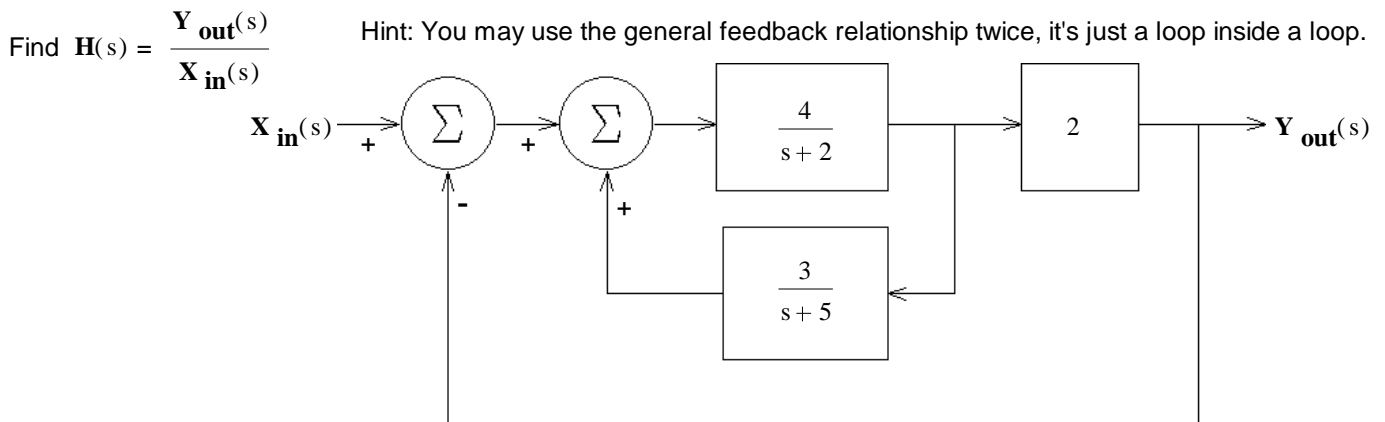
- b) Find the value of K to make the transfer function critically damped.
- c) If K is **more** than this value, will the system be underdamped or overdamped?
- d) List any zeroes of the transfer function.

3. a) For the feedback system shown below, find the transfer function of the whole system, with feedback.

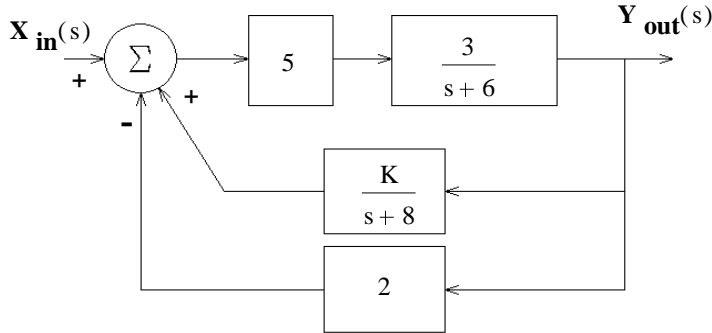


- b) Find the value of G to make the transfer function critically damped.
- c) If G is **less** than this value, will the system be underdamped or overdamped?
- d) List any zeroes of the transfer function.

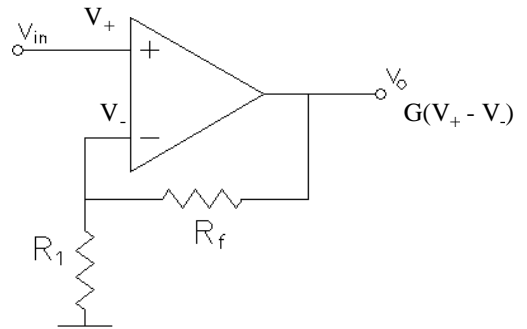
4. For the feedback system shown below, find the transfer function of the whole system, with feedback.



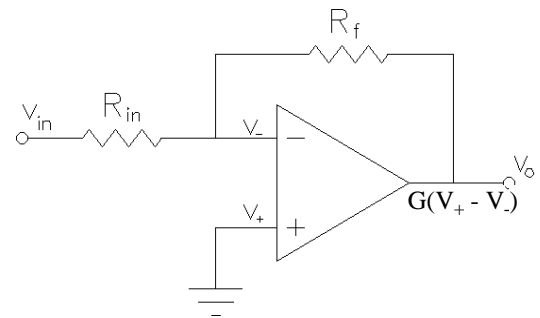
5. Redraw the feedback system below so that it is just one simple loop.



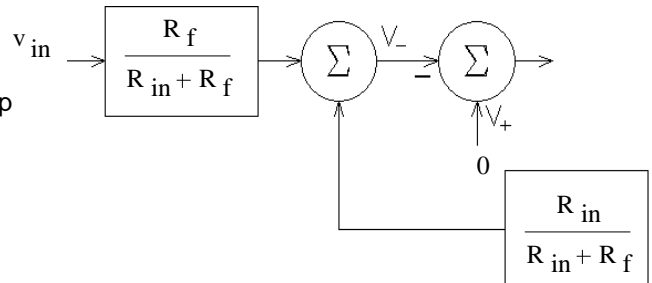
6. a) Draw a standard feedback loop for the noninverting op amp amplifier. Assume no current flows into the op-amp inputs.
 b) Use the standard feedback loop expression to find the transfer function for this amplifier.
 c) Show that this expression simplifies to the standard gain expression for this amplifier if G is very large.



7. a) Draw a standard feedback loop for the inverting op amp amplifier. There also will be an extra block before the loop. This amplifier is trickier than the noninverting amp, so I've done part of the loop for you. The first block determines v_in's contribution to V- (by superposition). The bottom block determines v_o's contribution to V- (by superposition). You will have to combine the summation circles together into one and complete the loop. Assume no current flows into the op-amp inputs.



- b) Combine the leading block with the standard feedback loop expression to find the transfer function for this amplifier.



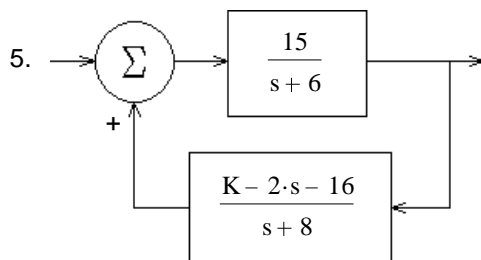
- c) Show that this expression simplifies to the standard gain expression for this amplifier if G is very large.

Answers

1. a) $\frac{30 \cdot s + 1800}{s^2 + 68 \cdot s + 510}$ 2. a) $\frac{K \cdot 24 \cdot s \cdot (s + 30)}{s^2 + 40 \cdot s + 300 + 2 \cdot K}$ b) 50 c) underdamped d) 0, -30

3. a) $K_1 \cdot \frac{G \cdot 80 \cdot s + G \cdot 4800}{s^2 + 90 \cdot s + 800 \cdot G + 1800}$ b) 0.28125 c) overdamped d) -60

4. $\frac{8 \cdot s + 40}{s^2 + 15 \cdot s + 38}$

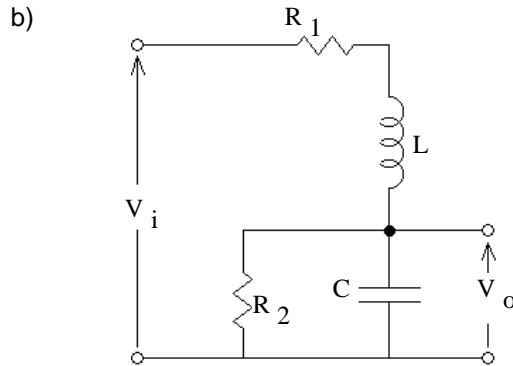
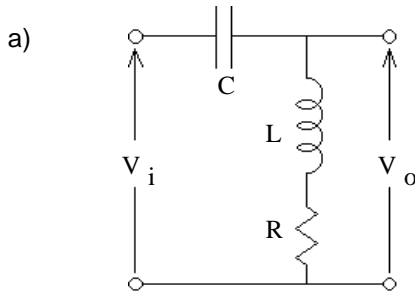


6. b) $\frac{G \cdot (R_1 + R_f)}{R_1 + R_f + R_1 \cdot G}$ c) $1 + \frac{R_f}{R_1}$

7. b) $-\frac{R_f \cdot G}{R_1 + R_f + R_1 \cdot G}$ c) $-\frac{R_f}{R_1}$

1. Problem 3.2b, p.51 in Bodson text.
2. Problem 3.3 in Bodson text. As part of your work to reach a solution, draw the pole diagram for each.

3. Find the transfer function $H(s) = \frac{V_o(s)}{V_i(s)}$ for these circuits. Properly simplify all your expressions for $H(s)$ like you did in HW 4.



4. Find the step response of: $H(s) = \frac{k}{(s+a_1)(s+a_2)}$ Step input: $x(t) = x_m \cdot u(t)$

$$X(s) = \frac{x_m}{s}$$

Show the steps necessary to arrive at the steady-state and transient responses shown as equation(s) 3.34 on p.33 of the text.

5. Find the step response of: $H(s) = \frac{k \cdot s}{(s+a)^2 + b^2} = \frac{k \cdot s}{s^2 + 2 \cdot s \cdot a + (a^2 + b^2)}$ where b is real

Show the steps necessary to arrive at the steady-state and transient responses.

6. For the transfer functions below, find the DC gain and the full step responses. You may use the results found in section 3.3.2 of the text as well as problem 3, above.

a) $H(s) = \frac{2}{s^2 + 2 \cdot s + 1}$

b) $H(s) = \frac{-s-2}{s^2 + 2 \cdot s + 2}$

Hint: Notice how easily this will split into two parts that you already have answers for.

Answers

1. $\frac{H_1 \cdot H_4 + H_2 \cdot H_4 - H_1 \cdot H_2 \cdot H_3 + H_1 \cdot H_3}{1 + H_1}$

2. Stable

Example of a Problem input

- a) yes
- b) no
- c) yes
- d) no
- e) no
- f) no

$\cos(2 \cdot t)$
any input, even noise

$\frac{1}{1} \text{ DC}$
 $u(t)$ is assumed

3. a) $\frac{s^2 + \frac{R}{L} \cdot s}{s^2 + \frac{R}{L} \cdot s + \frac{1}{L \cdot C}}$

b) $\frac{\frac{1}{L \cdot C}}{s^2 + \left(\frac{1}{C \cdot R_2} + \frac{R_1}{L}\right) \cdot s + \left(1 + \frac{R_1}{R_2}\right) \cdot \frac{1}{L \cdot C}}$

4. $y(\infty) = \frac{x_m \cdot k}{a_1 \cdot a_2}$

$y_{tr}(t) = x_m \cdot \left[\frac{k}{a_1 \cdot (a_1 - a_2)} \cdot e^{-a_1 \cdot t} + \frac{k}{a_2 \cdot (a_2 - a_1)} \cdot e^{-a_2 \cdot t} \right]$

OR: $y(t) = x_m \cdot \left[\frac{k}{a_1 \cdot a_2} + \frac{k}{a_1 \cdot (a_1 - a_2)} \cdot e^{-a_1 \cdot t} + \frac{k}{a_2 \cdot (a_2 - a_1)} \cdot e^{-a_2 \cdot t} \right]$

5. $y(\infty) = 0$

$y_{tr}(t) = x_m \cdot \frac{k}{b} \cdot e^{-a \cdot t} \cdot \sin(b \cdot t)$

OR:

$y(t) = 0 + x_m \cdot \frac{k}{b} \cdot e^{-a \cdot t} \cdot \sin(b \cdot t)$

6. a) $x_m \cdot (2 - 2 \cdot e^{-t} - 2 \cdot t \cdot e^{-t})$

b) $x_m \cdot (-1 + e^{-t} \cdot \cos(t))$

The 1st exam on Mon, 2/6/19 will include this material

- Given the conditions in example 3.4.3, p.40,
 - Show all the steps needed to find eq. 3.56.
 - Use the Laplace transform table to find the results in eq. 3.57 and 3.58 ($y_{ss}(t)$ part).
 - Show that equations 3.59 & 3.60 can be found from equations 3.58.
 - Show that equations 3.59 & 3.60 can be found from steady-state analysis of $H(s)$ (see eq. 3.52).
- Still referring to the system in example 3.4.3, p.40, the input is: $x(t) = x_m \cdot \sin(\omega_o \cdot t)$
 - Confirm eq. 3.62.
 - Use any method you want to find M and ϕ_2 in: $y_{ss}(t) = M \cdot x_m \cdot \cos(\omega_o \cdot t + \phi_2)$
 Hint:, you may want to recall that: $\sin(\omega_o \cdot t) = \cos(\omega_o \cdot t - 90 \cdot \text{deg})$
- This system: $H(s) = \frac{3}{s+8}$ Has a cosine input: $x(t) = 4 \cdot \cos(10 \cdot t) \cdot u(t)$
 - Express the output, $Y(s)$
 - This separates into 3 partial fractions that you can find in the laplace transform table without using complex numbers. Show what they are, but don't find the coefficients.
 - Continue with the partial fraction expansion just far enough to find the **transient** coefficient as a number.
 - Express the transient part as a function of time. $y_{tr}(t) = ?$
 - What is the time constant of this expression? $\tau = ?$
 - Use steady-state AC analysis to find the steady-state output in the form of a cosine with a magnitude and phase angle.
 $y_{ss}(t) = ?$
- This system: $H(s) = \frac{4}{s+12}$ Has this Cosine input: $x(t) = 5 \cdot \cos(8 \cdot t + 40 \cdot \text{deg}) \cdot u(t)$
 - Use steady-state AC analysis to find the steady-state response ($y_{ss}(t)$) of the system. $y_{ss}(t) = ?$
 - Separate the input $x(t)$ into a pure cosine part and a pure sine part.
 - Use the results of 1b) and 2a), above to find the transient responses to cosine and sine inputs and then add them together to find the total transient response.
- Find the steady-state (sinusoidal) magnitude and phase of the following transfer function.

$$|H(j\omega)| = ? \quad \angle H(j\omega) = ? \quad \omega := 20 \cdot \frac{\text{rad}}{\text{sec}} \quad H(s) = \frac{\frac{80}{\text{sec}} \cdot s - \frac{300}{\text{sec}^2}}{s^2 + \frac{90}{\text{sec}^2}}$$
- Express the following signal in the time domain, as a sum of cosine and sine with no phase angles:

$$\omega := 44 \cdot \frac{\text{rad}}{\text{sec}} \quad Y(j\omega) = 3 + 0.5 \cdot j$$

ECE 3510 homework # 7 p.2

7. The following questions refer to the general system whose output is given by eq. 3.70, p.42 in our text.

a) Can a system's response to initial conditions be calculated separately from its response to the input signal? Why or why not?

b) Can you expect a system's response to initial conditions to be similar to its response to a simple input signal? Why or why not?

c) To fully describe the state of the system, how many things do you need to know? List them.

d) If a system is BIBO stable, then what is its final response to initial conditions?

e) The output of a system with nonrepeated poles on the $j\omega$ -axis which is otherwise BIBO stable can be unbounded for some input signals. Is this also true for initial conditions alone when there is no input signal?

If no, why are the conditions for bounded output not as restrictive if there are only initial conditions and no input?

8. a) List 4 advantages of the state-space method over the frequency domain method we are using in this class.

b) List 2 advantages of the frequency domain method we are using in this class over the state-space method.

Answers

1 & 2a) Answers are right in the book

$$2.b) \frac{k}{\sqrt{\omega_o^2 + a^2}} \quad -\tan^{-1}\left(\frac{\omega_o}{a}\right) - 90\text{-deg}$$

$$3. a) \frac{3}{s+8} + \frac{4 \cdot s}{s^2 + 100}$$

$$b) \frac{A}{s+8} + \frac{B \cdot s}{(s^2 + 100)} + \frac{C \cdot 10}{(s^2 + 100)}$$

$$c) -0.585$$

$$d) -0.585 \cdot e^{-8 \cdot t}$$

$$e) 125 \cdot \text{ms}$$

$$f) 0.936 \cdot \cos(10 \cdot t - 51.34 \cdot \text{deg})$$

$$4. a) 1.385 \cdot \cos(8 \cdot t + 6.3 \cdot \text{deg})$$

$$b) x(t) = (3.83 \cdot \cos(8 \cdot t) - 3.214 \cdot \sin(8 \cdot t)) \cdot u(t)$$

$$c) -1.378 \cdot e^{-12 \cdot t}$$

$$5. \quad 5.251 \quad -79.38 \cdot \text{deg}$$

$$6. \quad 3 \cdot \cos\left(44 \cdot \frac{\text{rad}}{\text{sec}} \cdot t\right) - 0.5 \cdot \sin\left(44 \cdot \frac{\text{rad}}{\text{sec}} \cdot t\right)$$

$$7. c) 4 \quad y(0) \quad \frac{d}{dt}y(0) \quad x(0) \quad \frac{d}{dt}x(0) \quad d) 0$$

8. See section 3.1 in the Nise textbook.

Use the current-force analogy discussed in class for the following problems.

1. a) Find the equivalent electric circuit for the mechanical system shown. F_{in} is an input.

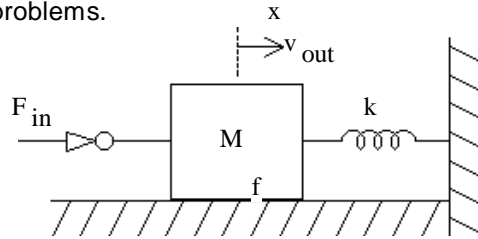
b) Find the transfer function for the system. Put it in the standard form. $\frac{v_{out}(s)}{F_{in}(s)}$

c) Check the units of all coefficients of the transfer function to make sure they agree and work out to the units of velocity over force.

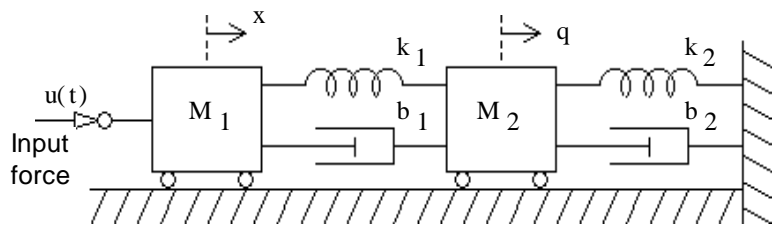
Recall that the units of $s = \frac{1}{\text{sec}}$

d) The resonant frequency of an electrical circuit can be found from $\frac{1}{\sqrt{L \cdot C}}$. What is it for this system?

e) Find the transfer function for the system. Put it in the standard form. $\frac{x_{out}(s)}{F_{in}(s)}$ Where x is the displacement of the mass rather than its velocity.

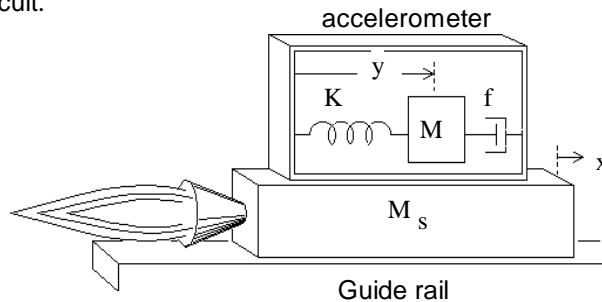
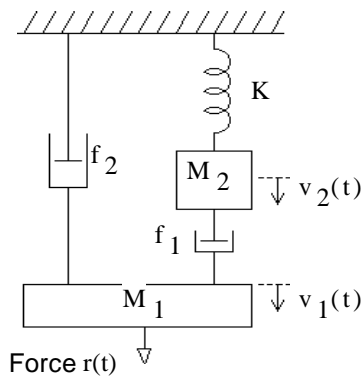


2. Find the equivalent electric circuit for the mechanical system shown. $u(t)$ is an input. Show x -velocity and q -velocity on the circuit.

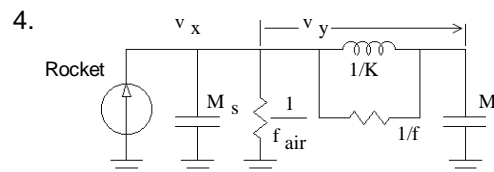
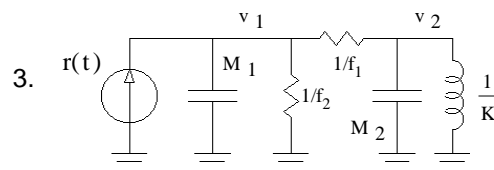
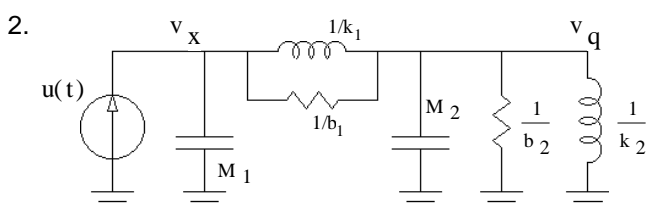
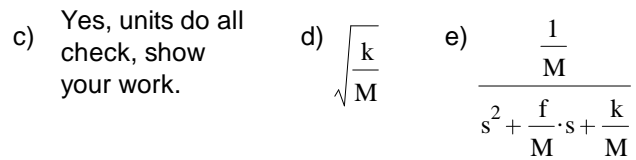
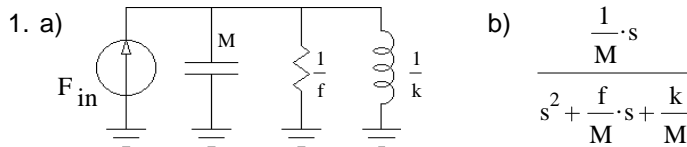


3. Find the equivalent electric circuit for the mechanical system shown. $r(t)$ is an input. Show v_1 & v_2 on the circuit.

4. Find the equivalent electric circuit for the levitated rocket sled shown. The rocket is a force input. There is no friction between the sled and guide rail, but there is air resistance (which can be modeled in exactly the same way as friction between the sled and guide rail) The accelerometer is firmly mounted onto the sled. Show x -velocity and y -velocity on the circuit.

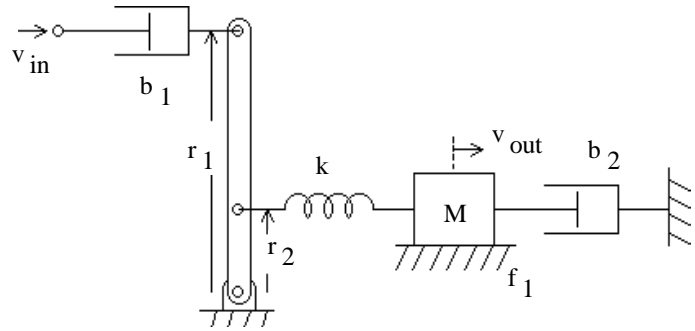


Answers

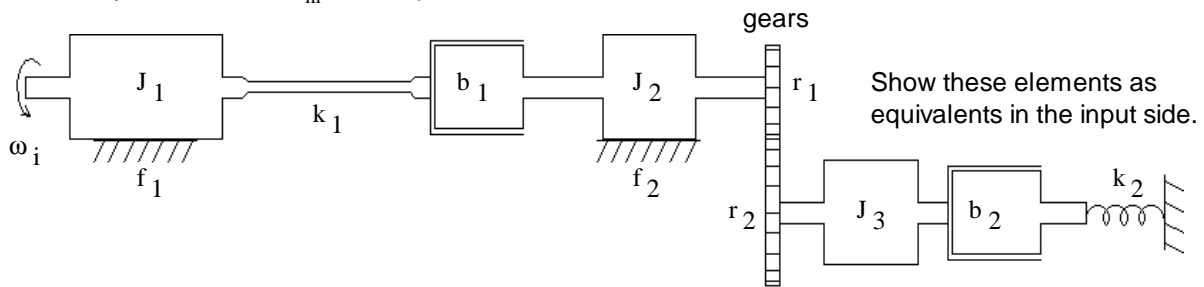


Use the current-force analogy discussed in class for the following problems.

1. Find the equivalent electric circuit for the mechanical system shown. v_{in} is the input.

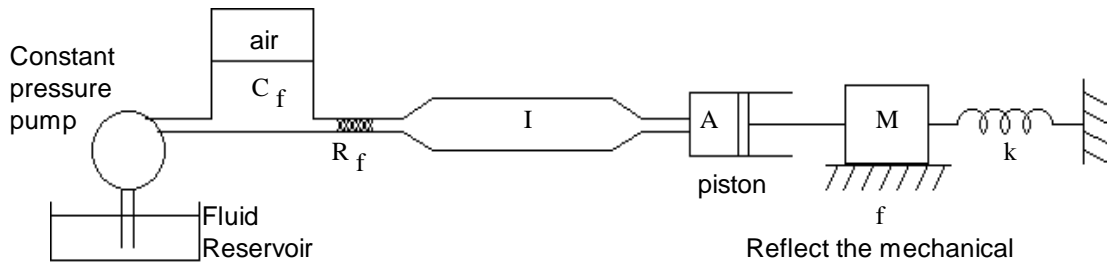


2. Find the equivalent electric circuit for the mechanical system shown. ω_{in} is the input.



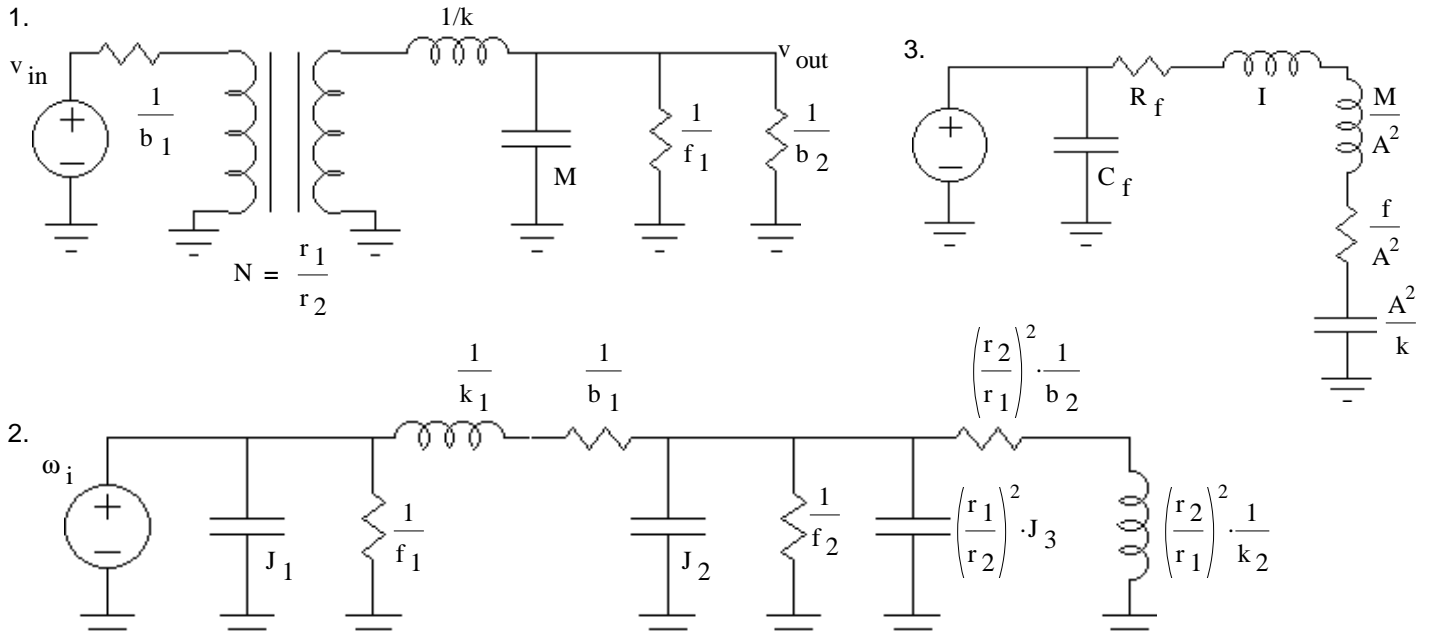
Show these elements as equivalents in the input side.

3. Find the equivalent electric circuit for the fluid system shown.



Reflect the mechanical elements into the fluid system.

Answers



- Draw a basic control system loop such as that shown in Fig 4.7 (Bodson), show all the items listed on p. 59 plus a feedback sensor labeled $F(s)$ and a disturbance input.
- Add $F(s)$ or $n_f(s)$ and $d_f(s)$ into the following equations: full $Y(s) =$
 With disturbance as zero: Eq. 4.5 Eq. 4.7 Eq. 4.10
 With input ($R(s)$) as zero: Eq. 4.13 Eq. 4.15
- List 5 measures of a control system's quality (see p. 59-60) and list one or two things that can be done to achieve each.
- The transfer functions of $C(s)$ and $P(s)$ are given below. In each case determine if the steady-state error will go to zero and whether disturbances will be completely rejected. Be sure to check for closed-loop stability when needed.

a) $C(s) = \frac{s+4}{s^2+3s+2}$	P(s) = $\frac{s+1}{s^2+3s}$	b) $C(s) = \frac{s+1}{s^2+3s}$	P(s) = $\frac{s+4}{s^2+3s+2}$
c) $C(s) = \frac{s(s+6)}{s^2+3s+2}$	P(s) = $\frac{s+8}{s^2+12s}$	d) $C(s) = \frac{s+9}{s^2+3s+2}$	P(s) = $\frac{s}{s+16}$
e) $C(s) = \frac{s+1}{s^2+5s+6}$	P(s) = $\frac{s+1}{s^2+8s+15}$	f) $C(s) = \frac{s+1}{s^3+7s^2+12s}$	P(s) = $\frac{s+1}{s+3}$
- Problem 4.2 (p.98) in the text. Use your calculator or Matlab to find the actual roots, or use the Routh-Hurwitz method.

6. EXTRA CREDIT

Characteristic equations of feedback systems are shown below. In each case, use the Routh-Hurwitz method to determine the value range of K that will produce a stable system. You must show your work.

a) $0 = s^4 + 20s^3 + 10s^2 + s + K$ b) $0 = s^4 + 2Ks^3 + 5s^2 + Ks + K$

Answers

1., 2., 3. Read sections 4.1 - 4.2 in text (Bodson). $Y(s) = \frac{P \cdot C \cdot R + P \cdot D}{1 + P \cdot C \cdot F}$

4. a) Yes No b) Yes Yes
 c) No No d) No Yes
 e) No No f) Yes Yes

5. a) Yes b) No c) No

6. EXTRA CREDIT a) $0 < K < 0.4975$ b) $0 < K < 2.25$