

Table of Discrete-Time Fourier Transform Pairs:

$$\text{Discrete-Time Fourier Transform} : X(\Omega) = \sum_{n=-\infty}^{\infty} x[n]e^{-j\Omega n}$$

$$\text{Inverse Discrete-Time Fourier Transform} : x[n] = \frac{1}{2\pi} \int_{2\pi} X(\Omega)e^{j\Omega n} d\Omega .$$

$x[n]$	$X(\Omega)$	condition
$a^n u[n]$	$\frac{1}{1 - ae^{-j\Omega}}$	$ a < 1$
$(n+1)a^n u[n]$	$\frac{1}{(1 - ae^{-j\Omega})^2}$	$ a < 1$
$\frac{(n+r-1)!}{n!(r-1)!} a^n u[n]$	$\frac{1}{(1 - ae^{-j\Omega})^r}$	$ a < 1$
$\delta[n]$	1	
$\delta[n - n_0]$	$e^{-j\Omega n_0}$	
$x[n] = 1$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$	
$u[n]$	$\frac{1}{1 - e^{-j\Omega}} + \sum_{k=-\infty}^{\infty} \pi \delta(\Omega - 2\pi k)$	
$e^{j\Omega_0 n}$	$2\pi \sum_{k=-\infty}^{\infty} \delta(\Omega - \Omega_0 - 2\pi k)$	
$\cos(\Omega_0 n)$	$\pi \sum_{k=-\infty}^{\infty} \{\delta(\Omega - \Omega_0 - 2\pi k) + \delta(\Omega + \Omega_0 - 2\pi k)\}$	
$\sin(\Omega_0 n)$	$\frac{\pi}{j} \sum_{k=-\infty}^{\infty} \{\delta(\Omega - \Omega_0 - 2\pi k) - \delta(\Omega + \Omega_0 - 2\pi k)\}$	
$\sum_{k=-\infty}^{\infty} \delta[n - kN]$	$\frac{2\pi}{N} \sum_{k=-\infty}^{\infty} \delta\left(\Omega - \frac{2\pi k}{N}\right)$	
$x[n] = \begin{cases} 1 & , n \leq N \\ 0 & , n > N \end{cases}$	$\frac{\sin(\Omega(N + 1/2))}{\sin(\Omega/2)}$	
$\frac{\sin(Wn)}{\pi n} = \frac{W}{\pi} \text{sinc}\left(\frac{Wn}{\pi}\right)$	$X(\Omega) = \begin{cases} 1 & , 0 \leq \Omega \leq W \\ 0 & , W < \Omega \leq \pi \end{cases}$	
$X(\Omega)$ is periodic with period 2π		

Table of Discrete-Time Fourier Transform Properties: For each property, assume

$$x[n] \xleftrightarrow{DTFT} X(\Omega) \quad \text{and} \quad y[n] \xleftrightarrow{DTFT} Y(\Omega)$$

Property	Time domain	DTFT domain
Linearity	$Ax[n] + By[n]$	$AX(\Omega) + BY(\Omega)$
Time Shifting	$x[n - n_0]$	$X(\Omega)e^{-j\Omega n_0}$
Frequency Shifting	$x[n]e^{j\Omega_0 n}$	$X(\Omega - \Omega_0)$
Conjugation	$x^*[n]$	$X^*(-\Omega)$
Time Reversal	$x[-n]$	$X(-\Omega)$
Convolution	$x[n] * y[n]$	$X(\Omega)Y(\Omega)$
Multiplication	$x[n]y[n]$	$\frac{1}{2\pi} \int_{2\pi} X(\theta)Y(\Omega - \theta)d\theta$
Differencing in Time	$x[n] - x[n - 1]$	$(1 - e^{-j\Omega})X(\Omega)$
Accumulation	$\sum_{k=-\infty}^{\infty} x[k]$	$\frac{1}{1 - e^{-j\Omega}} + \pi X(0) \sum_{k=-\infty}^{\infty} \delta(\Omega - 2\pi k)$
Frequency Differentiation	$nx[n]$	$j \frac{dX(\Omega)}{d\Omega}$
Parseval's Relation for Aperiodic Signals	$\sum_{k=-\infty}^{\infty} x[k] ^2$	$\frac{1}{2\pi} \int_{2\pi} X(\Omega) ^2 d\Omega$

Table of Z-Transform Pairs:

$$\text{Z-Transform} : X(z) = \sum_{n=-\infty}^{\infty} x[n]z^{-n}$$

$$\text{Inverse Z-Transform} : x[n] = \frac{1}{2\pi j} \oint_{\mathcal{C}} X(z)z^{n-1} dz .$$

$x[n]$	$X(\Omega)$
$a^n u[n]$	$\frac{1}{1 - az^{-1}}$
$-a^n u[-n - 1]$	$\frac{1}{1 - az^{-1}}$
$na^n u[n]$	$\frac{az^{-1}}{(1 - az^{-1})^2}$
$\delta[n]$	1
$\delta[n - n_0]$	z^{-n_0}
$u[n]$	$\frac{1}{1 - z^{-1}}$
$\cos(\Omega_0 n)u[n]$	$\frac{1 - z^{-1} \cos(\Omega_0)}{1 - 2z^{-1} \cos(\Omega_0) + z^{-2}}$
$\sin(\Omega_0 n)u[n]$	$\frac{z^{-1} \sin(\Omega_0)}{1 - 2z^{-1} \cos(\Omega_0) + z^{-2}}$