Table of Fourier Series Properties:

$$\text{Fourier Analysis} \quad : \quad c_k = \frac{1}{T_0}\int_{T_0} x(t)e^{-jk\omega_0 t}dt$$

$$\text{Fourier Synthesis} \quad : \quad x(t) = \sum_{k=-\infty}^{\infty} c_k e^{jk\omega_0 t}$$

(ω_0 is the *fundamental angular frequency* of $x(t)$ and T_0 is the *fundamental period* of $x(t)$)

For each property, assume $x(t) \overset{\mathcal{F}}{\longleftrightarrow} c_k$ and $y(t) \overset{\mathcal{F}}{\longleftrightarrow} d_k$

Property	Time domain	Fourier domain
Linearity	$Ax(t) + By(t)$	$Ac_k + Bd_k$
Time Shifting	$x(t - t_0)$	$c_k e^{-jk\omega_0 t_0}$
Frequency Shifting	$x(t)e^{jM\omega_0 t}$	c_{k-M}
Conjugation	$x^*(t)$	c^*_{-k}
Time Reversal	$x(-t)$	c_{-k}
Circular Conv.	$x(t) \circledast y(t)$	$T_0 c_k d_k$
Multiplication	$x(t)y(t)$	$c_k * d_k$
Differentiation	$\frac{d}{dt}x(t)$	$(jk\omega_0)c_k$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\left(\frac{1}{jk\omega_0}\right) c_k$
Conjugate Symmetry for		
Real Signals	$x(t)$ is real	$c_k = c^*_{-k}$
Real and Even Signals	$x(t)$ is real and even	c_k is real and even
Real and Odd Signals	$x(t)$ is real and odd	c_k is purely imaginary and odd
Parseval's Relation for		
Cont. Periodic signals	$\frac{1}{T_0}\int_{T_0} \lvert x(t)\rvert^2\, dt$	$\sum_{k=-\infty}^{\infty} \lvert c_k\rvert^2$

Table of Special Functions:

Function name	Expression	Notes
Sinc function	$\text{sinc}(x) = \frac{\sin(x)}{x}$	Note that there exists an alternative definition for $\text{sinc}(x)$
Rectangle function	$\text{rect}(x) = \begin{cases} 0 & if \quad x \geq 1/2 \quad , \quad x < -1/2 \\ 1 & if \quad -1/2 \leq x < 1/2 \end{cases}$	
	$\text{rect}(x) = u(x + \frac{1}{2}) - u(x - \frac{1}{2})$	Note that there are alternative definitions with different values for $\text{rect}(\pm 1/2)$
Unit triangle function	$\Delta(x) = \begin{cases} 0 & if \quad \lvert x \rvert \geq 1/2 \\ 1 - 2\lvert x \rvert & if \quad \lvert x \rvert < 1/2 \end{cases}$	

Table of Fourier Transform Pairs:

$$\text{Fourier Transform} \quad : \quad X(\omega) = \int_{-\infty}^{\infty} x(t) e^{-j\omega t}\, dt$$

$$\text{Inverse Fourier Transform} \quad : \quad x(t) = \frac{1}{2\pi} \int_{-\infty}^{\infty} X(\omega) e^{j\omega t}\, d\omega\ .$$

$x(t)$	$X(\omega)$	condition
$e^{-at}u(t)$	$\frac{1}{a+j\omega}$	$a > 0$
$e^{at}u(-t)$	$\frac{1}{a-j\omega}$	$a > 0$
$e^{-a\|t\|}$	$\frac{2a}{a^2+\omega^2}$	$a > 0$
$te^{-at}u(t)$	$\frac{1}{(a+j\omega)^2}$	$a > 0$
$t^n e^{-at}u(t)$	$\frac{n!}{(a+j\omega)^{n+1}}$	$a > 0$
$\delta(t)$	1	
1	$2\pi\delta(\omega)$	
$e^{j\omega_0 t}$	$2\pi\delta(\omega-\omega_0)$	
$\cos(\omega_0 t)$	$\pi\left(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right)$	
$\sin(\omega_0 t)$	$j\pi\left(\delta(\omega+\omega_0)-\delta(\omega-\omega_0)\right)$	
$u(t)$	$\pi\delta(\omega)+\frac{1}{j\omega}$	
$\text{sgn}(t)$	$\frac{2}{j\omega}$	
$\cos(\omega_0 t)u(t)$	$\frac{\pi}{2}\left(\delta(\omega-\omega_0)+\delta(\omega+\omega_0)\right)+\frac{j\omega}{\omega_0^2-\omega^2}$	
$\sin(\omega_0 t)u(t)$	$\frac{\pi}{2j}\left(\delta(\omega-\omega_0)-\delta(\omega+\omega_0)\right)+\frac{\omega_0}{\omega_0^2-\omega^2}$	
$e^{-at}\sin(\omega_0 t)u(t)$	$\frac{\omega_0}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
$e^{-at}\cos(\omega_0 t)u(t)$	$\frac{a+j\omega}{(a+j\omega)^2+\omega_0^2}$	$a > 0$
$\text{rect}\left(\frac{t}{T}\right)$	$T\text{sinc}\left(\frac{\omega T}{2}\right)$	
$\frac{W}{\pi}\text{sinc}(Wt)$	$\text{rect}\left(\frac{\omega}{2W}\right)$	
$\Delta\left(\frac{t}{T}\right)$	$\frac{T}{2}\text{sinc}^2\left(\frac{\omega T}{4}\right)$	
$\frac{W}{2\pi}\text{sinc}^2\left(\frac{WT}{2}\right)$	$\Delta\left(\frac{\omega}{2W}\right)$	
$\sum_{n=-\infty}^{\infty}\delta(t-nT)$	$\omega_0\sum_{n=-\infty}^{\infty}\delta(\omega-n\omega_0)$	$\omega_0=\frac{2\pi}{T_0}$
$e^{-t^2/(2\sigma^2)}$	$\sigma\sqrt{2\pi}e^{-\sigma^2\omega^2/2}$	

Table of Fourier Transform Properties: For each property, assume

$$x(t) \overset{\mathcal{F}}{\longleftrightarrow} X(\omega) \quad \text{and} \quad y(t) \overset{\mathcal{F}}{\longleftrightarrow} Y(\omega)$$

Property	Time domain	Fourier domain
Linearity	$Ax(t) + By(t)$	$AX(\omega) + BY(\omega)$
Time Shifting	$x(t - t_0)$	$X(\omega)e^{-j\omega t_0}$
Time Scaling	$x(\alpha t)$	$\frac{1}{\lvert\alpha\rvert} X\left(\frac{\omega}{\alpha}\right)$
Frequency Shifting	$x(t)e^{j\omega_0 t}$	$X(\omega - \omega_0)$
Conjugation	$x^*(t)$	$X^*(-\omega)$
Time Reversal	$x(-t)$	$X(-\omega)$
Convolution	$x(t) * y(t)$	$X(\omega)Y(\omega)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi} X(\omega) * Y(\omega)$
Differentiation	$\frac{d}{dt}x(t)$	$(j\omega)X(\omega)$
Integration	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\left(\frac{1}{j\omega}\right) X(\omega)$
Conjugate Symmetry for		
Real Signals	$x(t)$ is real	$X(\omega) = X^*(-\omega)$
Real and Even Signals	$x(t)$ is real and even	$X(\omega)$ is real and even
Real and Odd Signals	$x(t)$ is real and odd	$X(\omega)$ is purely imaginary and odd
Parseval's Relation for		
Aperiodic signals	$\int_{-\infty}^{\infty} \lvert x(t)\rvert^2\, dt$	$\frac{1}{2\pi} \int_{-\infty}^{\infty} \lvert X(\omega)\rvert^2\, d\omega$

Table of Laplace Transform Pairs:

$$\text{Bilateral Laplace Transform} \quad : \quad X(s) = \int_{-\infty}^{\infty} x(t)e^{-st}\, dt$$

$$\text{Unilateral Laplace Transform} \quad : \quad X(s) = \int_{0}^{\infty} x(t)e^{-st}\, dt$$

$$\text{Inverse Laplace Transform} \quad : \quad x(t) = \frac{1}{2\pi j}\int_{c-j\infty}^{c+j\infty} X(s)e^{st}\, ds\ .$$

$x(t)$	$X(s)$
$\delta(t)$	1
$u(t)$	$\frac{1}{s}$
$tu(t)$	$\frac{1}{s^2}$
$t^n u(t)$	$\frac{n!}{s^{n+1}}$
$e^{at}u(t)$	$\frac{1}{s-a}$
$te^{at}u(t)$	$\frac{1}{(s-a)^2}$
$t^n e^{at}u(t)$	$\frac{n!}{(s-a)^{n+1}}$
$\cos(bt)u(t)$	$\frac{s}{s^2+b^2}$
$\sin(bt)u(t)$	$\frac{b}{s^2+b^2}$
$e^{-at}\cos(bt)u(t)$	$\frac{s+a}{(s+a)^2+b^2}$
$e^{-at}\sin(bt)u(t)$	$\frac{b}{(s+a)^2+b^2}$
$re^{-at}\cos(bt+\theta)u(t)$	$\frac{(r\cos(\theta))s+(ar\cos(\theta)-br\sin(\theta))}{s^2+2as+(a^2+b^2)}$
$re^{-at}\cos(bt+\theta)u(t)$	$\frac{0.5re^{j\theta}}{s+a-jb}+\frac{0.5re^{-j\theta}}{s+a+jb}$
$re^{-at}\cos(bt+\theta)u(t) \quad , \quad r=\sqrt{\frac{A^2c+B^2-2ABa}{c-a^2}}$ $\theta=\tan^{-1}\left(\frac{Aa-B}{A\sqrt{c-a^2}}\right) \quad , \quad b=\sqrt{c-a^2}$	$\frac{As+B}{s^2+2as+c}$
$e^{-at}\left[A\cos(bt)+\frac{B-Aa}{b}\sin(bt)\right]u(t) \quad , \quad b=\sqrt{c-a^2}$	$\frac{As+B}{s^2+2as+c}$

Table of Laplace Transform Properties: For each property, assume

$$x(t) \overset{\mathcal{L}}{\longleftrightarrow} X(s) \quad \text{and} \quad y(t) \overset{\mathcal{L}}{\longleftrightarrow} Y(s)$$

Property	Time domain	Laplace domain
Linearity	$Ax(t) + By(t)$	$AX(s) + BY(s)$
Time Shifting	$x(t - t_0)$	$X(s)e^{-st_0}$
Time Scaling	$x(\alpha t)$, $\alpha \geq 0$	$\frac{1}{\alpha}X\left(\frac{s}{\alpha}\right)$
Frequency Shifting	$x(t)e^{s_0 t}$	$X(s - s_0)$
Time Reversal	$x(-t)$	$X(-s)$
Convolution	$x(t) * y(t)$	$X(s)Y(s)$
Multiplication	$x(t)y(t)$	$\frac{1}{2\pi j}X(s) * Y(s)$
Differentiation	$\frac{d}{dt}x(t)$	$sX(s) - x(0^-)$
	$\frac{d^2}{dt^2}x(t)$	$s^2X(s) - sx(0^-) - \dot{x}(0^-)$
	$\frac{d^3}{dt^3}x(t)$	$s^3X(s) - s^2x(0^-) - s\dot{x}(0^-) - \ddot{x}(0^-)$
Integration	$\int_{0^-}^{t} x(\tau)\, d\tau$	$\left(\frac{1}{s}\right) X(s)$
	$\int_{-\infty}^{t} x(\tau)\, d\tau$	$\left(\frac{1}{s}\right) X(s) + \frac{1}{s}\int_{-\infty}^{0^-} x(t)dt$
Frequency Differentiation	$-tx(t)$	$\frac{dX(s)}{ds}$
Frequency Integration	$\frac{x(t)}{t}$	$\int_{s}^{\infty} X(z)dz$
Initial value	$x(0^+)$	$\lim_{s\to\infty} sX(s)$
Final value	$x(\infty)$	$\lim_{s\to 0} sX(s)$ (if poles of $sX(s)$ are in left-hand plane)