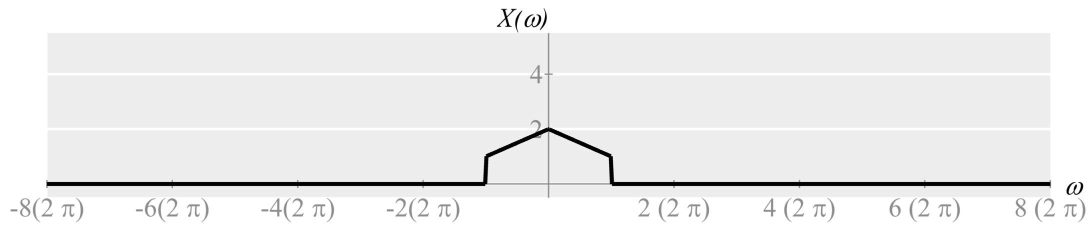
Full Name: ______________________ **Lab Section:** __________

ECE 3500 (Fall 2015) – Class #20 Examples **Date:** **Nov. 5, 2015**

Question #1: Consider the Fourier transform $X(\omega)$ of $x(t)$ illustrated below.



(a) Sketch the Fourier transform of the amplified and sampled signal $x_s(t) = x(t)\delta_{T_s}(t)$ with angular sampling rate $\omega_s = 8\pi$

(b) Sketch the Fourier transform of the amplified and sampled signal $x_s(t) = x(t)\delta_{T_s}(t)$ with angular sampling rate $\omega_s = 4\pi$

(c) Sketch the Fourier transform of the amplified and sampled signal $x_s(t) = x(t)\delta_{T_s}(t)$ with angular sampling rate $\omega_s = 2\pi$

Question #2: Consider the ideal low-pass filter $h(t)$ with cut-off frequency $\omega_s/2$ and gain of $\frac{2\pi}{\omega_s}$. Also, consider the signal

$$x(t) = \sin(\omega_0 t) \ .$$

Determine the sampled and reconstructed signal $y(t) = (x(t)\delta_{T_s}(t)) * h(t)$ for:

(a) $\omega_0 = 2$ and $\omega_s = 5$.

(b) $\omega_0 = 4$ and $\omega_s = 5$.

(c) $\omega_0 = 6$ and $\omega_s = 5$.

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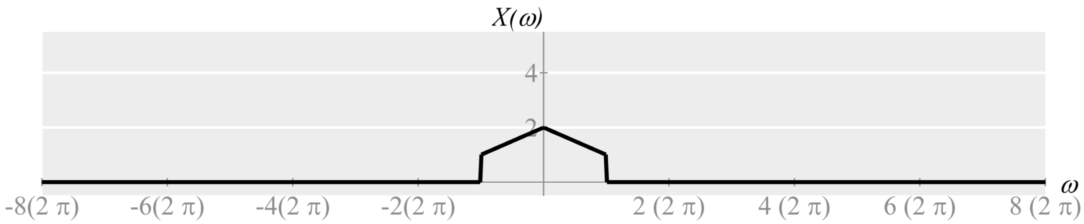
Question #1: Consider the Fourier transform $X(\omega)$ of $x(t)$ illustrated below.

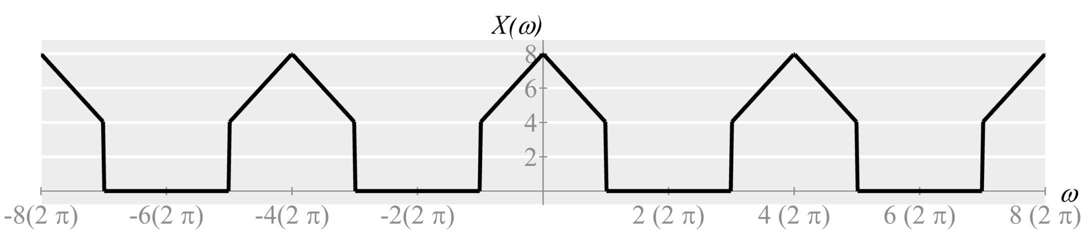


(a) Sketch the Fourier transform of the amplified and sampled signal $x_s(t) = x(t)\delta_{T_s}(t)$ with angular sampling rate $\omega_s = 8\pi$

Solution: From the notes, we get that

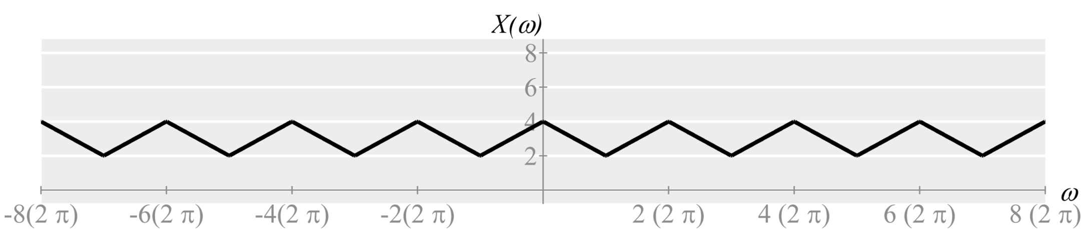
$$\begin{aligned} X_s(\omega) &= \frac{1}{T_s} X(\omega - \omega_s) \\ &= \frac{\omega_s}{2\pi} X(\omega - \omega_s) \\ &= 4X(\omega - 8\pi) \end{aligned}$$

Note that the solution below is periodic.



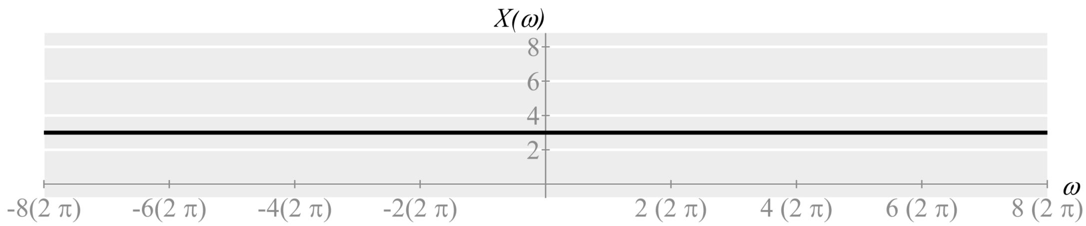
(b) Sketch the Fourier transform of the amplified and sampled signal $x_s(t) = x(t)\delta_{T_s}(t)$ with angular sampling rate $\omega_s = 4\pi$

Solution: Note that the solution below is periodic.



(c) Sketch the Fourier transform of the amplified and sampled signal $x_s(t) = x(t)\delta_{T_s}(t)$ with angular sampling rate $\omega_s = 2\pi$

Solution: Note that the solution below is periodic.



Question #2: Consider the ideal low-pass filter $h(t)$ with cut-off frequency $\omega_s/2$ and gain of $\frac{2\pi}{\omega_s}$. Also, consider the signal

$$x(t) = \sin(\omega_0 t) \; .$$

Determine the sampled and reconstructed signal $y(t) = (x(t)\delta_{T_s}(t)) * h(t)$ for:

(a) $\omega_0 = 2$ and $\omega_s = 5$.

Solution: The sinusoid $\sin(2t)$ is inside of the low-pass filter's frequency range, so

$$y(t) = \sin(2t)$$

(b) $\omega_0 = 4$ and $\omega_s = 5$.

Solution: The sinusoid $\sin(4t)$ is outside of the low-pass filter's frequency range, so

$$y(t) = \sin((4-5)t) = -\sin(t)$$

(c) $\omega_0 = 6$ and $\omega_s = 5$.

Solution: The sinusoid $\sin(6t)$ is outside of the low-pass filter's frequency range, so

$$y(t) = \sin((6-5)t) = \sin(t)$$