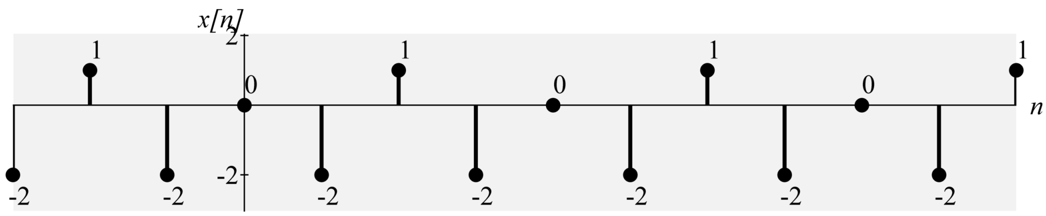
Full Name: ______________________________ **Lab Section:** ____________

ECE 3500 (Fall 2015) – Class #3 Examples **Date:** **Sept. 1, 2015**

Question #1: Consider the discrete-time signal $x[n]$ below. **Assume the periodic pattern shown in the plot continues forever to $n \to -\infty$ and $n \to \infty$.**



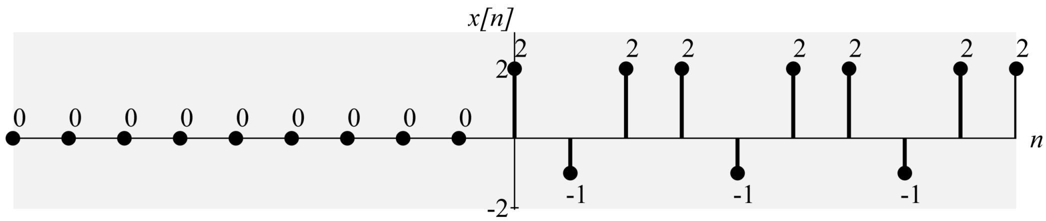
(a) Determine the fundamental period of $x[n]$.

(b) Compute the energy of $x[n]$.

(c) Compute the average power of $x[n]$.

(d) Is $x[n]$ causal? Also, is $x[n]$ even, odd, or neither?

Question #2: Consider the following signal $x[n]$. Assume the periodic pattern shown in the plot continues forever for $n \geq 0$. Also assume the zeros continue forever for $n < 0$.



(a) Express this signal (the entire signal) using step functions and/or impulse functions.

(b) The signal $y[n] = x[n] + x[?]$ is periodic. Determine a ? so that this statement is true.

(c) Determine the fundamental period of $y[n]$.

Question #3: Let $x_1(t)$, $x_2(t)$, and $x_3(t)$ be periodic signals with fundamental periods of 1, 3, and 10, respectively. Also let $x_1(t)$, $x_2(t)$, and $x_3(t)$ have powers of 1, 2, and 3, respectively.

(a) Compute the fundamental period of $z(t) = x_1(t) + x_2(t) + x_3(t)$

(b) Based on our knowledge, can we compute the power of $z(t)$? If so, what is the power? If not, why?

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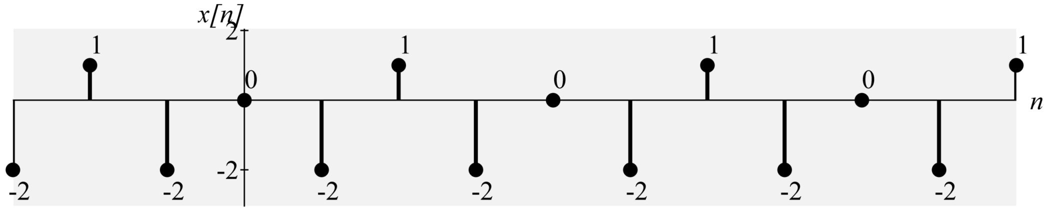
ECE 3500 (Fall 2015) – Class #3 Examples **Date: Sept. 1, 2015**

Question #1: Consider the discrete-time signal $x[n]$ below. **Assume the periodic pattern shown in the plot continues forever to $n \to -\infty$ and $n \to \infty$.**



(a) Determine the fundamental period of $x[n]$.

Solution: The signal repeats every 4 samples. Therefore, $N_0 = 4$.

(b) Compute the energy of $x[n]$.

Solution: The signal is infinite and periodic. Therefore, $|x[n]|^2$ is always positive from $-\infty < n < \infty$. Hence $E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2 = \infty$.

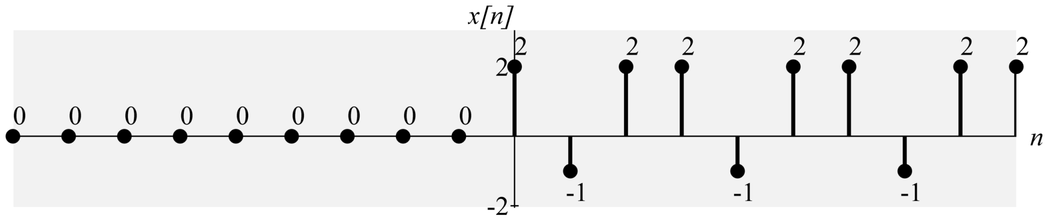
(c) Compute the average power of $x[n]$.

Solution: The signal is infinite and periodic, so it has a finite power. The power is the average energy of the signal in one period. Energy in one period is $(-2)^2 + (1)^2 + (-2)^2 = 9$. The fundamental period is $N_0 = 4$. Therefore the power is $9/4$.

(d) Is $x[n]$ causal? Also, is $x[n]$ even, odd, or neither?

Solution: The signal is non-zero for $n < 0$, so it is **not causal**. The signal is also **even**.

Question #2: Consider the following signal $x[n]$. Assume the periodic pattern shown in the plot continues forever for $n \geq 0$. Also assume the zeros continue forever for $n < 0$.



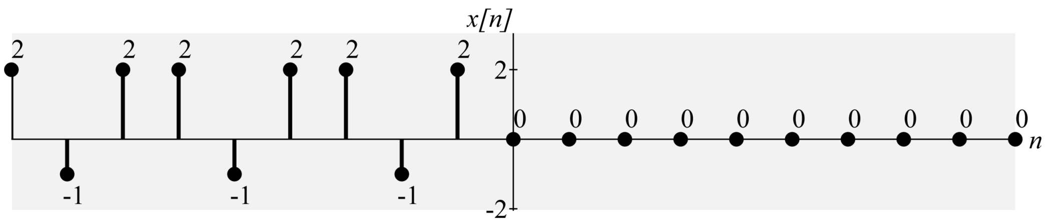
(a) Express this signal (the entire signal) using step functions and/or impulse functions.

Solution:

$$x[n] = \sum_{k=0}^{\infty} 2\delta[n-3k] - \delta[n-3k-1] + 2\delta[n-3k-2]$$

(b) The signal $y[n] = x[n] + x[?]$ is periodic. Determine a ? so that this statement is true.

Solution: The answer is $? = -n-1$. This is easiest to determine visually. If we plot the signal $x[-2n+1]$, we get



If we combine this with the plot above, we get a periodic signal across all time.

(c) Determine the fundamental period of $y[n]$.

Solution: The fundamental period is $N_0 = 3$.

Question #3: Let $x_1(t)$, $x_2(t)$, and $x_3(t)$ be periodic signals with fundamental periods of 1, 3, and 10, respectively. Also let $x_1(t)$, $x_2(t)$, and $x_3(t)$ have powers of 1, 2, and 3, respectively.

(a) Compute the fundamental period of $z(t) = x_1(t) + x_2(t) + x_3(t)$

Solution: The fundamental periods of each term are $T_1 = 1$, $T_2 = 3$, $T_3 = 10$. The least common multiple of the fundamental period is $LCM(T_1, T_2, T_3) = LCM(1, 3, 10) = 30$.

(b) Based on our knowledge, can we compute the power of $z(t)$? If so, what is the power? If not, why?

Solution: You cannot compute the power of $z(t)$. This is because

$$
\begin{aligned}
P_z &= \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |z(t)|^2 dt \\
&= \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |x_1(t) + x_2(t) + x_3(t)|^2 dt \\
&= \lim_{T\to\infty} \frac{1}{T} \int_{-\infty}^{\infty} |x_1(t)|^2 + |x_2(t)|^2 + |x_3(t)|^2 + 2x_1(t)x_2(t) + 2x_1(t)x_3(t) + 2x_2(t)x_3(t)\ dt
\end{aligned}
$$

We do not know each of the crossterms, e.g., $x_1(t)x_2(t)$, and therefore cannot compute the power of $z(t)$.