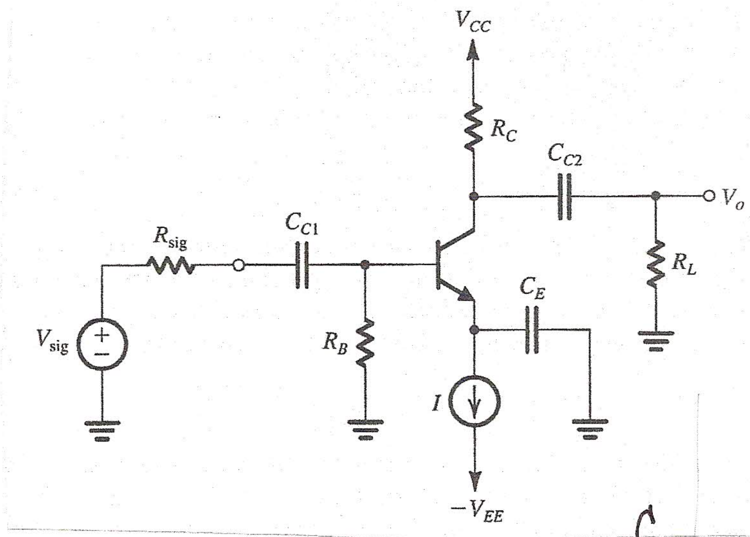
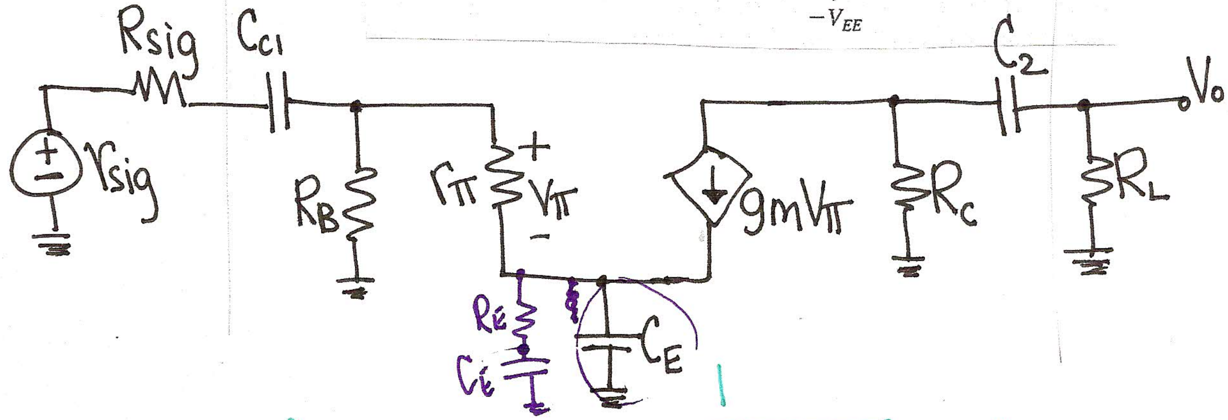




From C_{C1} : $\dfrac{1}{[\underbrace{R_{sig}+(R_B \| r_\pi)}_{R_{eq.}}]\,C_{C1}} = \omega_1$

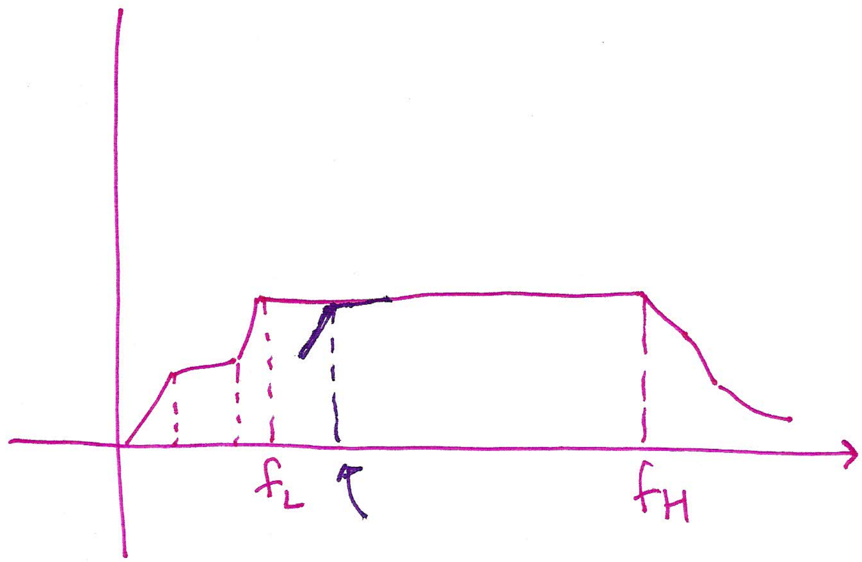
From C_E: $\dfrac{1}{C_E\left[\, r_\pi + \dfrac{R_B \| R_{sig}}{(\beta+1)} \right](+R_E)]} = \omega_2$ ← usually make dominant

From C_2 : $\dfrac{1}{[R_C+R_L]\,C_2} = \omega_3$

$\omega = 2\pi f$

Estimate: $(f_L \approx f_1 + f_2 + f_3)$

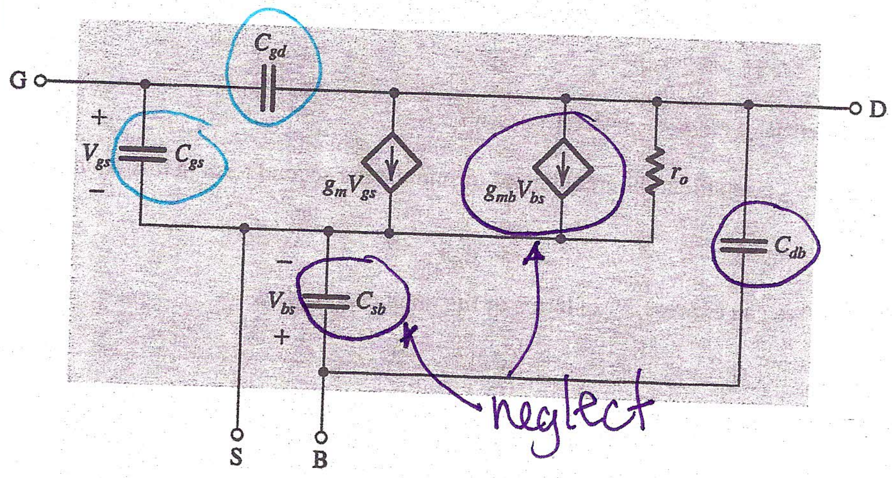
↑ if poles are close (a factor of 4)



f_L
f_H

Table 9.1 The MOSFET High-Frequency Model

Model



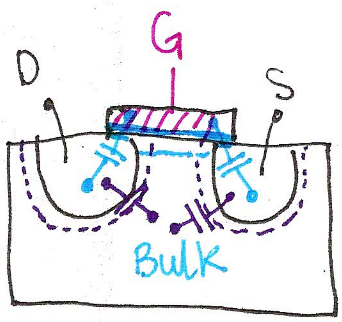
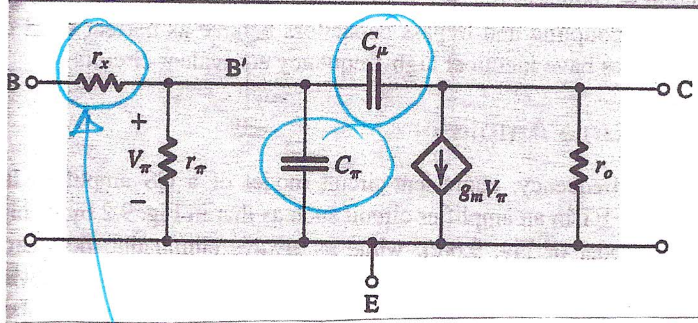
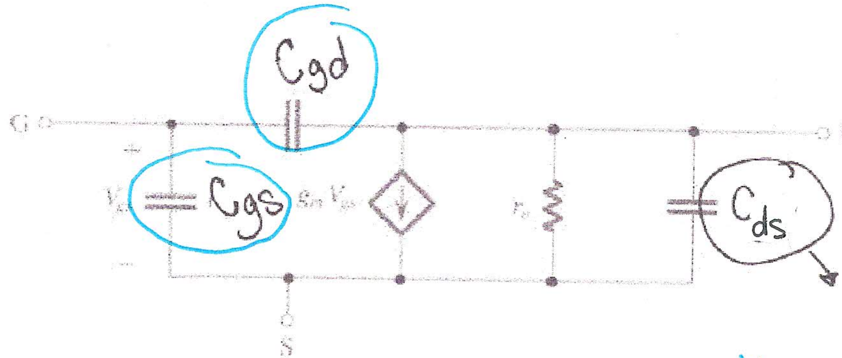


Table 9.2 The BJT High-Frequency Model

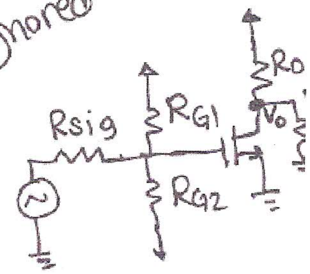


at high frequency → large effect

High Frequency: ·Controlled by parasitic capacitanc

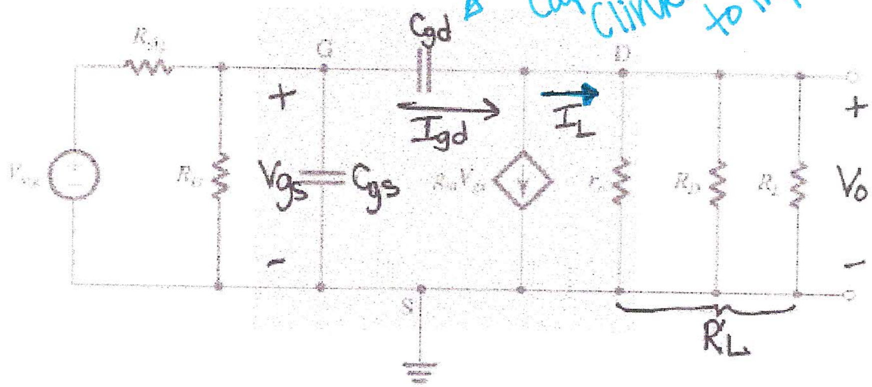


can be typically ignored



$R_G = R_{G1} \| R_{G2}$

miller capacitor (links output to input)



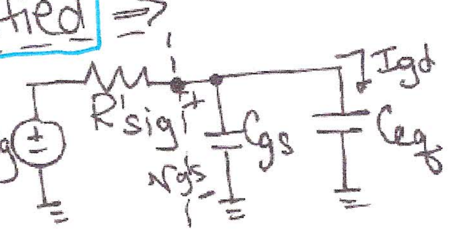
$I_L = g_m V_{gs} - I_{gd}$ (at frequencies $\sim f_H$ assume $I_{gd} \ll g_m V_{gs}$)

$\therefore V_o \approx -g_m R'_L V_{gs}$

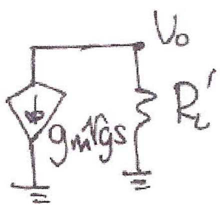
$$I_{gd} = \frac{V_G - V_o}{\frac{1}{C_{gd}\cdot s}} = C_{gd}\cdot s\left(V_{gs} - (-g_m R'_L V_{gs})\right)$$

$$I_{gd} = C_{gd}\cdot s\,(1 + g_m R'_L)V_{gs}$$

·Since I_{gd} does not influence the output, it can be modified ⇒



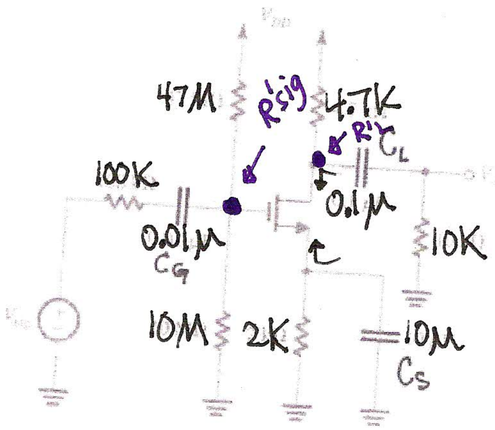
$\frac{R_G}{R_G + R_{sig}} V_{sig}$



where $R'_{sig} = R_{sig} \| R_G$

Thevenin equivalent

Example:



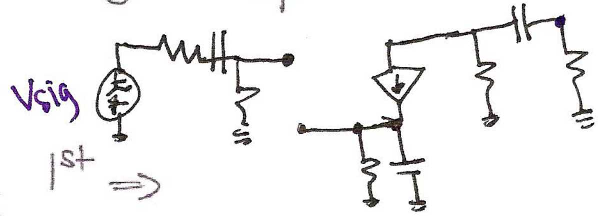
$g_m = 1\,mA/V$

$C_{gs} = 1pF$

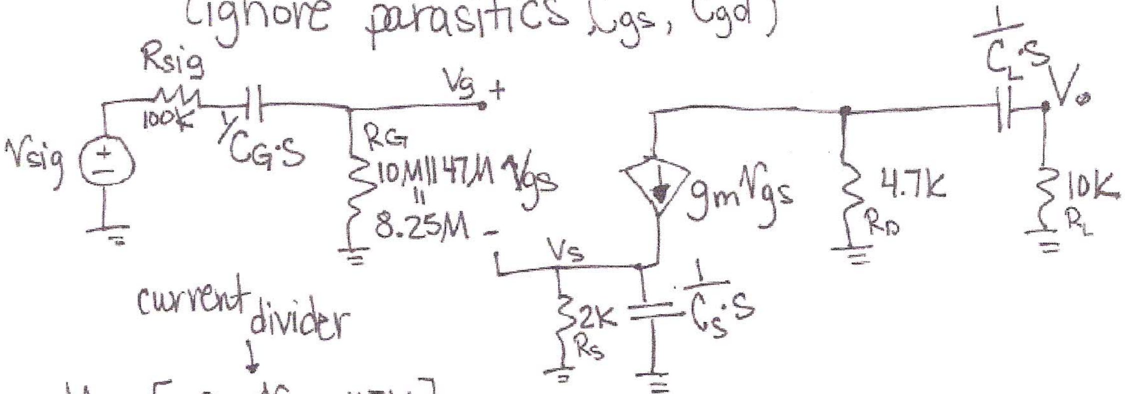
$C_{gd} = 0.2pF$

(short all caps)

$A_m = \frac{V_o}{V_{sig}}$



- Let us analyze low Freq. 1st ⇒ C_G, C_s, and C_L contribute to low response (ignore parasitics C_{gs}, C_{gd})



current divider

$$V_o = \left[\frac{-g_m V_{gs} \cdot 4.7K}{4.7k + \frac{1}{C_L S} + 10K}\right] \cdot 10K = \frac{-g_m 4.7k(10k) V_{gs} \cdot C_L S}{((14.7k)C_L S + 1)} =$$

$$V_{gs} = \frac{V_{sig}(8.25M)}{8.25M + 100K + \frac{1}{C_G \cdot S}} - g_m V_{gs}\left(2k \,\|\, \frac{1}{C_s \cdot s}\right)$$

$$V_{gs} = \frac{V_{sig}(8.25M) C_G \cdot S}{(8.35M\, C_G \cdot S + 1)} \cdot \frac{1}{1 + g_m\left(2k \,\|\, \frac{1}{C_s \cdot s}\right)}$$

$$V_{gs} = \frac{V_{sig}\,(8.25M)\,C_G\cdot S\,(2K\cdot C_S\cdot S+1)}{(8.35M\cdot C_G\cdot S+1)\,(1+g_m 2K)\left(\frac{2K\cdot C_S\cdot S}{1+g_m 2K}+1\right)}$$

$$\frac{V_o}{V_{sig}} = \frac{-4.7m\,(0.0825)\,(0.02\,s+1)\cdot s^2}{(1.47m\cdot s+1)(0.0835\cdot s+1)(3)(6.67m\cdot s+1)}$$

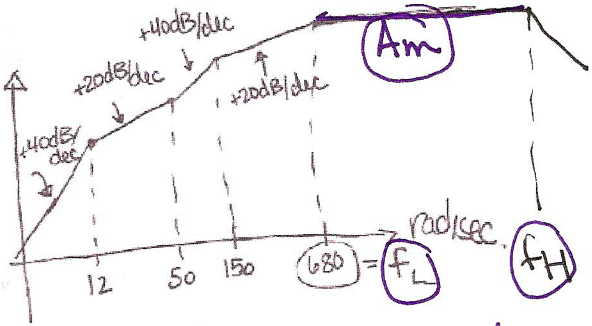
poles at $\omega \cong 680.3$ rad/sec $\rightarrow \frac{1}{(R_D+R_L)C_L} = \frac{1}{(14.7K)C_L}$

$\omega \cong 12$ rad/sec $\rightarrow \frac{1}{(R_G+R_{sig})\cdot C_G} = \frac{1}{[100K+(47M\|10M)]C_G}$

$\omega \cong 150$ rad/sec $\rightarrow \frac{1}{(1/g_m \| R_S)\cdot C_S} = \frac{1}{((1/g_m)\|2K)C_S}$

zero at $\omega \cong 50$ rad/sec

This circuit will start to work properly at $\boxed{f_L = 108\ Hz}$



High frequency ⇒ (Recall derivation) [Short external caps]

$\omega_H = \frac{1}{R'_{sig}(C_{eq}+C_{gs})}$ where $R'_{sig} = 100K \| 8.25M$

$R'_{sig} = 98.802K$

C_{eq} ⇒ (Short all external caps and only look at C_{gs} and C_{gd})

$\therefore\ C_{eq} = C_{gd}(1+g_m(4.7K\|10K)) = 0.84pF$

$\therefore\ \omega_H = \frac{1}{98.802K(0.84p+1p)} = 5.5M$ rad/sec

$\boxed{f_H \cong 876KHz}$

miller cap.

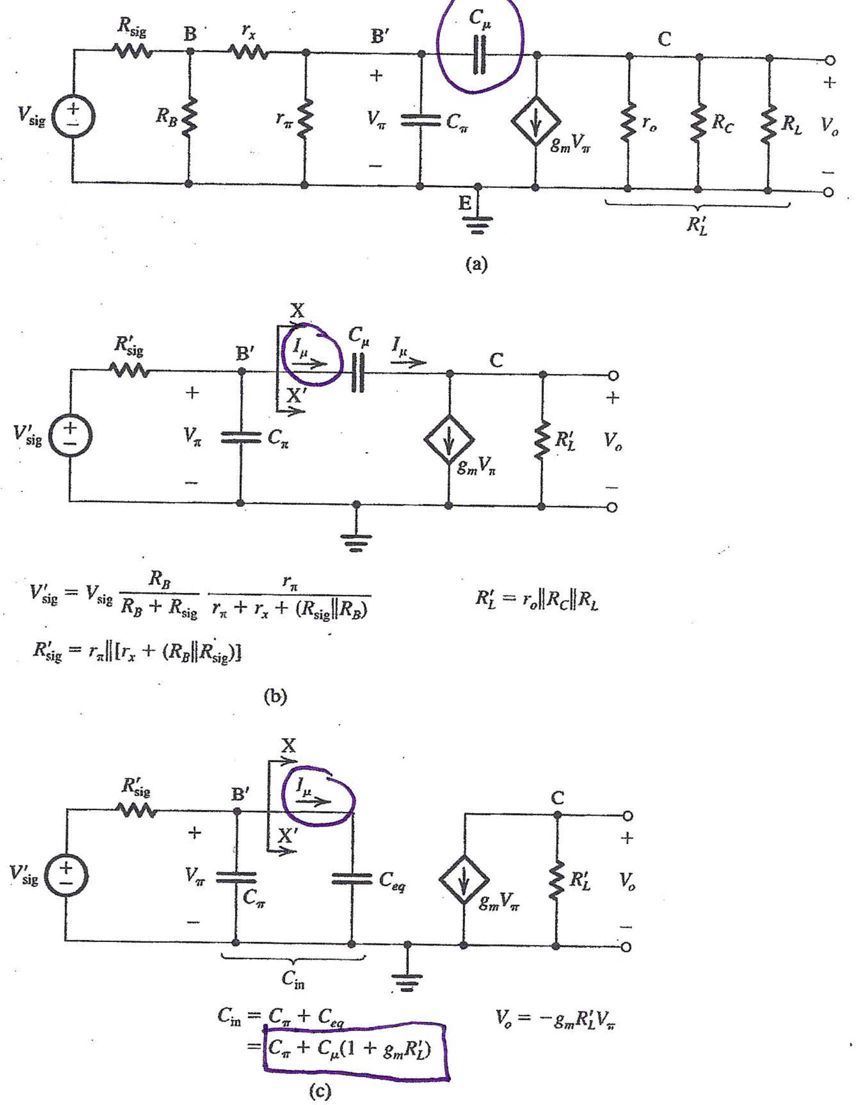


Figure 9.14 Determining the high-frequency response of the CE amplifier: (a) equivalent circuit; (b) the circuit of (a) simplified at both the input side and the output side; (c) equivalent circuit with C_μ replaced at the input side with the equivalent capacitance C_{eq}; (*continued*)

$\left|\frac{V_o}{V_{sig}}\right|$ (dB)

0

Figure 9.
STC circui

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Summary:

$\boxed{\text{BJT}}$

$C_{in} = C_\pi + C_{eq}$

$C_{eq} = C_\mu(1 + g_m R'_L)$ Req at Collector

$\omega_H = \dfrac{1}{C_{in} R'_{sig}}$

Req at base

$\boxed{\text{MOSFET}}$:

$C_{in} = C_{gs} + C_{eq}$

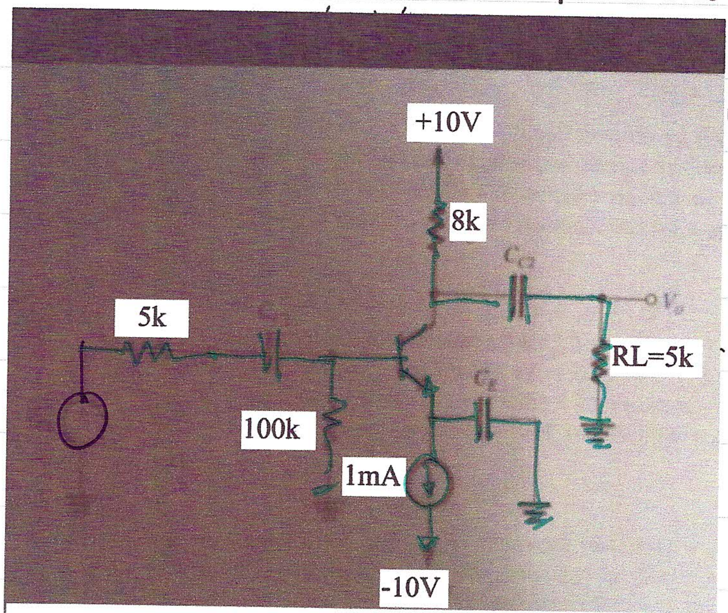
$C_{eq} = C_{gd}(1 + g_m R'_L)$ Req at drain

$\omega_H = \dfrac{1}{C_{in} R'_{sig}}$

Req at gate

Example:

It is required to find the midband gain and the upper 3-dB frequency of the common emitter amplifier shown below.



Use: $\beta = 100$

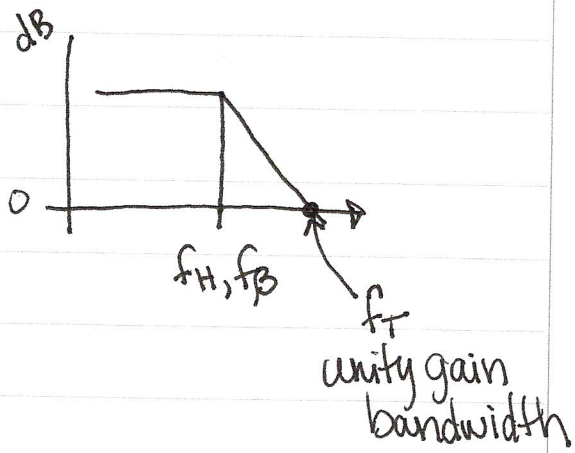
$V_A = 100$

$C_\mu = 1pF$

$f_T = 800MHz$

$r_x = 50\Omega$

$C_\pi = 7pF$

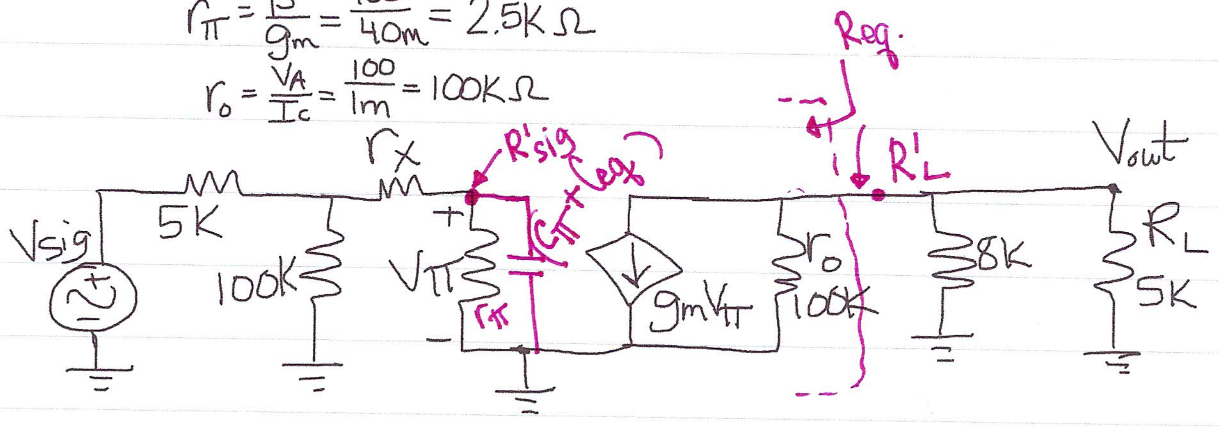


Biased at 1mA.

$$g_m = \frac{I_c}{V_T} = \frac{1m}{25m} = 40m\,A/V$$

$$r_\pi = \frac{\beta}{g_m} = \frac{100}{40m} = 2.5K\Omega$$

$$r_o = \frac{V_A}{I_c} = \frac{100}{1m} = 100K\Omega$$



$$V_{out} = -g_m V_\pi \underbrace{(8K \| 5K \| \overset{100K}{r_o})}_{2.9K = R_L'}$$

$$V_\pi = \frac{V_{sig}\left[100K \| \overset{50+2.5K}{(r_x + r_\pi)}\right]}{5K + \underbrace{100K \| \overset{50+2.5K}{(r_x + r_\pi)}}_{\approx 2.5K}} = 0.33 V_{sig}$$

$$V_{out} = -40m \cdot 0.33 V_{sig} \cdot 2.9K = \boxed{-39 \, V/V}$$

$$R'_{sig} = r_\pi \| [r_x + R_B \| R_{sig}] =$$

$$2.5K \| \underbrace{[50 + 100K \| 5K]}_{4.8K} = \underline{1.64K\,\Omega}$$

$$C_{in} = C_\pi + C_\mu(1 + g_m R_L') = 7p + 1p(1 + 40m \cdot 2.9K)$$

$$C_{in} = 124pF$$

$$\omega_H = \frac{1}{\underset{R'_{sig} \cdot C_{in}}{124p \cdot 1.64K}} = \boxed{\begin{array}{l} 4.9M \frac{rad}{sec} \\ 782kHz \end{array}}$$

A data sheet usually gives the behavior of β (or h_{fe}) versus frequency. A derivation is needed to relate this information to C_π.

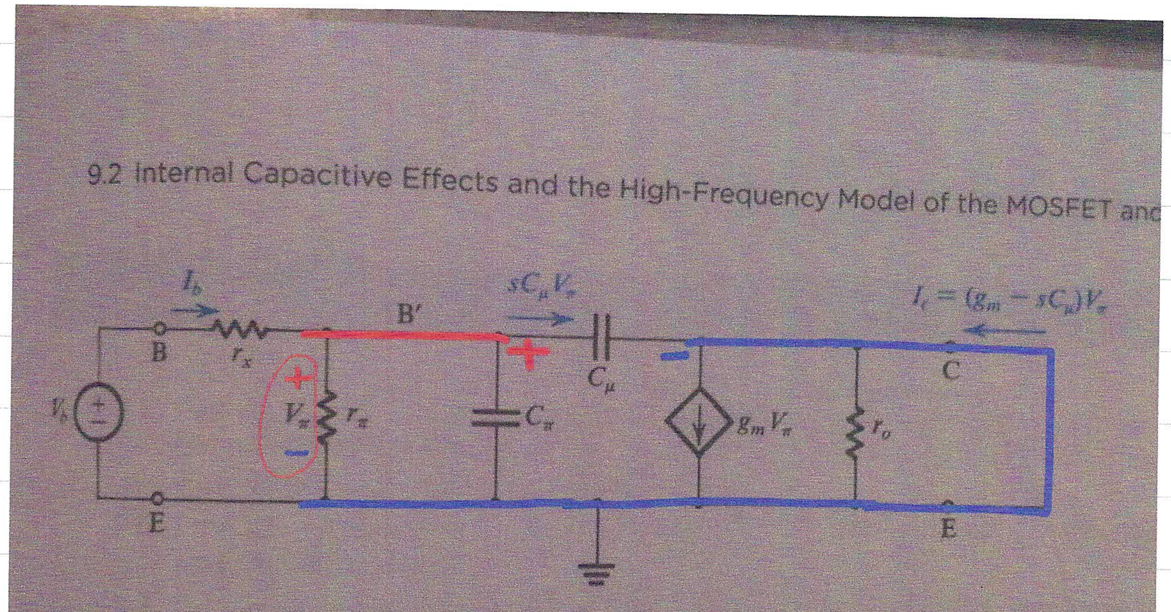


Figure 9.9 Circuit for deriving an expression for $h_{fe}(s) \equiv I_c/I_b$.

$$I_c = (g_m - sC_\mu)V_\pi$$

$$V_\pi = \frac{I_b}{\frac{1}{r_\pi} + s(C_\pi) + sC_\mu}$$

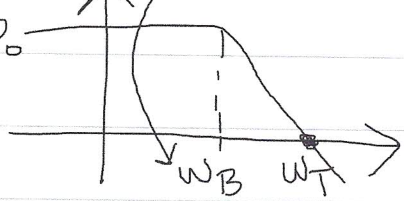
$$\left[h_{fe} = \frac{I_c}{I_b}\right] = \frac{\boxed{g_m r_\pi} - \cancel{sC_\mu}}{\frac{1}{\cancel{r_\pi}} + s(C_\pi + C_\mu) r_\pi}$$

neglect since $\omega C_\mu \ll g_m$

$$h_{fe} = \frac{\beta_0}{1 + s(C_\pi + C_\mu) r_\pi}$$

$\leftarrow$ low f value of β

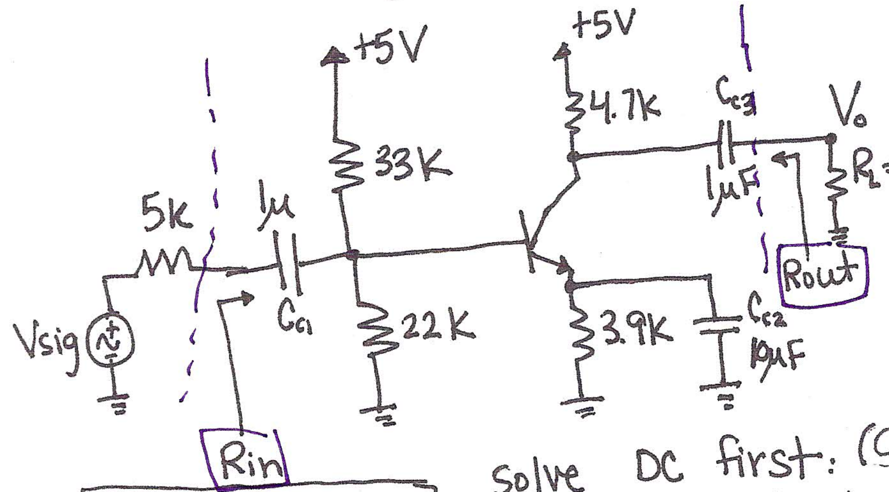
1 pole with $\omega_B = \frac{1}{(C_\pi + C_\mu) r_\pi}$



$$\omega_T = \beta_0 \omega_B = \frac{\beta_0}{(C_\pi + C_\mu) r_\pi} = g_m$$

$$C_\pi = \frac{g_m}{\omega_T} - C_\mu$$

~~High~~ Frequency Example:



$\beta = 120$

$r_o = 300k$

$r_x = 50\Omega$

$f_T = 700MHz$

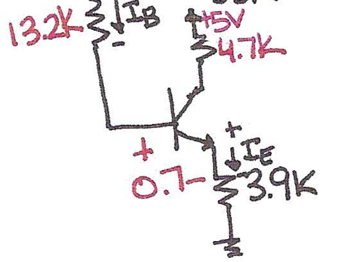
$C_\pi = 1.73pF$

$C_\mu = \dfrac{g_m}{\omega_T} - C_\pi$

Find:
- Rin
- Rout
- $A_m = \dfrac{V_o}{V_{sig}}$
- f_L
- f_H

Solve DC first: (Open caps)

$V_{Th} = \dfrac{5(22K)}{55K} = 2V$



$+2 - 13.2K\,I_B - 0.7 - 3.9K\,I_E = 0$

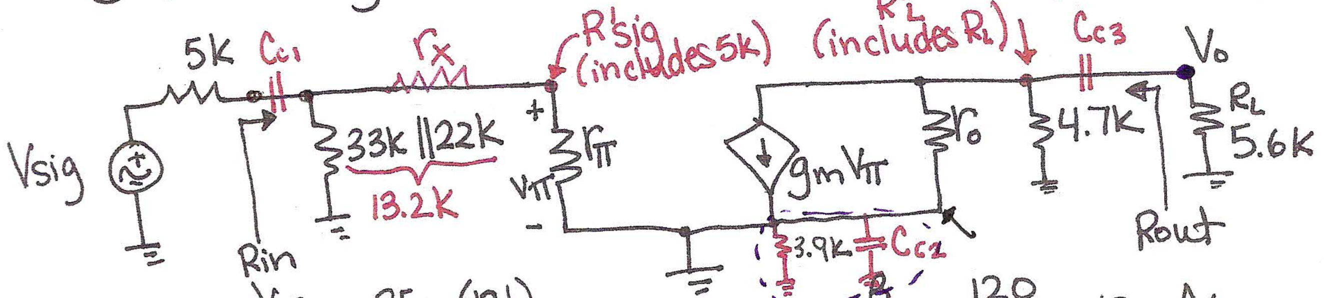
$I_B = \dfrac{I_E}{(\beta+1)}$

$I_E = \dfrac{1.3}{\left(\frac{13.2K}{121}\right) + 3.9K} \cong 0.3mA$

$V_E = 1.17,\ V_B = 1.87,\ V_C = 3.6$

$V_C > V_B \therefore$ ACTIVE

① Draw hybrid-π (short caps to find $\frac{V_o}{V_{sig}}$)



$r_\pi = \dfrac{V_T}{I_B} = \dfrac{25m(121)}{0.3m} = 10k, \quad g_m = \dfrac{\beta}{r_\pi} = \dfrac{120}{10k} = 12m\,A/V$

$V_o = -g_m V_\pi (4.7k \| 5.6k \| r_o)$

$V_\pi = \dfrac{V_{sig}\,(13.2k \| r_\pi)}{(13.2k \| r_\pi) + 5k}$

$\dfrac{V_o}{V_{sig}} \approx \boxed{-16\ V/V}$

$Rin = 13.2k \| (r_\pi + r_x) = \boxed{5.7k}$

$Rout = 4.7k \| \underset{300k}{r_o} = \boxed{4.6k}$

Low frequency: [without r_o]

C_{c1} : $\dfrac{1}{2\pi \cdot 1\mu \cdot (5k + 13.2k \| r_\pi)} = 14.87\ Hz$

C_{c2} : $\dfrac{1}{2\pi 10\mu \cdot \left[3.9k \| \dfrac{(r_\pi + 13.2k \| 5k)}{(\beta+1)}\right]} = \boxed{144.95\ Hz} = f_L$

highest value determines f_L

C_{c3}: $\dfrac{1}{2\pi \cdot 1\mu \cdot (4.7k \| r_o + 5.6k)} = 15.55\ Hz$

High frequency:

$$C_\mu = \frac{g_m}{2\pi f_T} - \overset{1.73p}{\overbrace{C_\pi}} \cong 1pF$$

$$C_{in} = C_\pi + C_\mu (1 + g_m R'_L)$$

$$C_{in} = 1.73p + 1p(1 + 12m \cdot \underbrace{\overset{300k}{r_o} \| 4.7k \| 5.6k}_{2.53k}) = 33pF$$

$$f_H = \frac{1}{2\pi \cdot C_{in} \cdot R'_{sig}} = \frac{1}{2\pi \cdot 33p \cdot [5k \| 13.2k + \underset{50}{r_x}] \| 10k}$$

$$f_H = \boxed{1.79\ MHz}$$

Definition

Dominant pole: (factor of 4) away from nearest pole or zero.

If transfer function is known:

* more accurate is $\Longrightarrow$ $H(s) = \dfrac{A\left(\frac{s}{\omega_{z_1}}+1\right)\left(\frac{s}{\omega_{z_2}}+1\right)\cdots}{\left(\frac{s}{\omega_{p1}}+1\right)\left(\frac{s}{\omega_{p2}}+1\right)\cdots}$

$$\omega_H \approx \frac{1}{\sqrt{\underbrace{\left(\frac{1}{\omega_{p1}^2}\right) + \left(\frac{1}{\omega_{p2}^2}\right)}_{\text{poles}} + \cdots - 2\left(\underbrace{\frac{1}{\omega_{z_1}^2} + \frac{1}{\omega_{z_2}^2} + \frac{1}{\omega_{z_3}^2}}_{\text{zeroes}} + \cdots\right)}}$$

If transfer function unknown:

open-circuit Time Constants

$$\omega_H \approx \frac{1}{\sum_i C_i R_i}$$ * Closest to actual value

Example: $$\frac{1}{\left[1\mu\left(\underbrace{5k + 3.2k \| r_\pi}_{R_{eq}}\right)\right] + \left[10\mu\left[\underbrace{3.9k \,\Big\|\, \frac{r_\pi + 3.2k \| 5k}{\beta+1}}_{R_{eq}}\right]\right] + \left[1\mu\left(\underbrace{4.7k \| r_o}_{R_{eq}} + 5.6\ldots\right.\right.}$$

· From new circuit,

$$I_{gd} = \frac{V_{gs}}{\frac{1}{C_{eq} \cdot s}} = C_{eq} \cdot s \cdot V_{gs}$$

· Setting old I_{gd} = new I_{gd}

$$\cancel{C_{eq}} \cdot \cancel{s} \cdot \cancel{V_{gs}} = C_{gd} \cdot s \cdot (1 + g_m R'_L) V_{gs}$$

$$\therefore \boxed{C_{eq} = C_{gd}(1 + g_m R'_L)}$$

R at drain

$$V_{gs} = \frac{\frac{V_{sig}(R_G)}{R_G + R'_{sig}} \left(\frac{1}{C_{gs} \cdot s} \parallel \frac{1}{C_{eq} \cdot s}\right)}{\left(\frac{1}{C_{gs} \cdot s} \parallel \frac{1}{C_{eq} \cdot s}\right) + R'_{sig}}$$

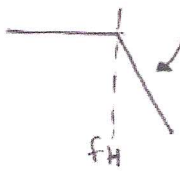
$$\frac{1}{C_{gs} \cdot s} \parallel \frac{1}{C_{eq} \cdot s} = \frac{1}{s(C_{eq} + C_{gs})}$$

$$\therefore V_{gs} = \frac{\frac{V_{sig} R_G}{(R_G + R'_{sig})} \cdot \frac{1}{s(C_{eq} + C_{gs})}}{\frac{1}{s(C_{eq} + C_{gs})} + R'_{sig}} = \frac{\frac{V_{sig} \cdot R_G}{(R_G + R'_{sig})}}{1 + R'_{sig}(s(C_{eq} + C_{gs}))}$$

$$V_{gs} = \frac{V_{sig} \cdot R_G}{(R_G + R'_{sig})(1 + s R'_{sig}(C_{eq} + C_{gs}))}$$

Recall $\rightarrow$ $V_o \cong -g_m R'_L V_{gs} \Rightarrow \frac{V_o}{V_{sig}} = \frac{-g_m R'_L R_G}{(R_G + R'_{sig})(1 + s R'_{sig}(C_{eq} + C_{gs}))}$

· with only 1 pole $\Longrightarrow$ (recall -20dB/dec)



$$\therefore \omega_H = \frac{1}{R'_{sig}(C_{eq} + C_{gs})}$$

R seen at the gate