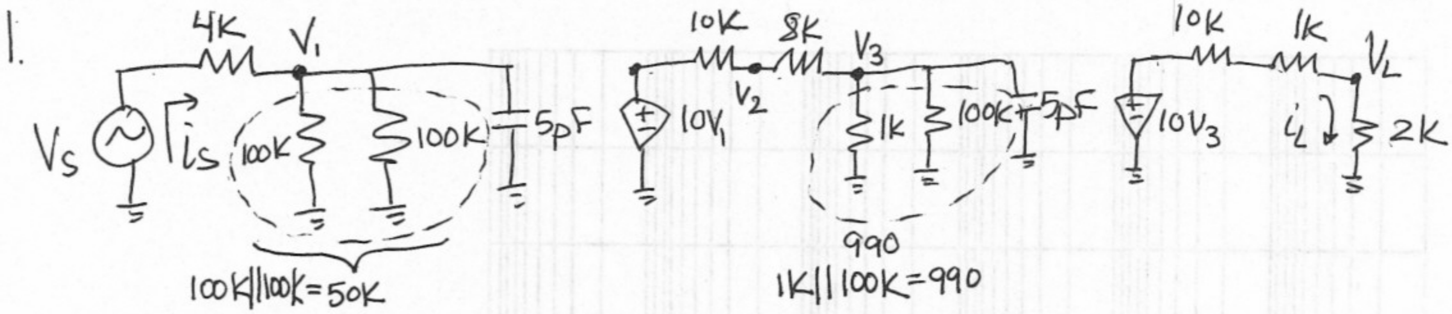
Homework 3 Solution

1.



$$R \parallel \frac{1}{CS} = \frac{R\left(\frac{1}{CS}\right)}{R+\frac{1}{CS}} = \frac{R}{RCS+1}$$

$$V_L = \frac{10V_3(2K)}{13K} = \frac{20}{13}V_3$$

$$V_3 = \frac{10V_1\left(990 \parallel \frac{1}{5p\cdot s}\right)}{\left(990 \parallel \frac{1}{5ps}\right)+18K} = \frac{\dfrac{10(990)}{990(5p)s+1}}{\dfrac{990}{990(5p)s+1} + \dfrac{18K(9905ps+1)}{(9905ps+1)}} = \frac{10(990)\cdot V_1}{990+8.91e^{-5}s+18K}$$

$$V_3 = \frac{9{,}900 \cdot V_1}{18{,}990\left(1+\frac{8.91e^{-5}s}{18{,}990}\right)} = \frac{0.52 \cdot V_1}{\left(1+\frac{s}{213M}\right)}$$

$$V_1 = \frac{V_s\left(50K \parallel \frac{1}{5ps}\right)}{\left(50K \parallel \frac{1}{5ps}\right)+4K} = \frac{\dfrac{V_s(50K)}{50K(5ps)+1}}{\dfrac{50K+4K(50K(5p)s+1)}{50K(5p)s+1}} = \frac{V_s(50K)}{54K\left(1+\frac{1e^{-3}s}{54K}\right)} = \frac{0.93\cdot V_s}{\left(1+\frac{s}{54M}\right)}$$

$$\frac{V_L}{V_s} = \frac{20}{13}\cdot\frac{0.52}{\left(1+\frac{s}{213M}\right)}\cdot\frac{0.93}{\left(1+\frac{s}{54M}\right)} = \boxed{\underbrace{\frac{0.74}{\left(1+\frac{s}{213M}\right)\left(1+\frac{s}{54M}\right)}}}$$

Note that these are in <u>rad/sec</u> - NOT HZ

3. b. $i_L = \frac{V_L}{2K}$

$$i_s = \frac{V_s}{4K + (50K \| \frac{1}{5ps})} = \frac{V_s}{4K + \frac{50K}{50K(5p)s+1}} = \frac{V_s}{\frac{4K(50K(5p)s+1)+50K}{(\frac{s}{4M}+1)}}$$

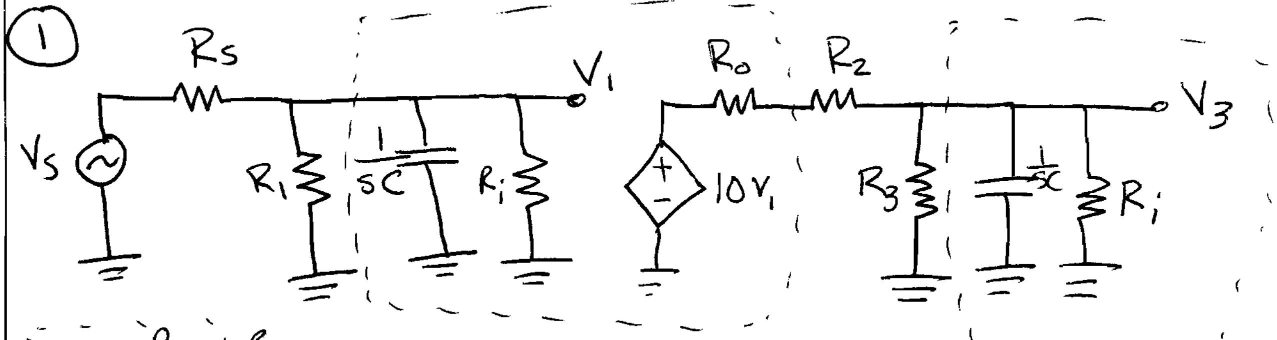
$$i_s = \frac{V_s(\frac{s}{4M}+1)}{54K(\frac{1e^{-3}}{54K}s+1)} = \frac{V_s(\frac{s}{4M}+1)}{54K(\frac{s}{54M}+1)}$$

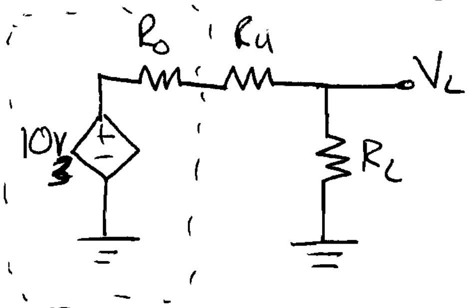
$$\frac{i_L}{i_s} = \frac{(\frac{V_L}{2K})}{\left(\frac{V_s(\frac{s}{4M}+1)}{54K(\frac{s}{54M}+1)}\right)} = \frac{V_L\, 54K(\frac{s}{54M}+1)}{2K\cdot V_s\cdot(\frac{s}{4M}+1)}$$

$$\frac{i_L}{i_s} = \frac{V_L}{V_s}\cdot\frac{27(\frac{s}{54M}+1)}{(\frac{s}{4M}+1)} = \frac{0.74\cdot 27\cancel{(\frac{s}{54M}+1)}}{(1+\frac{s}{213M})\cancel{(1+\frac{s}{54M})}(\frac{s}{4M}+1)}$$

$$\boxed{\frac{i_L}{i_s} \simeq \frac{20}{(1+\frac{s}{213M})(\frac{s}{4M}+1)}}$$

(1)





$R_S = 4k\Omega$ $R_1 = 100k\Omega$ $R_i = 100k\Omega$
$C = 5pF$ $R_o = 10k\Omega$ $R_2 = 8k\Omega$
$R_3 = 1k\Omega$ $R_4 = 1k\Omega$ $R_L = 2k\Omega$

$$\frac{V_1}{V_s} = \frac{R_1 \| R_i \| \frac{1}{sC}}{R_s + R_1 \| R_i \| \frac{1}{sC}} = \frac{\frac{1}{\frac{1}{R_1} + \frac{1}{R_i} + sC}}{R_s + \frac{1}{\frac{1}{R_1} + \frac{1}{R_i} + sC}} = \frac{1}{R_s\left(\frac{1}{R_1} + \frac{1}{R_i} + sC\right) + 1}$$

$$\frac{V_1}{V_s} = \frac{1}{sCR_s + \frac{R_sR_i}{R_1R_i} + \frac{R_sR_1}{R_iR_1} + \frac{R_1R_i}{R_1R_i}} = \frac{R_1R_i/(R_sR_i + R_sR_1 + R_1R_i)}{sCR_sR_1R_i/(R_sR_i + R_sR_1 + R_1R_i) + 1}$$

$$\frac{V_3}{V_1} = 10\,\frac{R_3 \| R_i \| \frac{1}{sC}}{(R_o + R_2) + R_3 \| R_i \| \frac{1}{sC}}$$ = Follow the same algebra as before where $R_1 = R_3$, $R_s = (R_o + R_2)$

$$\frac{V_3}{V_1} = 10\,\frac{R_3R_i/((R_o+R_2)R_i + (R_o+R_2)R_3 + R_3R_i)}{sC(R_o+R_2)R_iR_3/((R_o+R_2)R_i + (R_o+R_2)R_3 + R_3R_i) + 1}$$

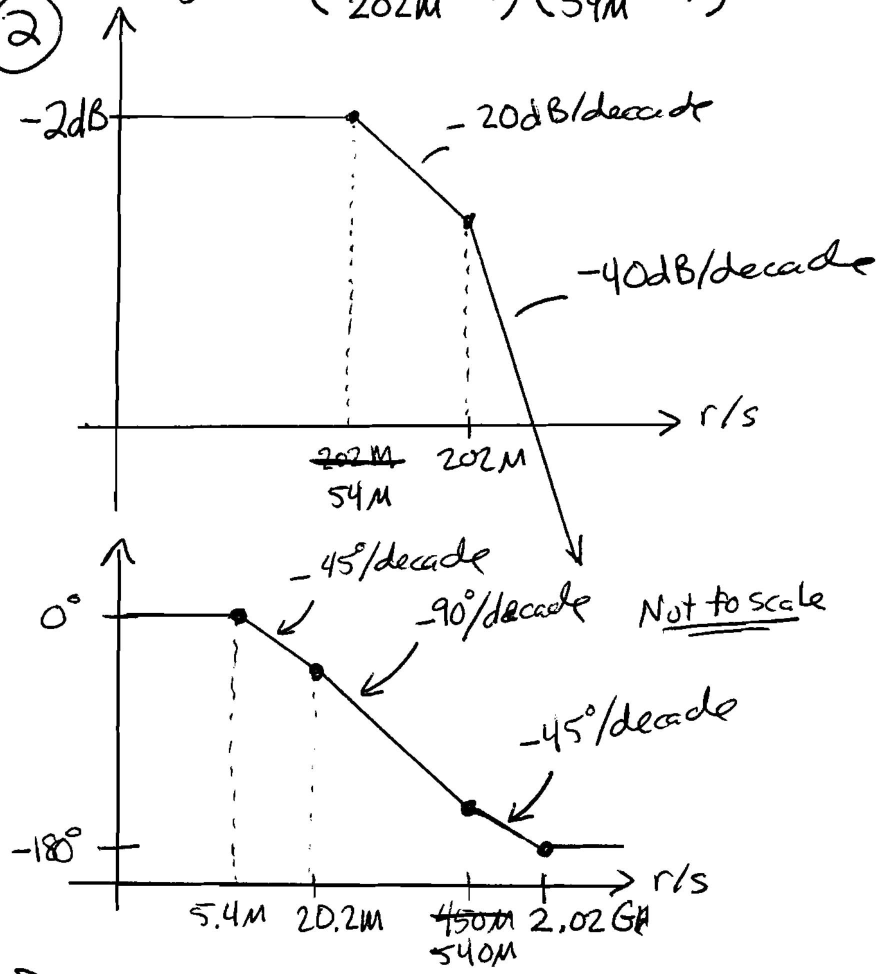
$$\frac{V_L}{V_3} = 10\,\frac{R_L}{R_o + R_4 + R_L}$$

$$\therefore \frac{V_L}{V_s} = 10\,\frac{R_L}{R_o + R_4 + R_L} \cdot 10\,\frac{\frac{R_3R_i}{(R_o+R_2)(R_i+R_3) + R_3R_i}}{s\,\frac{CR_iR_3(R_o+R_2)}{(R_o+R_2)(R_i+R_3)+R_3R_i} + 1} \cdot \frac{\frac{R_1R_i}{(R_s(R_i+R_1) + R_1R_i)}}{s\,\frac{CR_sR_1R_i}{(R_s(R_i+R_1)+R_1R_i} + 1}$$

$$\therefore \frac{V_L}{V_s} = \frac{20}{13} \cdot \frac{0.550}{\frac{s}{202M} + 1} \cdot \frac{0.926}{\frac{s}{54M} + 1}$$

$$\therefore \frac{V_L}{V_S} = \frac{0.784}{\left(\frac{s}{202M}+1\right)\left(\frac{s}{54M}+1\right)}$$

(2)



(3) a) Gain $= 0.784$ V/V or -2.11 dB

c) f_{3dB}: $\left| \dfrac{1}{\left(j\omega/202M + 1\right)\left(j\omega/54M + 1\right)} \right| = 0.707$

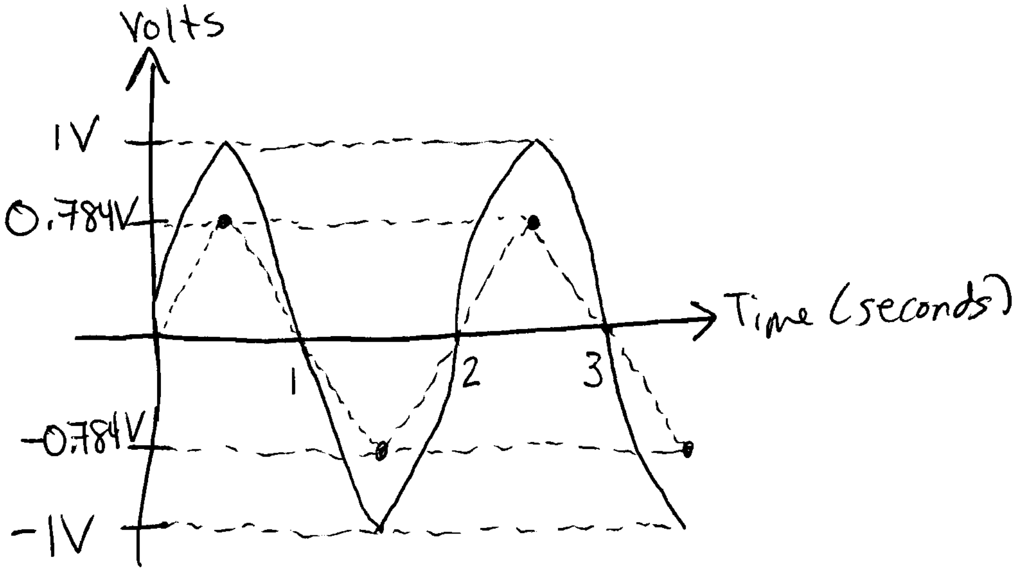
$$\Rightarrow \left(\frac{\omega^2}{(202M)^2} + 1\right)\left(\frac{\omega^2}{(54M)^2} + 1\right) = 2$$

$$\Rightarrow \omega^4 + 4.372\times10^{16}\omega^2 - 1.190\times10^{32} = 0$$

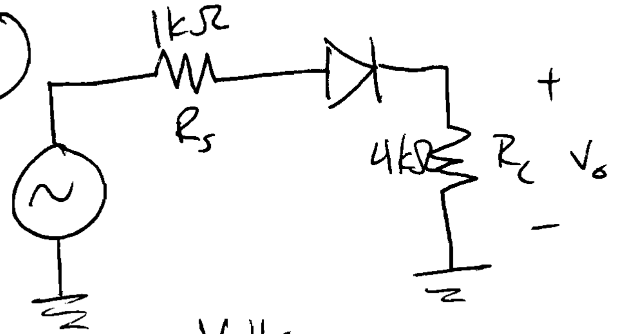
Applying the solution to the quadratic twice and taking the only positive, real result, we find that:

$$f_{3dB} = \frac{50.7M\,r/s}{2\pi\,r} = \boxed{8.07MHz}$$

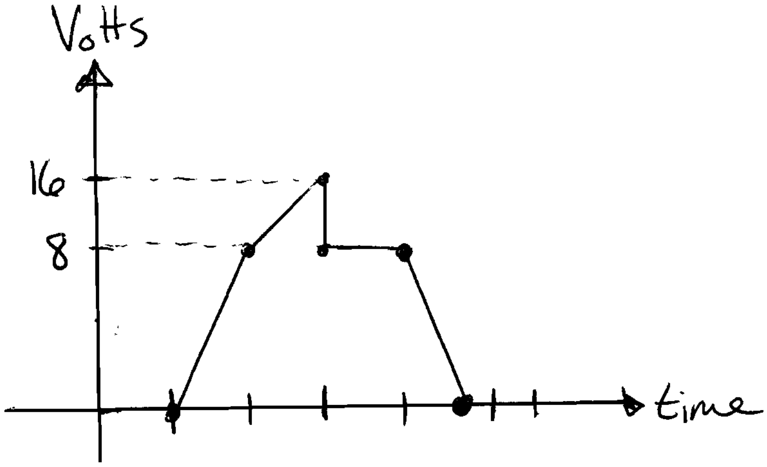
d) At $\pi \frac{rad}{s}$, both poles are negligible, so the amplification is an attenuation by the factor 0.784.



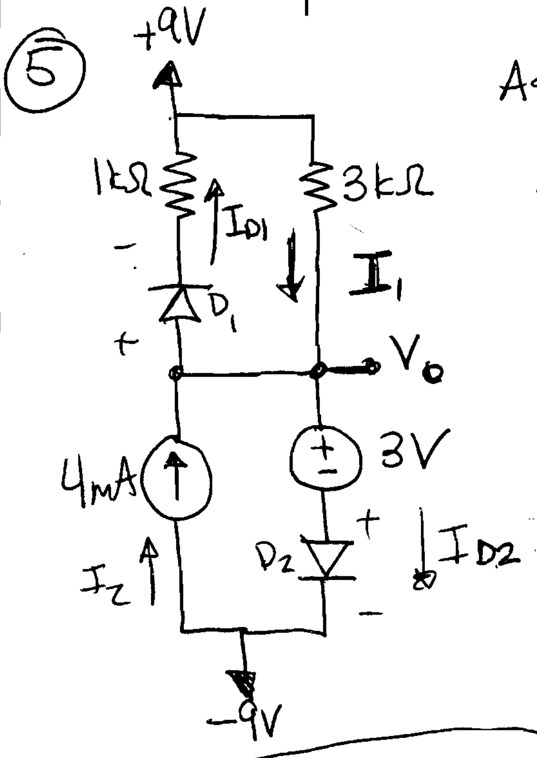
(4)



$$\frac{R_L}{R_s+R_L} = \frac{4}{5}$$



(5)



Assume D_1 off and D_2 ON

Then $I_1 = \dfrac{9V-(-5.3V)}{3k\Omega} = \dfrac{+14.3}{3k\Omega} = 4.6mA$

$V_{D2} = 0.7$, $I_{D2} = I_1 + 4mA = 8.6mA$

Diode on ☑

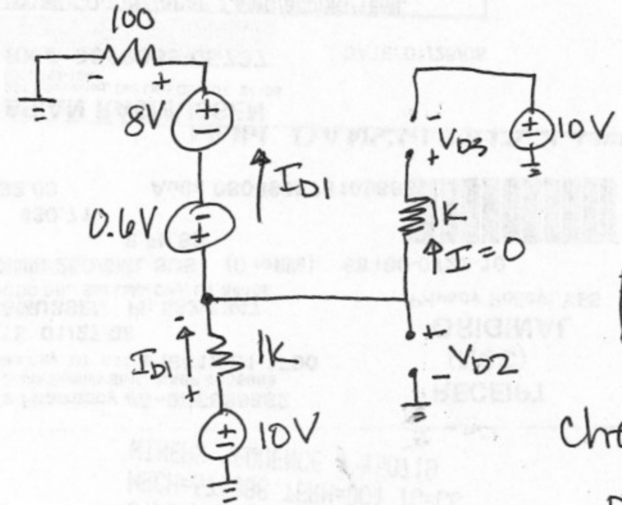
$V_{D1} = -5.3V - 9V = -14.3V$

$I_{D1} = 0A$ Diode off ☑

$I_1 = 4.6mA$
$I_2 = 4mA$
$V_o = -9V + 3.7 = -5.3V$
$I_{D1} = 0A$
$I_{D2} = 8.6mA$

Since this works, it must be the solution.

6. Assume D2, D3 off and D1 on



$$+10V - I_{D1}(1K) - 0.6 + 8 - I_{D1}(100) = 0$$

$$I_{D1} = \frac{10 - 0.6 + 8}{1K + 100} = \boxed{15.8mA}$$

$\boxed{I_{D3} = I_{D2} = 0}$

Check assumptions:

D1 on: $\underline{I_{D1} > 0}$ ✓ correct

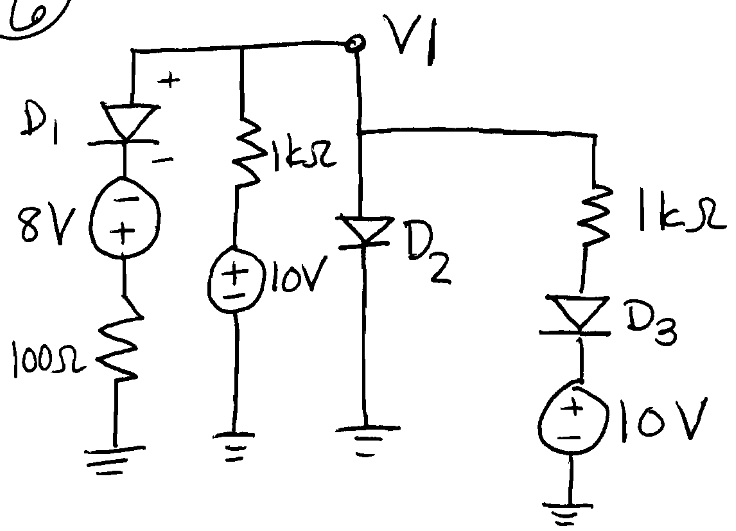
D2 off: $+10 - I_{D1}(1K) - V_{D2} = 0$

correct ✓ $V_{D2} = 10 - 15.8m(1K) = \underline{-5.8}V < 0.6V$

D3 off: $+10 - I_{D1}(1K) - 0 - V_{D3} - 10 = 0$

$V_{D3} = 10 - 15.8 - 10 = \underline{-15.8V} < 0.6V$ ✓ correct

(6)



Assumptions

D_3, D_2 off.

D_1 on.

With D_1 on, apply node voltage:

$$\frac{V_1 - 10V}{1k\Omega} + \frac{V_1 - \overset{0.6}{\cancel{0.7}} + 8V}{100\Omega} = 0A$$

$$V_1\left(\frac{1}{1k\Omega} + \frac{1}{100\Omega}\right) = \frac{10}{1k} + \frac{\overset{7.4}{-\cancel{7.3}}}{100} = \overset{-0.064}{\cancel{-0.063}}\ \frac{V}{\Omega}$$

$$\therefore V_1 = -5.73\ V$$

$$\boxed{\begin{array}{l} a)\ I_{D1} = \dfrac{V_1 - 0.6 + 8\ V}{100\Omega} = 16.7mA \\ b)\ I_{D2} = 0\ A \quad (D_2\ \text{off}) \\ c)\ I_{D3} = 0A \quad (D_3\ \text{off}) \end{array}}$$

Checks on Diodes: $D_1 \Rightarrow V_1 - (-8V + 16.7mA \cdot 100\Omega) = V_{D1}$

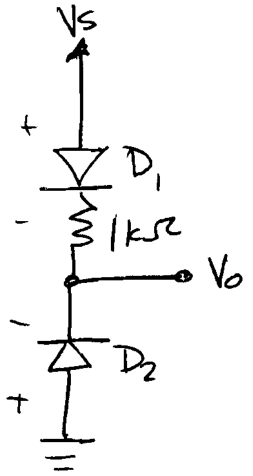
$\therefore V_{D1} = 0.6V$ I_{D1} correct ✓

$D_2 \Rightarrow I_{D2} = 0A$ $V_{D2} = V_1 - 0 = -5.73V$ correct ✓

$D_3 \Rightarrow I_{D3} = 0A$ $V_{D3} = V_1 - 10 = -15.73V$ correct ✓

(7)

DC-circuit



a) Since V_o is not attached to a load, and D_2 is reverse biased, there can be no current flowing through either diode, (no loops exist).

$$\therefore I_{D1} = I_{D2} = 0\,A$$

D_1 is forward-biased

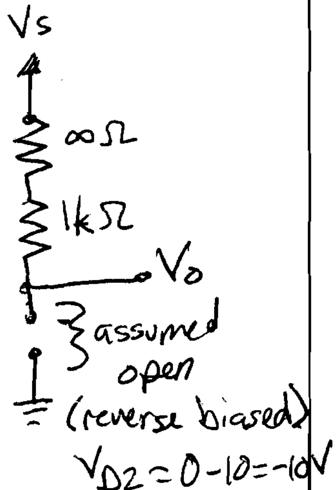
D_2 is reverse-biased

b) $V_{ODC} = 10\,V - 0.5\,V = 9.5\,V$

$$\therefore V_{ODC} = 9.5\,V$$

c) $r_d = \dfrac{nV_T}{I_D} = \dfrac{3(25mV)}{0} = \infty\,\Omega$ $\quad i_d = \dfrac{v_d}{\infty} = 0A$

AC circuit



d) $v_o = \sin(10kt)\,V$

e) Total Output for V_o

$$V_o = 9.5 + \sin(10kt)\ V$$